2023

1 Cairo Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1 The sum of any two side lengths of a triangle is .... the length of the third side.

(a) smaller than (b) equal to (c) greater than (d) twice

2 If the two circles M and N are touching externally and their radii lengths are 4 cm. and 9 cm. , then MN = ... cm.

(a) 4 (b) 5 (c) 9 (d) 13

3 The sum of measures of two supplementary angles equals ....

(a) 90° (b) 180° (c) 270° (d) 360°

4 The type of the inscribed angle opposite to an arc greater than the semicircle is .... angle.

(a) an acute (b) a right (c) an obtuse (d) a straight

5 If ABCD is a cyclic quadrilateral , m (∠ C) = 2 m (∠ A) , then m (∠ C) = ...°

(a) 30° (b) 60° (c) 90° (d) 120°

6 ABC is a triangle in which m (∠ A) = 40° , m (∠ C) = 70° , then the number of axes of symmetry of this triangle equals ....

(a) 1 (b) 2 (c) 3 (d) 4

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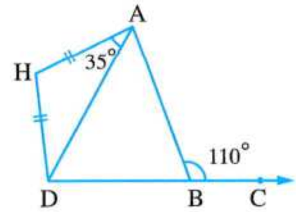
2 [a] In the opposite figure :

HA = HD , m (∠ DAH) = 35° , m (∠ ABC) = 110°



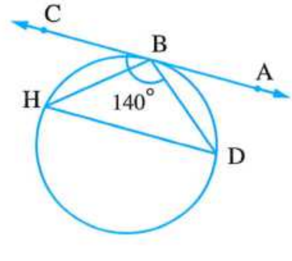
1 **Find with proof :** m (∠ H)

2 **Prove that :** ABDH is a cyclic quadrilateral.

[b] In the opposite figure :

$\overleftrightarrow{AC}$ is a tangent to the circle at B

, m (∠ DBC) = 140°

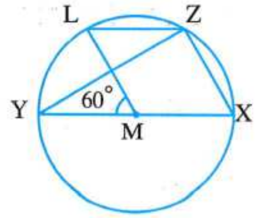


Find with proof :

1 m (∠ ABD)

2 m (∠ H)

3 [a] **In the opposite figure :**



$\overline{XY}$ is a diameter in the circle M , m (∠ LMY) = 60°

Find with proof : 1 m (∠ XZY)

2 m (∠ YZL)

[b] Using the geometric tools , draw the equilateral triangle whose side length is 5 cm. , then draw the circumcircle of it.

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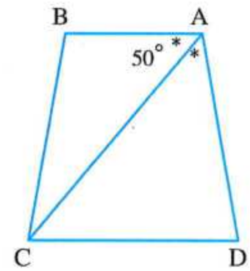
4 [a] **In the opposite figure :**



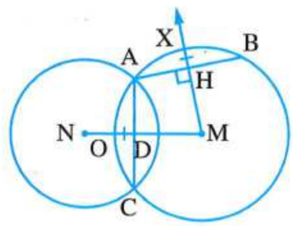
ABCD is a cyclic quadrilateral

, $\overrightarrow{AC}$ bisects ∠ BAD

and m (∠ BAC) = 50°

Find with proof : m (∠ BCD)

[b] **In the opposite figure :**



M and N are two intersecting circles at

A and C , $\overrightarrow{MH} \perp \overline{AB}$

and intersects the circle M at X , HX = DO

Prove that : AB = AC

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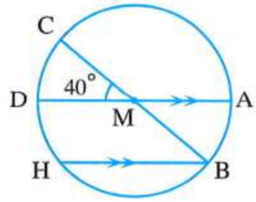
5 [a] **In the opposite figure :**



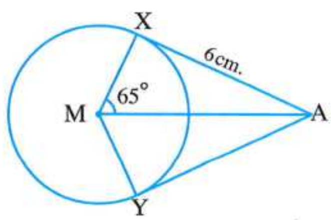
$\overline{AD}$, $\overline{BC}$ are two diameters in the circle M

, m (∠ CMD) = 40° , $\overline{AD}$ // $\overline{BH}$

Find with proof : 1 m (∠ AMB)

2 m ($\overset{\frown}{DH}$)

[b] **In the opposite figure :**



$\overline{AX}$ and $\overline{AY}$ are two tangent-segments

to the circle M at X and Y respectively

, m (∠ AMX) = 65° and AX = 6 cm.

Find with proof : 1 The length of $\overline{AY}$

2 m (∠ AXM)

3 m (∠ XAY)

2 Giza Governorate

Answer the following questions :

1 Choose the correct answer :

1 The two diagonals are equal in length and non-perpendicular in the ....

(a) square. (b) rhombus. (c) rectangle. (d) parallelogram.

2 If the straight line is a tangent to the circle of diameter length 8 cm. , then the distance between the straight line and the centre is .... cm.

(a) 3 (b) 4 (c) 6 (d) 8

3 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

4 The inscribed angle which is drawn in a semicircle is ....

(a) acute. (b) obtuse. (c) straight. (d) right.

5 The point of concurrence of the medians of the triangle divides each of them in the ratio of .... from the base.

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 2

6 If ABCD is a cyclic quadrilateral , m (∠ A) = 60° , then m (∠ C) = ...

(a) 60° (b) 30° (c) 90° (d) 120°

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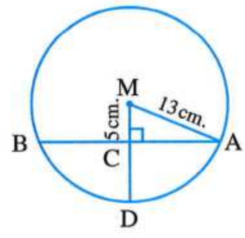
2 [a] In the opposite figure :

$\overline{MC} \perp \overline{AB}$, AM = 13 cm.

, MC = 5 cm.

Find : The length of each of $\overline{AB}$ and $\overline{CD}$

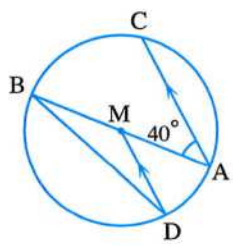


[b] In the opposite figure :

$\overline{AC}$ // $\overline{MD}$

, m (∠ CAB) = 40°

Find with proof : m (∠ ABD)

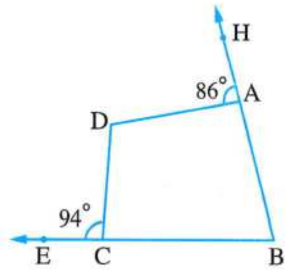


3 [a] **In the opposite figure :**

m (∠ HAD) = 86°

, m (∠ DCE) = 94°

Prove that : ABCD is a cyclic quadrilateral.



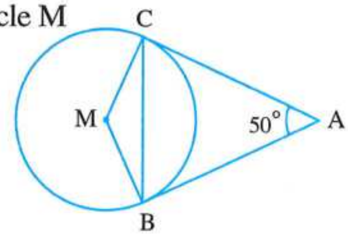
[b] **In the opposite figure :**

m (∠ A) = 50° , $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M

Find : 1 m (∠ ABC)

2 m (∠ MCB)

3 m (∠ CMB)



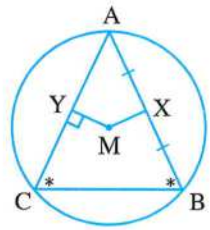
4 [a] **In the opposite figure :**

ABC is an inscribed triangle in a circle M

, m (∠ B) = m (∠ C)

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

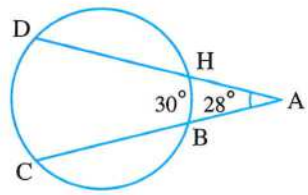
Prove that : MX = MY



[b] **In the opposite figure :**

m (∠ A) = 28° , m $(\widehat{BH})$ = 30°

Find : m $(\widehat{DC})$

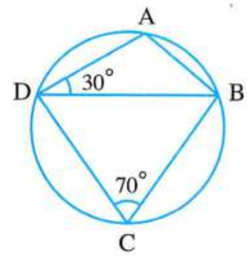


5 [a] **In the opposite figure :**

m (∠ BCD) = 70°

, m (∠ ADB) = 30°

Find with proof : m (∠ ABD)



[b] **In the opposite figure :**

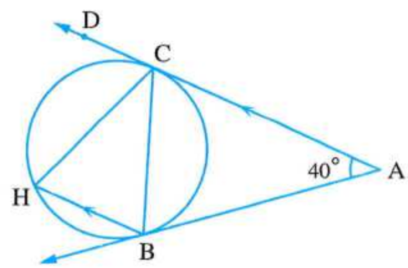
$\overrightarrow{AC}$ and $\overrightarrow{AB}$ are two tangents to the circle at C and B

, m (∠ A) = 40°

, $\overrightarrow{AC}$ // $\overline{BH}$

Find with proof : 1 m (∠ CHB)

2 m $(\widehat{BH})$



3 Alexandria Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1 The inscribed angle in a semicircle is .... angle.

(a) an acute (b) an obtuse (c) a straight (d) a right

2 ABCD is a cyclic quadrilateral in which m (∠ A) = 60° , then m (∠ C) = ...

(a) 60° (b) 30° (c) 90° (d) 120°

3 If the straight line L is a tangent to the circle M of diameter length 8 cm. , then the distance between L and the centre of the circle equals .... cm.

(a) 3 (b) 4 (c) 5 (d) 6

4 The area of the rhombus with diagonal lengths 6 cm. and 8 cm. is .... cm^2.

(a) 2 (b) 14 (c) 24 (d) 48

5 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

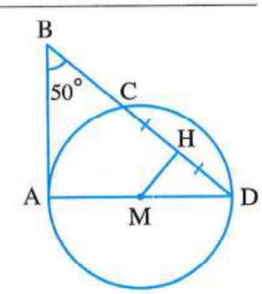
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

6 In Δ ABC , if $(AC)^2 > (AB)^2 + (BC)^2$, then Δ ABC is ....

(a) right-angled. (b) acute-angled.

(c) obtuse-angled. (d) equilateral.

2 [a] In the opposite figure :



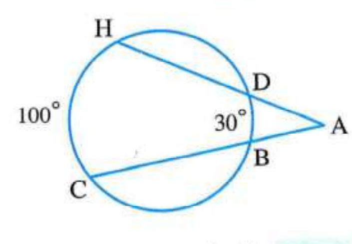
$\overline{AB}$ is a tangent-segment to the circle M at A

, H is the midpoint of $\overline{CD}$

, m (∠ B) = 50°

Find : m (∠ AMH)

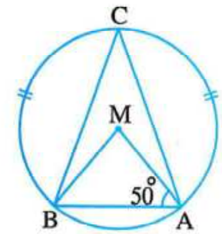
[b] In the opposite figure :



m $(\overset{\frown}{HC})$ = 100° , m $(\overset{\frown}{BD})$ = 30°

Find with proof : m (∠ A)

3 [a] **In the opposite figure :**

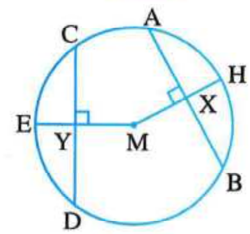


m $(\overset{\frown}{AC})$ = m $(\overset{\frown}{BC})$

, m (∠ MAB) = 50°

Find : m (∠ CAM)

[b] **In the opposite figure :**



AB = CD , $\overline{MX} \perp \overline{AB}$

, $\overline{MY} \perp \overline{CD}$

Prove that : HX = EY

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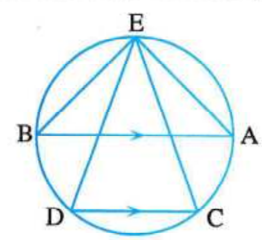
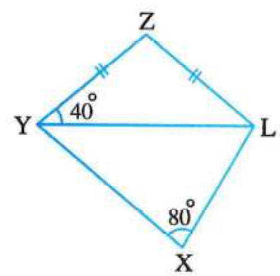
4 [a] **In the opposite figure :**



$\overline{AB}$ // $\overline{CD}$

Prove that : m (∠ AED) = m (∠ CEB)

[b] **In the opposite figure :**



XYZL is a quadrilateral in which

ZL = ZY , m (∠ ZYL) = 40°

, m (∠ X) = 80°

Prove that : XYZL is a cyclic quadrilateral.

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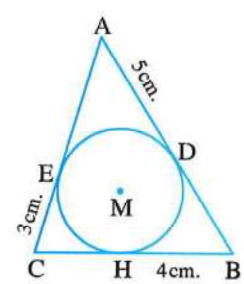
5 [a] **In the opposite figure :**



A circle M inscribed in Δ ABC

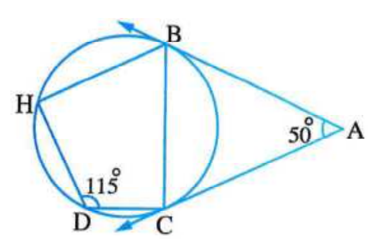
where AD = 5 cm.

, BH = 4 cm.

, CE = 3 cm.

Find : The perimeter of Δ ABC

[b] **In the opposite figure :**



$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m (∠ A) = 50°

, m (∠ HDC) = 115°

Prove that : $\overrightarrow{BC}$ bisects ∠ ABH

4 El-Kalyoubia Governorate

Answer the following questions :

1 Choose the correct answer from the given answers :

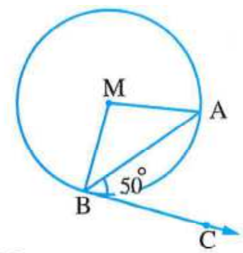
1 There are .... axes of symmetry to the circle.

(a) 1 (b) 2 (c) 3 (d) an infinite number of.

2 If ABCD is a cyclic quadrilateral , then m (∠ A) + m (∠ C) = ...

(a) 90° (b) 120° (c) 180° (d) 270°

3 **In the opposite figure :**



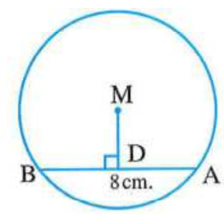
$\overrightarrow{BC}$ is a tangent

, m (∠ ABC) = 50°

, then m (∠ AMB) = ...

(a) 60° (b) 100° (c) 120° (d) 150°

4 **In the opposite figure :**



$\overline{MD} \perp \overline{AB}$

, AB = 8 cm.

, then BD = ... cm.

(a) 2 (b) 3 (c) 4 (d) 5

5 The measure of the arc which represents $\frac{1}{4}$ the measure of the circle equals ....

(a) 360° (b) 270° (c) 180° (d) 90°

6 The number of circles that can be drawn and passes through the terminals of the line segment $\overline{AB}$ equals ....

(a) 1 (b) 2 (c) 3 (d) an infinite number.

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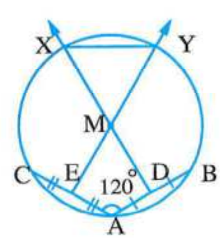
2 [a] **In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two chords in the circle M

, E is the midpoint of $\overline{AC}$, D is the midpoint of $\overline{AB}$

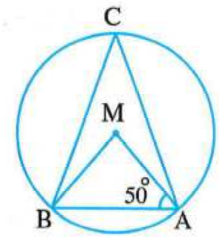
and m (∠ BAC) = 120°

Prove that : Δ MXY is an equilateral triangle.

[b] In the opposite figure :

m (∠ MAB) = 50°

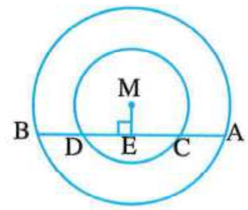
Find : m (∠ ACB)



3 **[a] In the opposite figure :**

Two concentric circles with centre M , $\overline{AB}$ is a chord of the greater circle and intersects the smaller circle at C , D , $\overline{ME} \perp \overline{AB}$

Prove that : AC = BD

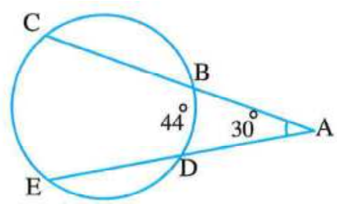


[b] In the opposite figure :

$\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$

, m (∠ A) = 30° , m $(\widehat{BD})$ = 44°

Find : m $(\widehat{EC})$



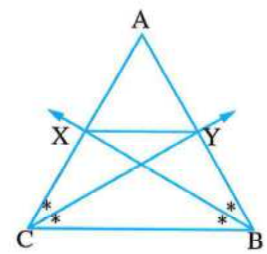
4 **[a] In the opposite figure :**

ABC is a triangle , AB = AC

, $\overrightarrow{CY}$ bisects ∠ ACB

, $\overrightarrow{BX}$ bisects ∠ ABC

Prove that : BCXY is a cyclic quadrilateral.

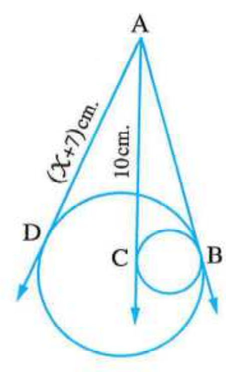


[b] In the opposite figure :

Two circles are touching at B , $\overrightarrow{AB}$ is a common tangent to the two circles , $\overrightarrow{AC}$ is a tangent to the smaller circle , $\overrightarrow{AD}$ is a tangent to the greater circle , AC = 10 cm.

, AD = ($\mathcal{X}$ + 7) cm.

Find : The value of $\mathcal{X}$

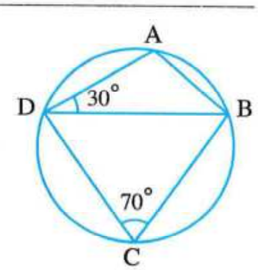


5 **[a] In the opposite figure :**

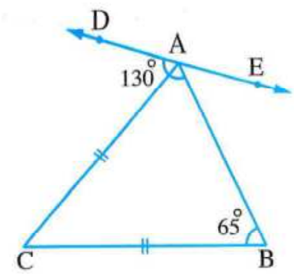
m (∠ ADB) = 30°

, m (∠ C) = 70°

Find : m (∠ ABD)



[b] In the opposite figure :



m (∠ DAB) = 130°

, m (∠ B) = 65° , AC = BC

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC

5 El-Sharkia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 The number of symmetry axes of the semicircle is ....

(a) zero. (b) 1 (c) 2 (d) an infinite number.

2 A circle is of circumference 6 π cm. , and the straight line L is distant from its centre by 3 cm. , then the straight line L is ....

(a) a tangent. (b) a secant.

(c) outside the circle. (d) a diameter.

3 The number of circles which passes through three collinear points is ....

(a) an infinite number. (b) two.

(c) one. (d) zero.

4 If the area of a square equals 50 cm^2 , then the length of its diagonal equals .... cm.

(a) 10 (b) 8 (c) 6 (d) 4

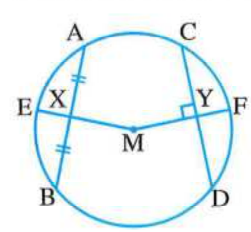
5 If ABCD is a cyclic quadrilateral in which m (∠ A) = 3 m (∠ C) , then m (∠ A) = ...

(a) 45° (b) 90° (c) 120° (d) 135°

6 The number of common tangents of two circles touching externally is ....

(a) 4 (b) 3 (c) 2 (d) 1

2 [a] In the opposite figure :



$\overline{AB}$ and $\overline{CD}$ are two chords equal in length in the circle M

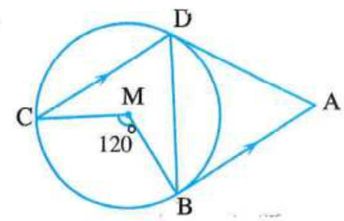
, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{CD}$

Prove that : XE = YF

[b] In the opposite figure :

$\overline{AB}$, $\overline{AD}$ are two tangent-segments to the circle M and $\overline{AB}$ // $\overline{DC}$, if m (∠ BMC) = 120°

, prove that : Δ ABD is an equilateral triangle.



3 **[a] In the opposite figure :**

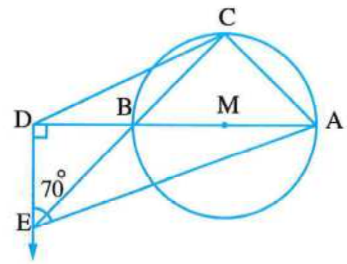
$\overline{AB}$ is a diameter of the circle M

, D ∈ $\overrightarrow{AB}$, D ∉ $\overline{AB}$, $\overrightarrow{DE} \perp \overrightarrow{AB}$, C ∈ $\widehat{AB}$

, $\overrightarrow{CB} \cap \overrightarrow{DE} = \{E\}$, m (∠ AED) = 70°

1 **Prove that :** The figure ACDE is a cyclic quadrilateral.

2 **Find :** m (∠ DCE)



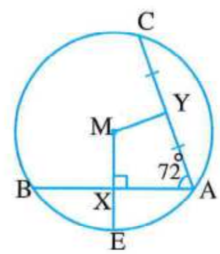
[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two chords in the circle M

If its radius length is 10 cm. , $\overrightarrow{MX} \perp \overline{AB}$ and intersects $\overline{AB}$ at X and intersects the circle at E , Y is the midpoint of $\overline{AC}$

, AB = 16 cm. , m (∠ CAB) = 72°

, find : 1 m (∠ XMY) 2 The length of $\overline{XE}$



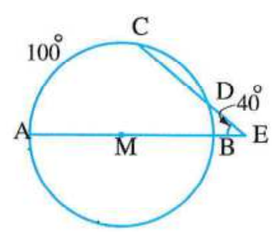
4 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$

, m (∠ AEC) = 40° , m ($\widehat{AC}$) = 100°

Find : m ($\widehat{CD}$)

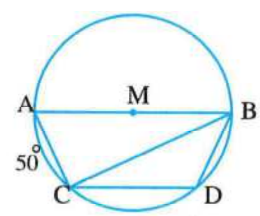


[b] In the opposite figure :

$\overline{AB}$ is a diameter of the circle M

, m ($\widehat{AC}$) = 50°

Find : m (∠ CDB)

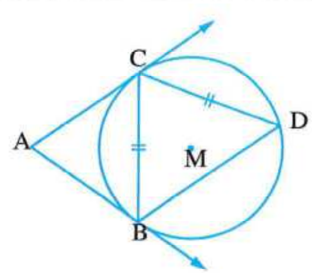


5 **[a] In the opposite figure :**

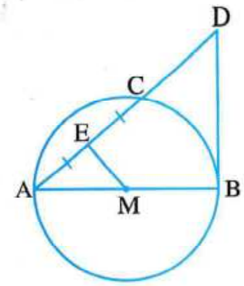
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M

, CB = CD

Prove that : $\overrightarrow{CD}$ is a tangent to the circle passing through the vertices of Δ ABC



[b] **In the opposite figure :**



$\overline{AB}$ is a diameter of the circle M

, $\overline{BD}$ is a tangent-segment to the circle at B

, E is the midpoint of $\overline{AC}$

Prove that : The figure MEDB is a cyclic quadrilateral.

6 El-Monofia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 The number of the axes of symmetry of an equilateral triangle equals ....

(a) 1 (b) 2 (c) 3 (d) 4

2 The sum of the measures of the interior angles of the quadrilateral equals ....

(a) 90° (b) 180° (c) 270° (d) 360°

3 A circle of circumference 44 cm. , then its area is .... cm^2. $\left(\pi = \frac{22}{7}\right)$

(a) 22 (b) 49 (c) 88 (d) 154

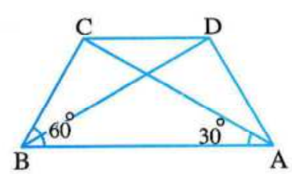
4 M and N are two intersecting circles , their radii lengths are 3 cm. and 5 cm. , then MN ∈ ....

(a)]8 , ∞[(b)]2 , ∞[(c)]0 , 2[(d)]2 , 8[

5 The number of the circles that can be drawn through three non-collinear points is ....

(a) zero. (b) only one. (c) three. (d) infinite.

6 **In the opposite figure :**



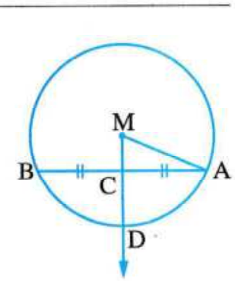
ABCD is a cyclic quadrilateral. If m (∠ BAC) = 30°

, m (∠ ABC) = 60° , then m (∠ ADB) = ...

(a) 50° (b) 60° (c) 80° (d) 90°

---

2 [a] **In the opposite figure :**



M is a circle with radius length 13 cm.

, $\overline{AB}$ is a chord of length 24 cm. , C is the midpoint of $\overline{AB}$

, $\overrightarrow{MC}$ intersects the circle at D

Find : The length of $\overline{CD}$

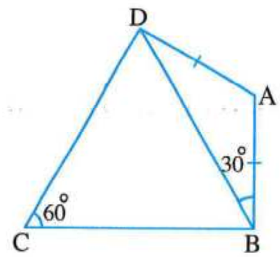
[b] In the opposite figure :

ABCD is a quadrilateral in which AB = AD

, m (∠ ABD) = 30°

and m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.



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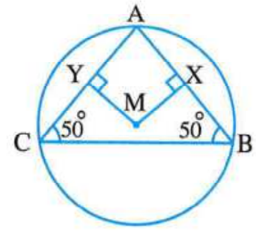
3 **[a] In the opposite figure :**

ABC is a triangle inscribed in a circle M

, m (∠ B) = m (∠ C) = 50° , AX = 3 cm.

, $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{AC}$

1 **Prove that :** MX = MY **2** **Find :** The length of $\overline{AC}$

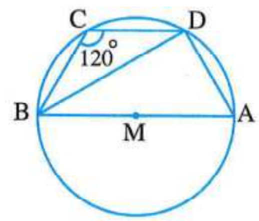


[b] In the opposite figure :

ABCD is a quadrilateral drawn in a circle M

, $\overline{AB}$ is a diameter in the circle M , m (∠ BCD) = 120°

Find : **1** m (∠ A) **2** m (∠ ABD)



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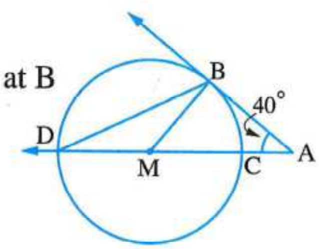
4 **[a] In the opposite figure :**

A is a point outside the circle M , $\overrightarrow{AB}$ is a tangent to the circle at B

, $\overrightarrow{AM}$ intersects the circle M at C , D respectively

and m (∠ A) = 40°

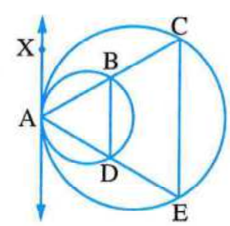
Find : m (∠ BDC)



[b] In the opposite figure :

Two circles are touching internally at A , $\overleftrightarrow{AX}$ is the common tangent to them at A , $\overleftrightarrow{AB}$ and $\overleftrightarrow{AD}$ intersect the small circle at B , D and the great circle at C , E

Prove that : $\overline{BD}$ // $\overline{CE}$



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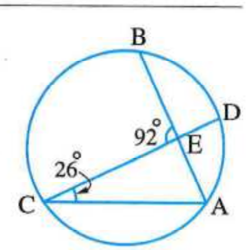
5 **[a] In the opposite figure :**

If $\overline{AB} \cap \overline{CD} = \{E\}$

, m (∠ ACD) = 26° , m (∠ BEC) = 92°

, then find : **1** m ($\overset{\frown}{AD}$) **2** m ($\overset{\frown}{BC}$)



[b] ABCD is a parallelogram in which AC = BC

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle passing through the vertices of the triangle ABC

7 El-Gharbia Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1 If the straight line L is a tangent to the circle of diameter length 8 cm. , then the distance between L and the centre of the circle equals .... cm.

(a) 3 (b) 4 (c) 6 (d) 8

2 The area of the rectangle whose length is 3 cm. and its width is 2 cm. equals .... cm^2.

(a) 4 (b) 5 (c) 6 (d) 10

3 The measure of the inscribed angle equals .... the measure of the central angle subtended by the same arc.

(a) half (b) third (c) quarter (d) twice

4 ABCD is a cyclic quadrilateral in which m (∠ A) = 50° , then m (∠ C) = ...

(a) 25° (b) 50° (c) 100° (d) 130°

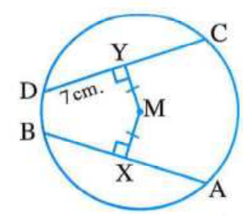
5 The number of symmetry axes of the equilateral triangle is ....

(a) 1 (b) 2 (c) 3 (d) 0

6 The measure of the inscribed angle in a semicircle equals ....

(a) 45° (b) 135° (c) 90° (d) 150°

2 [a] **In the opposite figure :**



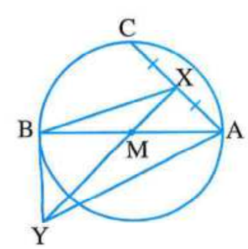
$\overline{AB}$ and $\overline{CD}$ are two chords in a circle M

, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

, MX = MY and YD = 7 cm.

Find : The length of $\overline{AB}$

[b] **In the opposite figure :**



$\overline{AB}$ is a diameter in a circle M , X is the midpoint of $\overline{AC}$

and $\overrightarrow{XM}$ intersects the tangent to the circle at B in Y

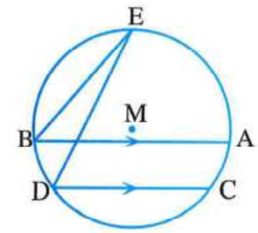
Prove that : The figure AXBY is a cyclic quadrilateral.

3 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two parallel chords in a circle M

, m $(\widehat{AC}) = 30°$

Find : m (∠ BED)

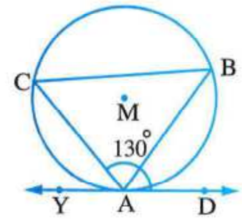


[b] In the opposite figure :

$\overleftrightarrow{DY}$ is a tangent to the circle M at A

, m (∠ DAC) = 130°

Find : m (∠ ABC)



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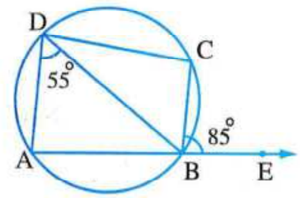
4 **[a] In the opposite figure :**

E ∈ $\overrightarrow{AB}$, E ∉ $\overline{AB}$, m (∠ ADB) = 55°

, m (∠ CBE) = 85°

Find : m (∠ CDB)

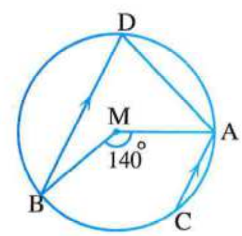


[b] In the opposite figure :

M is a circle , $\overline{AC}$ // $\overline{DB}$

, m (∠ AMB) = 140°

Find : m (∠ CAD)



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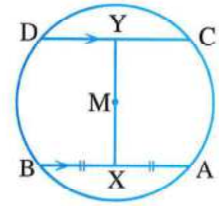
5 **[a] In the opposite figure :**

M is a circle , $\overline{AB}$ // $\overline{CD}$, X is the midpoint of $\overline{AB}$

, $\overrightarrow{XM}$ is drawn to intersect $\overline{CD}$ at Y

Prove that : Y is the midpoint of $\overline{CD}$



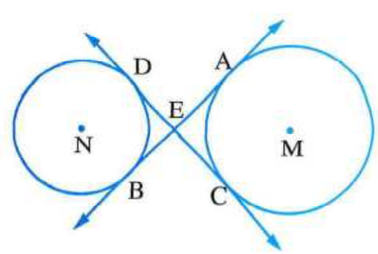
[b] In the opposite figure :

$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two common

tangents to the two circles M and N

, $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$

Prove that : AB = CD



8 El-Dakahlia Governorate

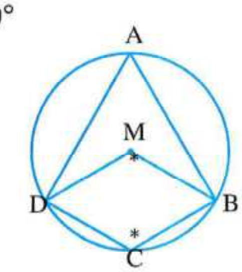
Answer the following questions : *(Calculator is permitted)*

1 [a] **Choose the correct answer :**

1. The sum of measures of the interior angles of the cyclic quadrilateral equals ....
   (a) 90° (b) 180° (c) 360° (d) 720°

2. The area of a circle is 25π cm^2, the straight line L is of distance 5 cm. of its centre , then L is ....
   (a) outside the circle. (b) a tangent to the circle.
   (c) a secant of the circle. (d) passing through the centre.

3. If ABCDEF is a regular hexagon drawn inside a circle , then m $(\widehat{AB})$ = ...
   (a) 60° (b) 90° (c) 180° (d) 360°



[b] **In the opposite figure :**

ABCD is a quadrilateral drawn inside the circle M

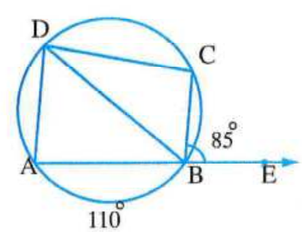
, m (∠ BMD) = m (∠ BCD)

Find : m (∠ A) in degrees.

---

2 [a] **Choose the correct answer :**

1. **In the opposite figure :**



   If E ∈ $\overrightarrow{AB}$, m (∠ EBC) = 85°
   , m $(\widehat{AB})$ = 110° , then m (∠ BDC) = ...
   (a) 30° (b) 55°
   (c) 85° (d) 110°

2. The altitudes of the obtuse-angled triangle intersect at a point lying ....
   (a) inside the triangle.
   (b) on one of its vertices.
   (c) outside the triangle.
   (d) at the midpoint of the opposite side to the obtuse angle.

3. The length of the arc representing half the circle equals .... length unit.
   (a) $2\pi r$ (b) πr (c) $\frac{1}{2}\pi r$ (d) $\frac{1}{3}\pi r$

[b] ABCD is a parallelogram , AC = BC

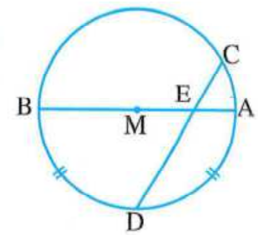
Prove that : $\overleftrightarrow{CD}$ is a tangent to the circumcircle of Δ ABC

3 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M , $\overline{AB} \cap \overline{CD} = \{E\}$

, m $(\widehat{AD})$ = m $(\widehat{BD})$ = 3 m $(\widehat{AC})$

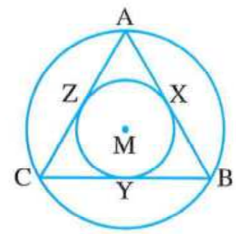
Find : m (∠ AEC)



[b] **In the opposite figure :**

two concentric circles , Δ ABC is drawn where its vertices lie on the greater circle and its sides touch the smaller circle at X , Y , Z

Prove that : Δ ABC is an equilateral triangle.



4 [a] **In the opposite figure :**

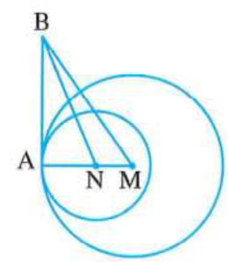
Two circles M and N , their radii lengths are 10 cm.

, 6 cm. respectively and touching internally at A

, $\overline{AB}$ is a common tangent-segment at A

, the area of Δ BMN = 24 cm^2

Find : The length of $\overline{AB}$



[b] $\overline{AB}$ and $\overline{CD}$ are two parallel chords in the circle M , $\overline{AD} \cap \overline{CB} = \{E\}$

Prove that : Δ EAB is an isosceles triangle.

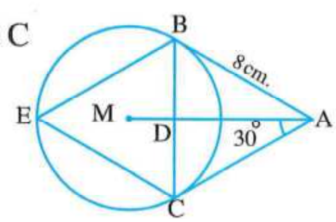
5 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M at B and C

, $\overline{AM} \cap \overline{BC} = \{D\}$, AB = 8 cm. , m (∠ CAM) = 30°

Find : 1 The perimeter of Δ ABC

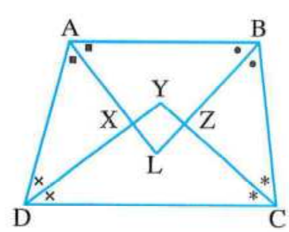
2 m (∠ E)



[b] **In the opposite figure :**

ABCD is a quadrilateral , $\overrightarrow{AX}$, $\overrightarrow{BZ}$, $\overrightarrow{CZ}$, $\overrightarrow{DX}$ bisect ∠ A , ∠ B , ∠ C , ∠ D respectively

Prove that : The figure XYZL is a cyclic quadrilateral.



9 Ismailia Governorate

Answer the following questions : *(Calculators are allowed)*

1 Choose the correct answer from those given :

1 The sum of measures of the interior angles of a triangle equals ....

(a) 90° (b) 120° (c) 180° (d) 360°

2 The measure of the arc which represents the quarter of a circle equals ....

(a) 360° (b) 180° (c) 120° (d) 90°

3 If the perimeter of a square is 20 cm. , then its area equals .... cm^2.

(a) 25 (b) 10 (c) 20 (d) 50

4 In a cyclic quadrilateral , each two opposite angles are ....

(a) complementary. (b) supplementary. (c) alternate. (d) equal in measure.

5 The number of circles that can pass through a given point is ....

(a) one circle. (b) two circles. (c) three circles. (d) an infinite number.

6 The centre of the circumcircle of the triangle is the point of intersection of ....

(a) its altitudes. (b) its medians.

(c) the symmetry axes of its sides. (d) the bisectors of its interior angles.

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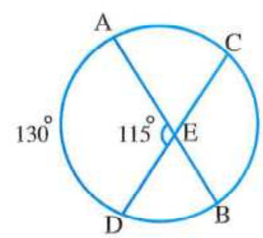
2 [a] In the opposite figure :



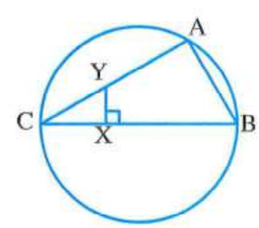
$\overline{AB} \cap \overline{CD} = \{E\}$

, m (∠ AED) = 115°

, m $(\overset{\frown}{AD})$ = 130°

Find with proof : m $(\overset{\frown}{BC})$

[b] In the opposite figure :



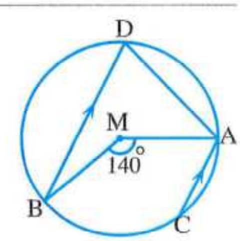
ABXY is a cyclic quadrilateral

, $\overline{YX} \perp \overline{CB}$

Prove that : $\overline{CB}$ is a diameter of the given circle.

---

3 [a] In the opposite figure :



$\overline{CA} \parallel \overline{BD}$

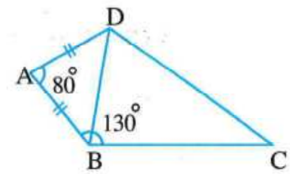
, m (∠ BMA) = 140°

Find with proof : m (∠ CAD)

[b] In the opposite figure :

AB = AD , m (∠ A) = 80° , m (∠ ABC) = 130°

Prove that : $\overline{BC}$ is a tangent-segment to the circle which passes through the points A , B and D



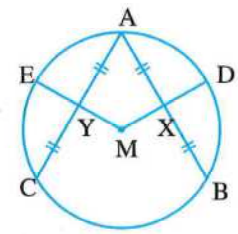
4 **[a] In the opposite figure :**

AB = AC in the circle M

and X , Y are the midpoints

of $\overline{AB}$, $\overline{AC}$ respectively.

Prove that : DX = EY



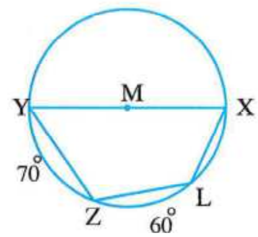
[b] In the opposite figure :

$\overline{XY}$ is a diameter of the circle M

, m $(\widehat{YZ})$ = 70°

, m $(\widehat{ZL})$ = 60°

Find with proof : The measures of the angles of the figure XYZL

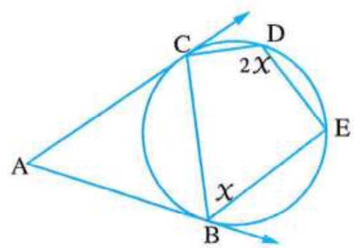


5 **[a] In the opposite figure :**

$\overrightarrow{BC}$ bisects ∠ ABE , $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C respectively

, m (∠ D) = 2 x , m (∠ CBE) = x

Prove that : Δ ABC is an equilateral triangle.

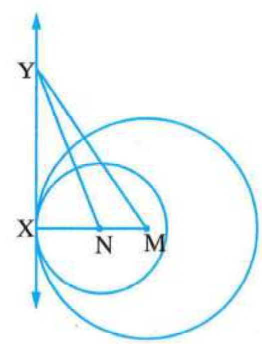


[b] In the opposite figure :

M and N are two circles with radii of lengths 10 cm. and 6 cm. respectively and they are touching internally at X , $\overrightarrow{XY}$ is a common tangent at X

If the area of Δ YMN = 24 cm.^{2}

, find the proof : The length of $\overline{MY}$



10 Suez Governorate

Answer the following questions : ***(Calculators are allowed)***

1 **Choose the correct answer from those given :**

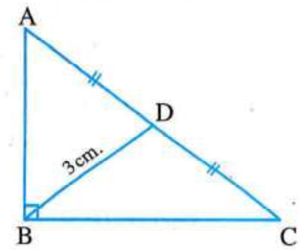
1 The inscribed angle in a semicircle is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

2 **In the opposite figure :**

ABC is a right-angled triangle at B , D is the midpoint of $\overline{AC}$

, BD = 3 cm. , then AC = …………… cm.



(a) 3 (b) 6

(c) 9 (d) 12

3 If the circle M ⋂ the circle N = {A , B} , then the two circles M and N are ……………

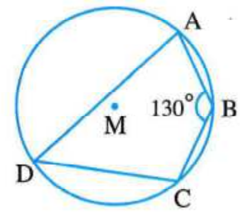
(a) distant. (b) concentric.

(c) touching externally. (d) intersecting.

4 **In the opposite figure :**

If M is a circle , m (∠ B) = 130°

, then m (∠ D) = ……………



(a) 130° (b) 60°

(c) 50° (d) 65°

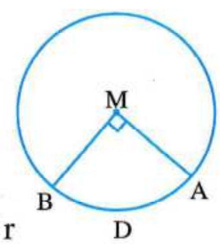
5 If ∠ A and ∠ B are two complementary angles , then m (∠ A) + m (∠ B) = ……………

(a) 90° (b) 180° (c) 360° (d) 120°

6 **In the opposite figure :**

If M is a circle , m (∠ AMB) = 90°

, then the length of $\overset{\frown}{ADB}$ = ……………



(a) $2\pi r$ (b) πr (c) $\frac{1}{2}\pi r$ (d) $\frac{1}{4}\pi r$

---

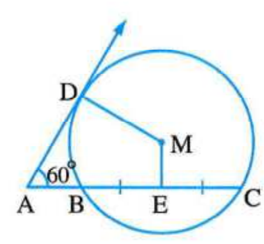
2 [a] **In the opposite figure :**

$\overrightarrow{AD}$ is a tangent to the circle M

, $\overrightarrow{AC}$ intersects the circle M at B and C

, E is the midpoint of $\overline{BC}$, m (∠ A) = 60°

Find with proof : m (∠ DME)

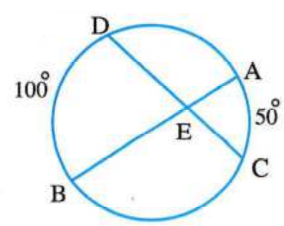


[b] **In the opposite figure :**

$\overline{AB} \cap \overline{CD}$ = {E}

, m ($\overset{\frown}{AC}$) = 50° , m ($\overset{\frown}{BD}$) = 100°

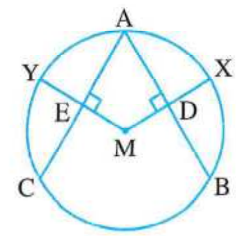
Find with proof : m (∠ AEC)



3 [a] **In the opposite figure :**

M is a circle , AB = AC , $\overrightarrow{MD} \perp \overline{AB}$ intersecting the circle at X , $\overrightarrow{ME} \perp \overline{AC}$ intersecting the circle at Y

Prove that : XD = YE

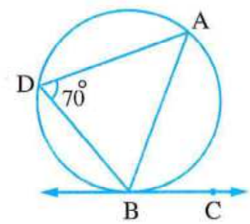


[b] **In the opposite figure :**

$\overleftrightarrow{BC}$ is a tangent to the circle at B

, m (∠ ADB) = 70°

Find with proof : 1 m (∠ ABC) 2 m ($\overset{\frown}{AB}$)



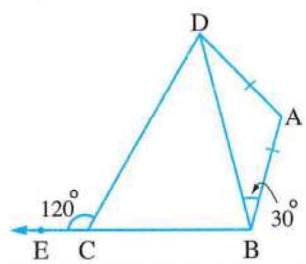
4 [a] State two cases of cyclic quadrilateral.

[b] **In the opposite figure :**

ABCD is a quadrilateral , E ∈ $\overrightarrow{BC}$, AB = AD

, m (∠ ABD) = 30° , m (∠ DCE) = 120°

Prove that : ABCD is a cyclic quadrilateral.

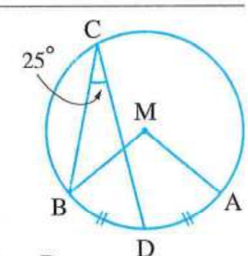


5 [a] **In the opposite figure :**

M is a circle , D is the midpoint of $\overset{\frown}{AB}$

, m (∠ C) = 25°

Find with proof : m (∠ AMB)



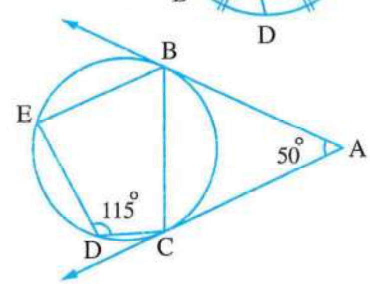
[b] **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 50°

, m (∠ CDE) = 115°

Prove that : $\overrightarrow{BC}$ bisects ∠ ABE



11 Port Said Governorate

Answer the following questions :

First Objective Questions

Choose the correct answer from those given :

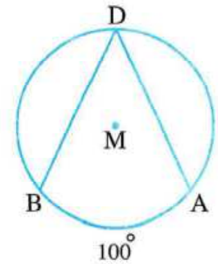
1 The centre of the circumcircle of a triangle is the point of intersection of ....

(a) its interior angles bisectors.
(b) its medians.
(c) its heights.
(d) axes of symmetry of its sides.

2 A tangent to a circle of diameter length 6 cm. is at a distance of .... cm. from its centre.

(a) 2 (b) 3 (c) 6 (d) 12

3 **In the opposite figure :**



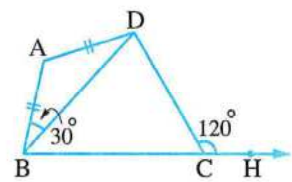
A circle of centre M and m $(\widehat{AB}) = 100°$

, then m (∠ ADB) = ...

(a) 150° (b) 100°

(c) 50° (d) 25°

4 **In the opposite figure :**



ABCD is a quadrilateral in which m (∠ ABD) = 30°

, m (∠ DCH) = 120°

, AB = AD , then the shape ABCD is called a ....

(a) rectangle. (b) rhombus.

(c) cyclic quadrilateral. (d) parallelogram.

5 The measure of the inscribed angle which is drawn in a semicircle equals ....

(a) 45° (b) 90° (c) 135° (d) 180°

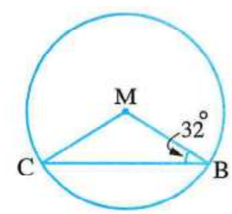
6 If M , N are two touching circles internally , their radii lengths are 5 cm. , 9 cm.
, then MN = ... cm.

(a) 14 (b) 4 (c) 5 (d) 9

7 If the measure of an arc of a circle equals 60° , then its length equals ....
of the circumference.

(a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

8 **In the opposite figure :**



M is the centre of the circle , m (∠ MBC) = 32°

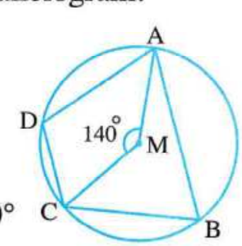
, then m $(\widehat{BC})$ = ...

(a) 116° (b) 32° (c) 58° (d) 64°

9 We can draw a circle passing through the vertices of a ....

(a) rhombus. (b) square. (c) trapezium. (d) parallelogram.

10 **In the opposite figure :**



m (∠ CMA) = 140°

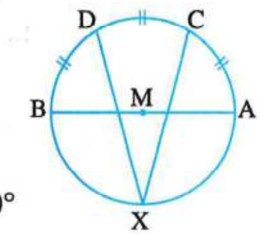
, then m (∠ CDA) = ...

(a) 70° (b) 110° (c) 40° (d) 140°

11 The area of the square whose side length equals 6 cm. is .... cm^2

(a) 12 (b) 24 (c) 36 (d) 60

12 **In the opposite figure :**



$\overline{AB}$ is a diameter of the circle M , m $(\widehat{AC})$ = m $(\widehat{CD})$ = m $(\widehat{DB})$

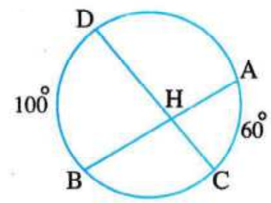
, then m (∠ CXD) = ...

(a) 15° (b) 30° (c) 45° (d) 60°

13 Two tangents drawn from the end points of a diameter of a circle are ....

(a) perpendicular. (b) concident. (c) parallel. (d) intersecting.

14 **In the opposite figure :**



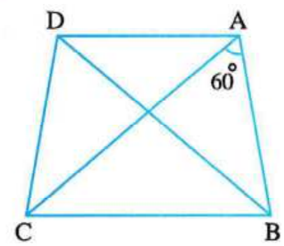
$\overline{AB} \cap \overline{CD} = \{H\}$

, m $(\widehat{AC})$ = 60° , m $(\widehat{BD})$ = 100°

, then m (∠ DHB) = ...

(a) 16° (b) 100° (c) 80° (d) 60°

15 **In the opposite figure :**



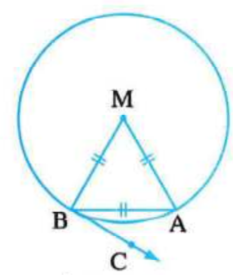
ABCD is a cyclic quadrilateral

, m (∠ BAC) = 60° , then m (∠ BDC) = ...

(a) 300° (b) 120°

(c) 60° (d) 30°

16 **In the opposite figure :**



Δ MAB is equilateral , $\overrightarrow{BC}$ is a tangent at B

, then m (∠ ABC) = ...

(a) 120° (b) 90°

(c) 60° (d) 30°

17 The smallest radius length of a circle can be drawn passing through the two points A and B where AB = 6 cm. is .... cm.

(a) 1 (b) 2 (c) 3 (d) 4

18 The circumference of a circle whose diameter length equals 7 cm. is .... cm.

(a) 7 π (b) 14 π (c) 49 π (d) $\frac{7}{2}$ π

19 The number of common tangents for two distant circles is ....

(a) one. (b) two. (c) three. (d) four.

20 The sum of the measures of all interior angles of any triangle equals ....

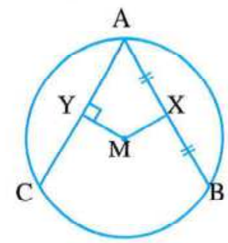
(a) 180° (b) 360° (c) 540° (d) 720°

21 The diameter is .... passing through the centre of the circle.

(a) a straight line (b) a ray (c) a tangent. (d) a chord

Second Essay questions

22 **In the opposite figure :**

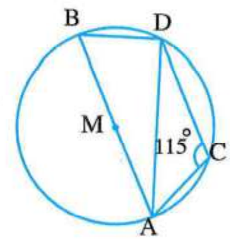


M is the centre of a circle

, AB = AC , $\overline{MY} \perp \overline{AC}$, X is the midpoint of $\overline{AB}$

Show that : MX = MY

23 **In the opposite figure :**

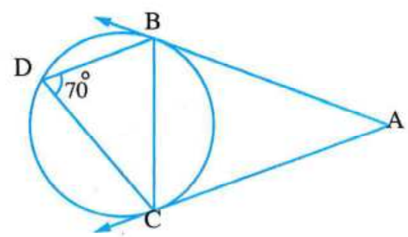


$\overline{AB}$ is a diameter of the circle M

, m (∠ ACD) = 115°

Find : m (∠ DAB)

24 **In the opposite figure :**



$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents touching the circle at B , C respectively and m (∠ BDC) = 70°

Find : m (∠ A)

12 Damietta Governorate

Answer the following questions : *(Calculators are allowed)*

1 Choose the correct answer from those given :

1 The corresponding angles of the two similar polygons are .... in measure.

(a) different (b) proportional (c) alternate (d) equal

2 The inscribed angle drawn in a semicircle is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

3 The image of the point (– 3 , 4) by reflection in the y-axis is ....

(a) (3 , 4) (b) (3 , – 4) (c) (– 3 , – 4) (d) (4 , – 3)

4 M and N are two circles , their radii lengths are 5 cm. and 3 cm. , if MN = 6 cm. , then the two circles are ....

(a) distant. (b) touching externally.

(c) intersecting. (d) one inside the other.

5 ABCD is a cyclic quadrilateral , where m (∠ A) = 2 m (∠ C) , then m (∠ C) = ...

(a) 50° (b) 60° (c) 90° (d) 120°

6 A rectangle , its length is 5 cm. and its perimeter is 16 cm. , then its area is .... cm^2

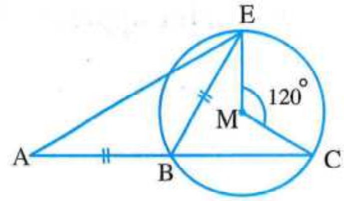
(a) 15 (b) 40 (c) 55 (d) 80

2 **[a] In the opposite figure :**

M is a circle , m (∠ EMC) = 120°

and AB = EB

Find with proof : m (∠ A)



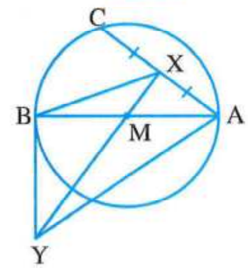
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, X is the midpoint of $\overline{CA}$ and $\overline{BY}$

is a tangent-segment to the circle M at B

Prove that : The figure AXBY is a cyclic quadrilateral.



3 **[a] In the opposite figure :**

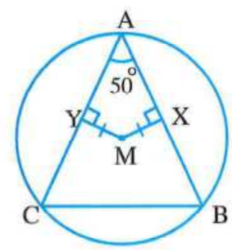
A triangle ABC is inscribed in the circle M , in which :

$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

, MX = MY and

m (∠ A) = 50°

Find with proof : m (∠ B)

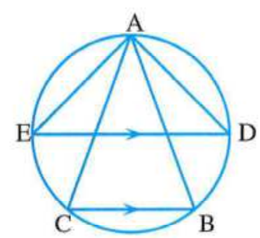


[b] In the opposite figure :

ABC is an inscribed triangle in a circle

, $\overline{ED}$ // $\overline{CB}$

Prove that : m (∠ DAC) = m (∠ EAB)

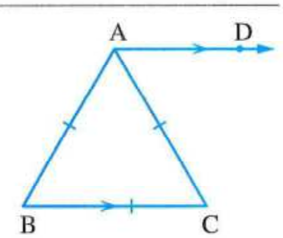


4 **[a] In the opposite figure :**

$\overrightarrow{AD}$ // $\overline{BC}$

, AB = BC = CA

Prove that : $\overrightarrow{AD}$ is a tangent to the circumcircle of Δ ABC



[b] In the opposite figure :

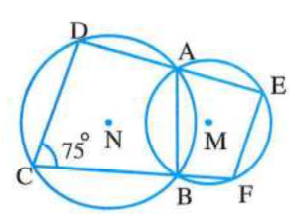
M and N are two intersecting circles at A and B

, $\overleftrightarrow{AD}$ is drawn to intersect circle M at E and circle N at D

, $\overleftrightarrow{BC}$ is drawn to intersect circle M at F and circle N at C

, and m (∠ C) = 75°

1 **Find :** m (∠ F) **2** **Prove that :** $\overline{CD}$ // $\overline{EF}$



5 **[a]** Draw $\overline{AB}$ where AB = 6 cm. , then draw a circle passing through the two points A and B , the length of its radius is 4 cm. , using your geometric instruments.

How many circles can be drawn ? (Don't remove the arcs)

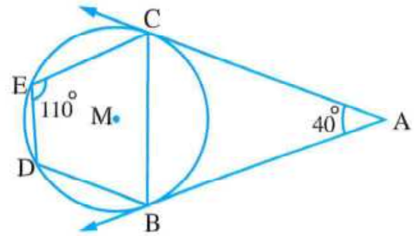
[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ E) = 110°

, m (∠ A) = 40°

Prove that : $\overrightarrow{BC}$ bisects ∠ ABD



13 Kafr El-Sheikh Governorate

Answer the following questions : *(Calculators are permitted)*

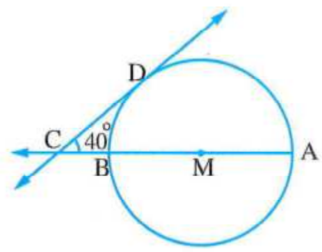
1 Choose the correct answer from those given :

1 In the opposite figure :

m ($\widehat{DB}$) = ...

(a) 25° (b) 50°

(c) 80° (d) 130°



2 The measure of the supplementary angle of an angle whose measure is 60° equals ....

(a) 30° (b) 90° (c) 120° (d) 60°

3 The inscribed angle drawn in a semicircle is ....

(a) acute. (b) obtuse. (c) straight. (d) right.

4 If M and N are two touching circles externally , their radii lengths are 3 cm. and 5 cm. , then MN = ... cm.

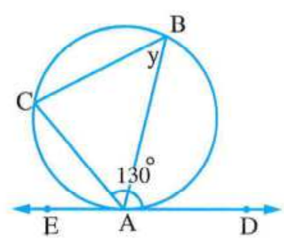
(a) 9 (b) 8 (c) 2 (d) 6

5 In the opposite figure :

y = ...

(a) 50° (b) 25°

(c) 100° (d) 130°



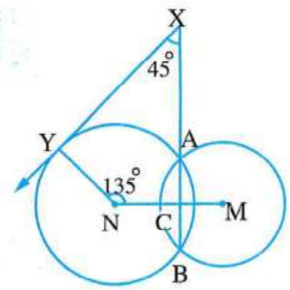
6 The cyclic quadrilateral from the following figures is ....

(a) a rhombus. (b) a rectangle.

(c) a trapezium. (d) a parallelogram.

2 [a] **In the opposite figure :**

Prove that : $\overleftrightarrow{XY}$ is a tangent to the circle N at Y



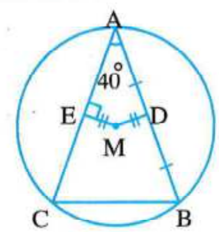
[b] A is a point outside the circle M , $\overrightarrow{AB}$ is a tangent to the circle at B , $\overrightarrow{AM}$ intersects the circle at C and D respectively , m (∠ A) = 40° **Find with proof :** m (∠ BDC)

---

3 [a] **In the opposite figure :**

MD = ME

Find : m (∠ B)



[b] $\overline{AB}$ and $\overline{AC}$ are two chords in a circle M , X and Y are the midpoints of $\overset{\frown}{AB}$ and $\overset{\frown}{AC}$ respectively , $\overline{XY}$ was drawn and intersected $\overline{AB}$ and $\overline{AC}$ at D and E respectively.

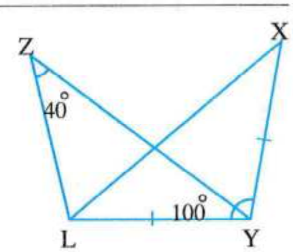
Prove that : AD = AE

---

4 [a] **In the opposite figure :**

XY = YL , m (∠ XYL) = 100° , m (∠ Z) = 40°

Prove that : The points X , Y , L and Z have only one circle passing through them.

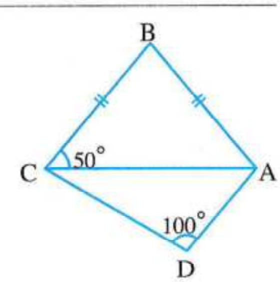


[b] Using the geometrical tools , draw Δ XYZ which has XY = 5 cm. , YZ = 3 cm. and ZX = 7 cm. , then draw the outer circle of Δ XYZ , then find by measuring the length of its radius. (Don't remove the arcs)

---

5 [a] **In the opposite figure :**

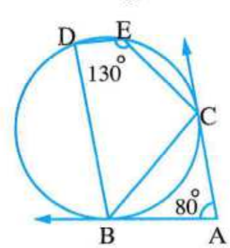
Prove that : ABCD is a cyclic quadrilateral.



[b] **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

Prove that : [1] $\overrightarrow{BC}$ bisects ∠ ABD [2] $\overleftrightarrow{BD}$ // $\overleftrightarrow{AC}$



14 El-Beheira Governorate

Answer the following questions : ***(Calculator is permitted)***

1 Choose the correct answer from the given ones :

1 In Δ ABC , if $(AB)^2 + (BC)^2 < (AC)^2$, then $\angle$ B is ....

(a) obtuse. (b) right. (c) acute. (d) straight.

2 The measure of the exterior angle of the equilateral triangle equals ....

(a) 30° (b) 45° (c) 60° (d) 120°

3 A square its area is 50 cm^2 , then its diagonal length equals .... cm.

(a) 5 (b) 10 (c) 15 (d) 25

4 The number of circles which can be drawn passing through the end points of the line segment $\overline{AB}$ equals ....

(a) 1 (b) 2 (c) 3 (d) infinite.

5 XYZL is a cyclic quadrilateral , m ($\angle$ X) = 65° , then m ($\angle$ Z) = ...

(a) 30° (b) 60° (c) 90° (d) 115°

6 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 45° (b) 90° (c) 120° (d) 180°

---

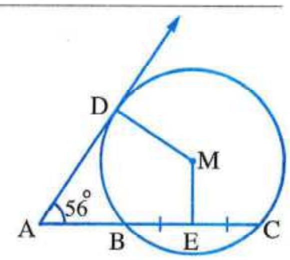
2 [a] In the opposite figure :

$\overrightarrow{AD}$ is a tangent to the circle M at D

, $\overrightarrow{AC}$ intersects the circle M at B and C

, E is the midpoint of $\overline{BC}$, m ($\angle$ A) = 56°

Find : m ($\angle$ DME)

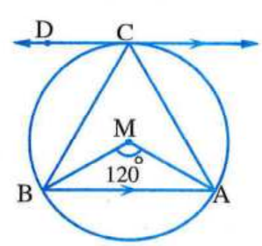


[b] In the opposite figure :

$\overleftrightarrow{CD}$ is a tangent to the circle at C , $\overleftrightarrow{CD}$ // $\overline{AB}$

, m ($\angle$ AMB) = 120°

Prove that : The triangle CAB is an equilateral triangle.



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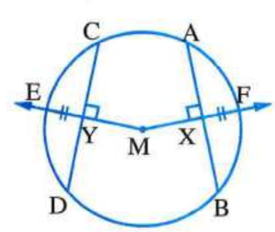
3 [a] In the opposite figure :

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M , $\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle at F , $\overrightarrow{MY} \perp \overline{CD}$ and intersects the circle at E , FX = EY

Prove that : AB = CD



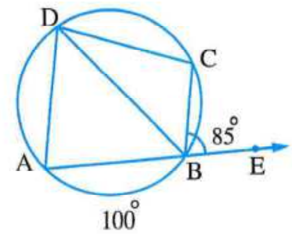
[b] In the opposite figure :

$E \in \overrightarrow{AB}$, $E \notin \overline{AB}$

, m $(\widehat{AB}) = 100°$

, m $(\angle CBE) = 85°$

Find : m $(\angle BDC)$

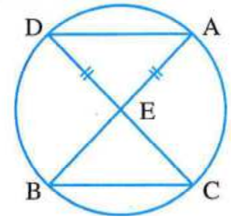


4 **[a] In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{E\}$

, EA = ED

Prove that : EB = EC



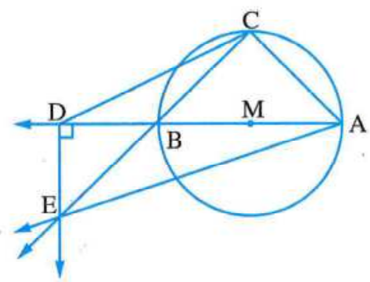
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $D \in \overrightarrow{AB}$, $D \notin \overline{AB}$, $\overrightarrow{DE} \perp \overrightarrow{AB}$

, $C \in \widehat{AB}$ and $\overrightarrow{CB} \cap \overrightarrow{DE} = \{E\}$

Prove that : ACDE is a cyclic quadrilateral.



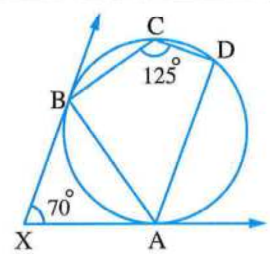
5 **[a] In the opposite figure :**

$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

, m $(\angle AXB) = 70°$

, m $(\angle DCB) = 125°$

Prove that : $\overrightarrow{AB}$ bisects $\angle$ DAX



[b] ABC is a triangle inscribed in a circle , $\overleftrightarrow{AD}$ is a tangent to the circle at A , $X \in \overline{AB}$, $Y \in \overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

15 El-Fayoum Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer :**

1. A triangle has only one symmetry axis , and its side lengths are 8 cm. , 4 cm. , x cm. , then x = ...

   (a) 12 (b) 8 (c) 4 (d) 2

2. M and N are two intersecting circles , their radii lengths are 3 cm. and 5 cm. , then MN $\in$ ....

   (a) $]0 , 2[$ (b) $]2 , 8[$ (c) $]8 , \infty[$ (d) $]2 , \infty[$

3 In Δ ABC , if $(AB)^2 - 3 = (AC)^2 + (BC)^2$, then $\angle$ C is ....

(a) obtuse. (b) right. (c) straight. (d) acute.

4 The number of common tangents of two distant circles is ....

(a) 1 (b) 2 (c) 3 (d) 4

5 If the area of a square is 25 cm^2 , then its perimeter is .... cm.

(a) 4 (b) 10 (c) 14 (d) 20

6 A circle with diameter length $(2X + 5)$ cm. , the straight line L is at a distance $(X + 2)$ cm. from its centre , then the straight line L is ....

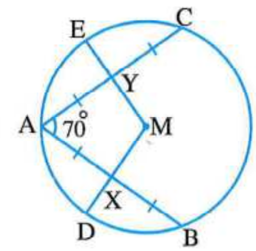
(a) a tangent. (b) a secant.

(c) outside the circle. (d) a diameter.

---

2 [a] **In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$, m ($\angle$ CAB) = 70°



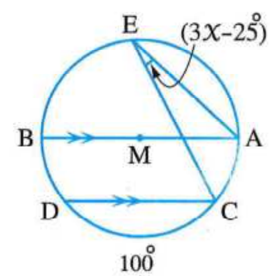
1 **Find :** m ($\angle$ EMD)

2 **Prove that :** XD = YE

[b] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M , $\overline{AB}$ // $\overline{CD}$, m ($\widehat{CD}$) = 100°

, m ($\angle$ AEC) = $3X - 25°$



Find : 1 The value of X 2 m ($\widehat{BD}$)

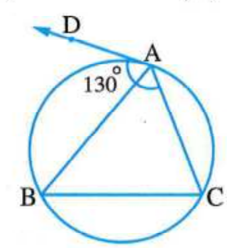
---

3 [a] Find the measure of the arc which represents $\frac{1}{4}$ the measure of the circle , then calculate the length of this arc if the radius length of the circle is 14 cm. $\left(\text{Where } \pi = \frac{22}{7}\right)$

[b] **In the opposite figure :**

$\overrightarrow{AD}$ is a tangent to the circle at A

, m ($\angle$ DAC) = 130°



Find by proof : m ($\angle$ B)

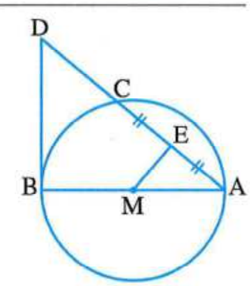
---

4 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M

, $\overline{BD}$ is a tangent-segment to the circle at B

, E is the midpoint of $\overline{AC}$



Prove that : The figure EMBD is a cyclic quadrilateral.

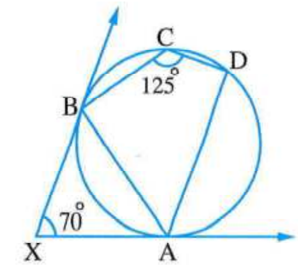
[b] In the opposite figure :

$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

, m (∠ X) = 70°

, m (∠ DCB) = 125°

Prove that : $\overrightarrow{AB}$ bisects ∠ DAX

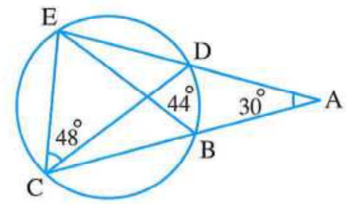


5 **[a] In the opposite figure :**

m (∠ A) = 30° , m $(\overset{\frown}{BD})$ = 44°

, m (∠ DCE) = 48°

Find : m $(\overset{\frown}{EC})$, m $(\overset{\frown}{BC})$

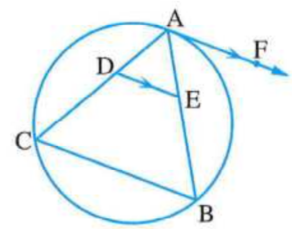


[b] In the opposite figure :

$\overrightarrow{AF}$ is a tangent to the circle at A

, $\overrightarrow{AF}$ // $\overline{DE}$

Prove that : BCDE is a cyclic quadrilateral.



16 Beni Suef Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer from those given :**

1. The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

   (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 2

2. The inscribed angle which is drawn in a semicircle is ....

   (a) right. (b) acute. (c) obtuse. (d) reflex.

3. The two diagonals are equal in length and not perpendicular in the ....

   (a) parallelogram. (b) rectangle. (c) rhombus. (d) square.

4. Two circles with centres M and N are intersecting , their radii lengths are 3 cm. and 5 cm. , then MN ∈ ....

   (a)]2 , 8[(b)]8 , ∞[(c)]0 , 2[(d)]2 , ∞[

5. ABCD is a cyclic quadrilateral in which m (∠ A) = 2 m (∠ C) , then m (∠ C) = ...

   (a) 60° (b) 30° (c) 90° (d) 120°

6 If two polygons are similar , the ratio between the lengths of two corresponding sides is 1 : 3 and the perimeter of the smaller polygon is 15 cm. , then the perimeter of the greater polygon is .... cm.

(a) 30 (b) 45 (c) 60 (d) 75

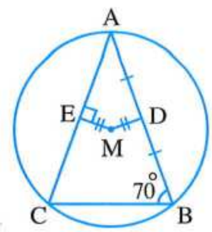
2 [a] **In the opposite figure :**

A circle of centre M , D is the midpoint of $\overline{AB}$

, $\overline{ME} \perp \overline{AC}$

, MD = ME and m (∠ B) = 70°

Find with proof : m (∠ A)

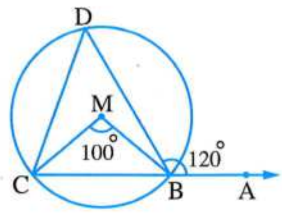


[b] **In the opposite figure :**

A circle of centre M , m (∠ BMC) = 100°

, m (∠ ABD) = 120°

Find with proof : m (∠ DCM)



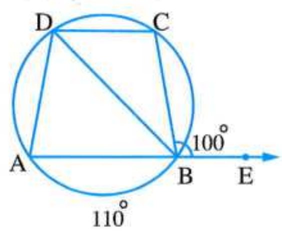
3 [a] **In the opposite figure :**

$E \in \overrightarrow{AB}$, $E \notin \overline{AB}$

, m ($\overset{\frown}{AB}$) = 110°

and m (∠ CBE) = 100°

Find : m (∠ BDC)

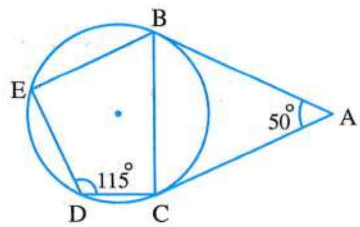


[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C

, m (∠ A) = 50° , m (∠ D) = 115°

Prove that : $\overrightarrow{BC}$ bisects ∠ ABE



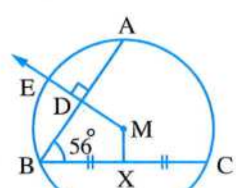
4 [a] **In the opposite figure :**

$\overline{AB}$, $\overline{CB}$ are two chords in the circle whose radius length is 5 cm.

, $\overrightarrow{MD} \perp \overline{AB}$ intersecting $\overline{AB}$ at D and intersecting the circle at E

, X is the midpoint of $\overline{BC}$, AB = 8 cm. , m (∠ ABC) = 56°

Find : m (∠ DMX) , the length of $\overline{DE}$

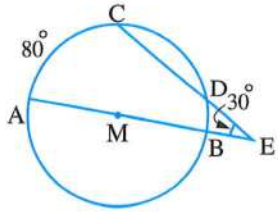


[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle , $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$

, m (∠ AEC) = 30° and m $(\widehat{AC})$ = 80°

Find : m $(\widehat{CD})$



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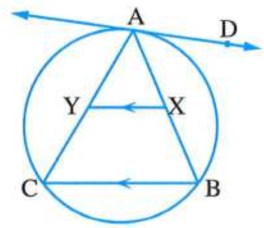
5 **[a] In the opposite figure :**

ABC is a triangle drawn inside a circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A

, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y



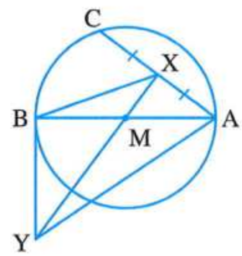
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, X is the midpoint of $\overline{AC}$ and $\overrightarrow{XM}$ intersects the tangent of the circle at B in Y

Prove that : 1 The figure AXBY is a cyclic quadrilateral.

2 Determine the centre of the circle passing through the vertices of the quadrilateral AXBY



17 El-Menia Governorate

Answer the following questions : ***(Calculators are allowed)***

1 **Choose the correct answer from those given :**

1 If ABCD is a cyclic quadrilateral , where m (∠ B) = 50° , then m (∠ D) = ...°

(a) 25° (b) 50° (c) 100° (d) 130°

2 The point of concurrence of the medians of the triangle divides the median by the ratio .... from the base.

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

3 The measure of the arc which represents half the measure of the circle equals ....

(a) 180° (b) 90° (c) 120° (d) 240°

4 In Δ ABC , if $(BC)^2 = (AB)^2 + (AC)^2$, then m (∠ A) = ...°

(a) 40° (b) 50° (c) 90° (d) 120°

5 The inscribed angle drawn in a semicircle is ....

(a) acute. (b) obtuse. (c) right (d) straight.

6 The angle of measure 20° is the complementary angle of the angle of measure ....

(a) 20° (b) 40° (c) 70° (d) 120°

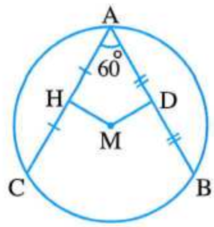
---

2 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords in the circle M

, D is the midpoint of $\overline{AB}$, H is the midpoint of $\overline{AC}$

, m (∠ BAC) = 60° **Find :** m (∠ DMH)



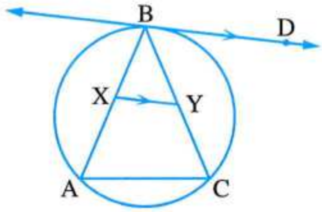
[b] **In the opposite figure :**

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B , X ∈ $\overline{AB}$

and Y ∈ $\overline{BC}$, where $\overline{XY}$ // $\overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.



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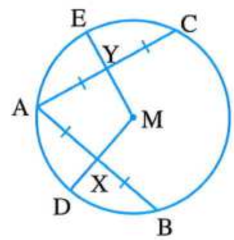
3 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$

and Y is the midpoint of $\overline{AC}$

Prove that : XD = YE

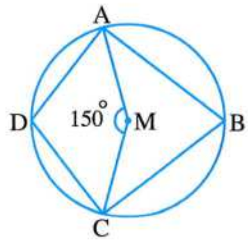


[b] **In the opposite figure :**

M is a circle

, m (∠ CMA) = 150°

Find : m (∠ CDA)



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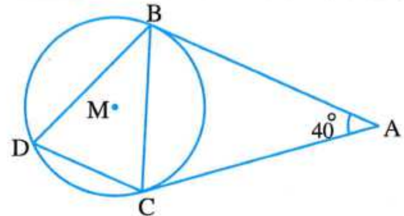
4 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C , m (∠ A) = 40°

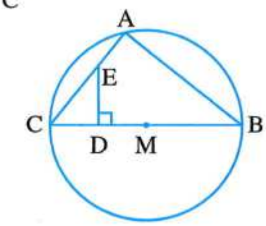
Find with proof : m (∠ D)



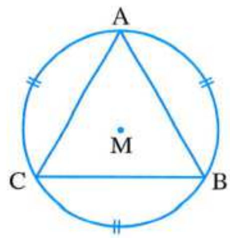
[b] **In the opposite figure :**

$\overline{BC}$ is a diameter in the circle M , $\overline{ED}$ ⊥ $\overline{BC}$

Prove that : ABDE is a cyclic quadrilateral.



5 [a] **In the opposite figure :**

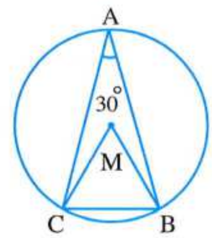


A , B and C are three points lie on the circle M where

m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{CA})$

Find by proof : m (∠ A)

[b] **In the opposite figure :**



ABC is a triangle inscribed in the circle M

, m (∠ A) = 30°

Prove that : Δ MBC is an equilateral triangle.

18 Assiut Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer :**

1 The number of circles which passes through three collinear points is ....

(a) zero. (b) 1 (c) 3 (d) infinite.

2 A square has a surface area of 50 cm^2 , then the length of its diagonal is .... cm.

(a) 5 (b) 10 (c) 15 (d) 25

3 ABC is a triangle , $(AC)^2 > (AB)^2 + (BC)^2$, then ∠ ABC is ....

(a) obtuse. (b) acute. (c) right. (d) straight.

4 The measure of the arc which equals third the measure of the circle is ....

(a) 60° (b) 90° (c) 120° (d) 240°

5 ABCD is a cyclic quadrilateral , m (∠ A) = 3 m (∠ C) , then m (∠ A) = ...

(a) 90° (b) 45° (c) 135° (d) 120°

6 The measure of the reflex angle of the angle that is measured 100° equals ....

(a) 80° (b) 90° (c) 200° (d) 260°

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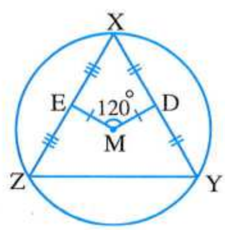
2 [a] **In the opposite figure :**



XYZ is a triangle inscribed in a circle M

, D , E are the midpoints of $\overline{XY}$, $\overline{XZ}$ respectively

, MD = ME , m (∠ DME) = 120°

Prove that : Δ XYZ is an equilateral triangle.

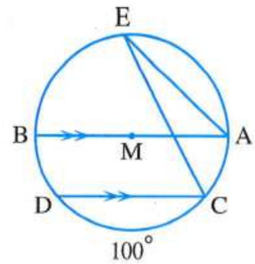
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overline{AB} // \overline{CD}$

, m $(\overset{\frown}{CD}) = 100°$

Find with proof : m (∠ AEC)



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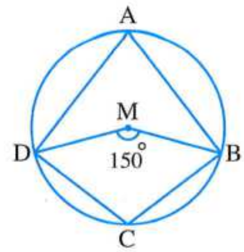
3 **[a] In the opposite figure :**

A circle with centre M

, m (∠ BMD) = 150°

Find with proof : m (∠ BCD)



[b] In the opposite figure :

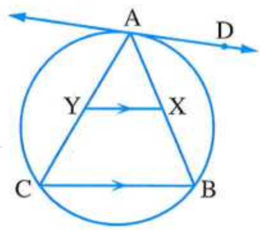
$\overleftrightarrow{AD}$ is a tangent to the circle at A

, ABC is a triangle inscribed in the circle

, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$

, $\overline{XY} // \overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y



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4 **[a]** Two circles M and N with radii lengths 8 cm. and 6 cm. respectively

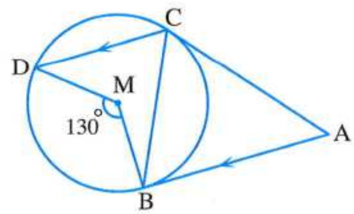
Find the length of $\overline{MN}$ in each of the following cases :

1. The two circles are touching externally.
2. The two circles are touching internally.
3. The two circles are concentric.

[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M at B , C , $\overline{AB} // \overline{CD}$, m (∠ BMD) = 130°

Find with proof : m (∠ A)



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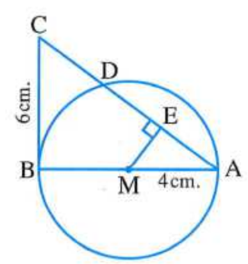
5 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overline{BC}$ is a tangent-segment to the circle at B

, $\overline{ME} \perp \overline{AD}$, AM = 4 cm. , BC = 6 cm.

1. **Prove that :** EMBC is a cyclic quadrilateral.
2. **Find :** The length of $\overline{AC}$

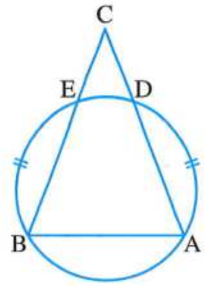


[b] **In the opposite figure :**

m $(\widehat{AD})$ = m $(\widehat{BE})$

, $\overrightarrow{AD} \cap \overrightarrow{BE} = \{C\}$

Prove that : AC = BC



19 Souhag Governorate

Answer the following questions : ***(Calculators are allowed)***

1 Choose the correct answer :

1 The rhombus in which the lengths of its diagonals are 12 cm. , 18 cm. , its area is .... cm^2.

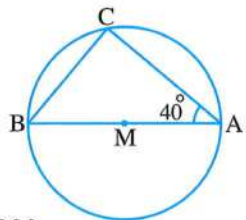
(a) 108 (b) 216 (c) 54 (d) 30

2 **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M

, m (∠ CAB) = 40°

, then m $(\widehat{AC})$ = ...



(a) 50° (b) 40° (c) 100° (d) 80°

3 If M , N are two circles touching externally , the lengths of their radii are 3 cm. and 5 cm. , then MN = ... cm.

(a) 2 (b) 4 (c) 8 (d) 15

4 The number of axes of symmetry of a circle is ....

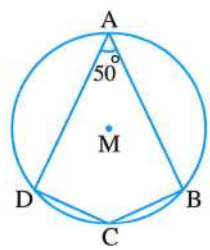
(a) an infinite number. (b) zero.

(c) single axis. (d) three axes.

5 **In the opposite figure :**

If M is a circle , m (∠ BAD) = 50°

, then m (∠ BCD) = ...



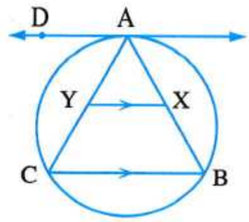
(a) 50° (b) 130°

(c) 260° (d) 65°

6 The length of the opposite side of the angle with measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

2 [a] **In the opposite figure :**

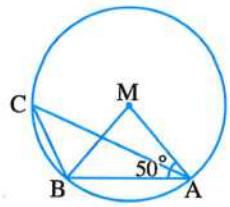


Δ ABC is inscribed in a circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A , $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

[b] **In the opposite figure :**

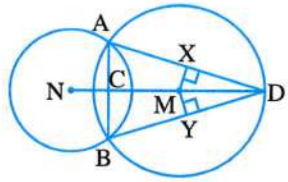


If M is a circle

, m ($\angle$ MAB) = 50°

, find with proof : 1 m ($\angle$ ACB) 2 m ($\widehat{ACB}$)

3 [a] **In the opposite figure :**



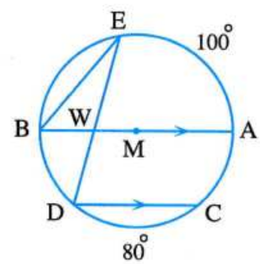
The circle M $\cap$ the circle N = $\{A , B\}$

, $\overline{AB} \cap \overleftrightarrow{MN} = \{C\}$, D $\in \overleftrightarrow{MN}$

, $\overline{MX} \perp \overline{AD}$ and $\overline{MY} \perp \overline{BD}$

Prove that : MX = MY

[b] **In the opposite figure :**



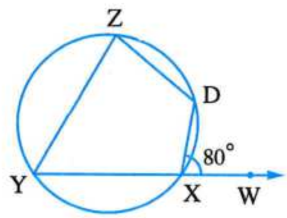
$\overline{AB}$ is a diameter of the circle M

, $\overline{AB}$ // $\overline{CD}$, m ($\widehat{CD}$) = 80°

and m ($\widehat{AE}$) = 100°

Find with proof : m ($\angle$ DEB) , m ($\angle$ AWE)

4 [a] **In the opposite figure :**



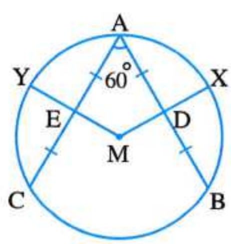
XYZD is a cyclic quadrilateral

, W $\in \overrightarrow{YX}$ where m ($\angle$ WXD) = 80°

, m ($\angle$ Y) = $\frac{1}{2}$ m ($\angle$ D)

Find with proof : 1 m ($\angle$ Z) 2 m ($\angle$ D)

[b] **In the opposite figure :**



$\overline{AB}$, $\overline{AC}$ are two chords equal in length in the circle M

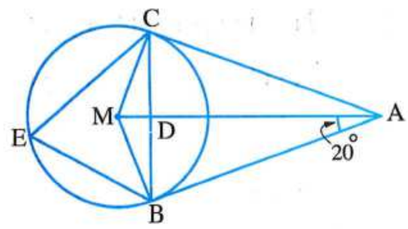
, D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

, m ($\angle$ BAC) = 60°

1 **Find with proof :** m ($\angle$ XMY)

2 **Prove that :** XD = YE

5 [a] **In the opposite figure :**



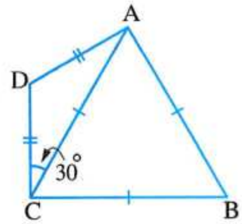
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AM} \cap \overline{BC} = \{D\}$, m (∠ BAM) = 20°

Find with proof : 1 m (∠ ACB)

2 m (∠ BEC)

[b] **In the opposite figure :**



ABCD is a quadrilateral , AB = BC = AC

, AD = DC , m (∠ ACD) = 30°

Prove that : ABCD is a cyclic quadrilateral.

20 Qena Governorate

Answer the following questions : ***(Calculators are permitted)***

1 **Choose the correct answer from those given :**

1 A tangent to a circle of diameter length 8 cm. is at a distance of .... cm. from its centre.

(a) 3 (b) 4 (c) 6 (d) 8

2 The sum of measures of the interior angles of the quadrilateral equals ....

(a) 180° (b) 270° (c) 360° (d) 720°

3 The inscribed angle opposite to the greatest arc in a circle is ....

(a) acute. (b) right. (c) obtuse. (d) reflex.

4 The number of diagonals of the pentagon is ....

(a) 3 (b) 5 (c) 7 (d) 9

5 A circle can be drawn passing through the vertices of a ....

(a) rectangle. (b) trapezoid. (c) rhombus. (d) parallelogram.

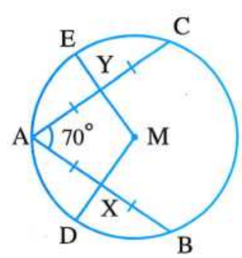
6 The area of a square is 100 cm^2 , then its perimeter is .... cm.

(a) 10 (b) 20 (c) 30 (d) 40

---

2 [a] Find the length and the measure of the arc which is opposite to an inscribed angle of measure 45° in a circle of radius length 7 cm. $\left(\pi \simeq \frac{22}{7}\right)$

[b] **In the opposite figure :**



$\overline{AB}$, $\overline{AC}$ are two chords equal in length in the circle M

, X , Y are the midpoints of $\overline{AB}$, $\overline{AC}$

respectively , m (∠ CAB) = 70°

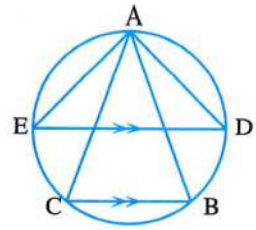
1 **Find :** m (∠ DME) 2 **Prove that :** XD = YE

3 [a] **In the opposite figure :**

ABC is a triangle drawn in a circle

, $\overline{DE} \parallel \overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)



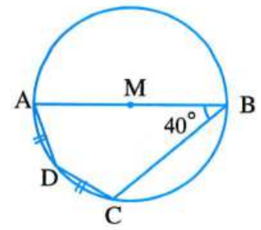
[b] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, D is the midpoint of $\overset{\frown}{AC}$

, m (∠ ABC) = 40°

Find by proof : m (∠ DAB)



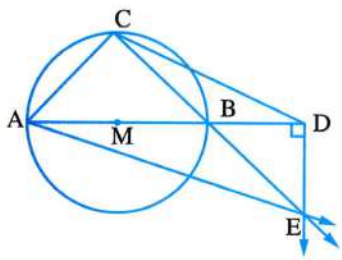
4 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $D \in \overrightarrow{AB}$, $D \notin \overline{AB}$, $\overrightarrow{DE} \perp \overrightarrow{AB}$

, $C \in \overset{\frown}{AB}$, $\overrightarrow{CB} \cap \overrightarrow{DE} = \{E\}$

Prove that : ACDE is a cyclic quadrilateral.

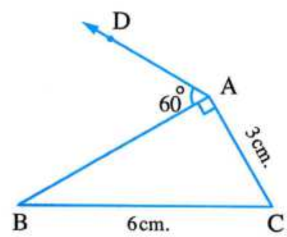


[b] **In the opposite figure :**

Δ ABC is a right-angled triangle at A

, AC = 3 cm. , BC = 6 cm. , m (∠ BAD) = 60°

Prove that : $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC



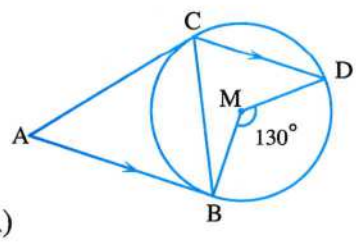
5 [a] **In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M , $\overline{AB} \parallel \overline{CD}$

, m (∠ BMD) = 130°

1 **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD

2 **Find :** m (∠ A)



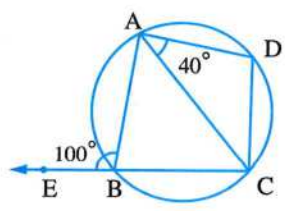
[b] **In the opposite figure :**

ABCD is a quadrilateral drawn inside a circle

, m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{AD}$)



21 Luxor Governorate

Answer the following questions :

1 Choose the correct answer :

1 If the two circles M and N are touching internally , the radius length of the circle N = 3 cm. and MN = 5 cm. , then the radius length of the circle M = ... cm.

(a) 2 (b) 8 (c) 5 (d) 9

2 The number of the common tangents of two distant circles is ....

(a) 1 (b) 2 (c) 3 (d) 4

3 If the figure ABCD is a cyclic quadrilateral , m (∠ A) = 70° , then m (∠ C) = ...

(a) 70° (b) 110° (c) 90° (d) 180°

4 The area of a triangle = 30 cm^2 and one of its heights = 6 cm. , then the length of the base which is corresponding to this height = ... cm.

(a) 30 (b) 6 (c) 10 (d) 12

5 If the length of the diagonal of a square is 12 cm. , then its area is .... cm^2

(a) 72 (b) 144 (c) 12 (d) 24

6 The sum of the exterior angles of the triangle equals ....

(a) 120° (b) 180° (c) 270° (d) 360°

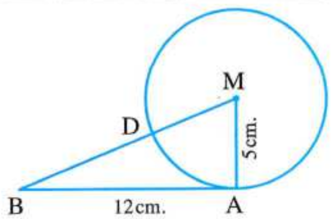
---

2 [a] In the opposite figure :

MA = 5 cm. , AB = 12 cm.

, $\overline{AB}$ is a tangent-segment to the circle M at A

Find : The length of $\overline{BD}$



[b] Using your geometric tools , draw $\overline{AB}$ with length 6 cm. , then draw a circle passing through the two points A and B whose radius length is 5 cm. How many circles can be drawn ?

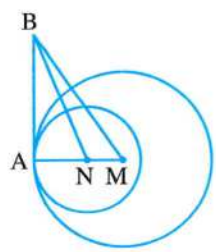
---

3 [a] In the opposite figure :

M and N are two circles with radii lengths 10 cm. and 6 cm. respectively and they are touching internally at A

, $\overline{AB}$ is a common tangent-segment for both.

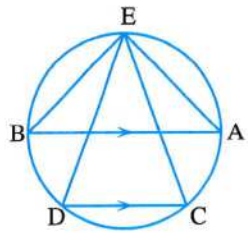
If the area of Δ BMN = 24 cm^2 , **find :** the length of $\overline{AB}$



[b] In the opposite figure :

$\overline{AB} \parallel \overline{CD}$

Prove that : m (∠ AED) = m (∠ BEC)



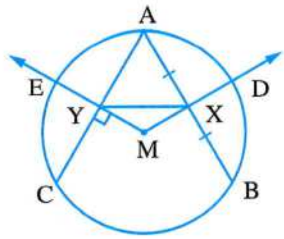
4 **[a] In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two chords equal in length in the circle M

, X is the midpoint of $\overline{AB}$, $\overrightarrow{MX}$ intersects the circle at D

, $\overrightarrow{MY} \perp \overline{AC}$ intersecting it at Y and intersecting the circle at E

Prove that : XD = YE , m (∠ YXB) = m (∠ XYC)



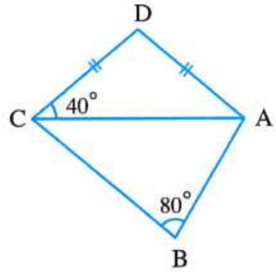
[b] In the opposite figure :

AD = CD

, m (∠ ACD) = 40°

, m (∠ B) = 80°

Prove that : ABCD is a cyclic quadrilateral.



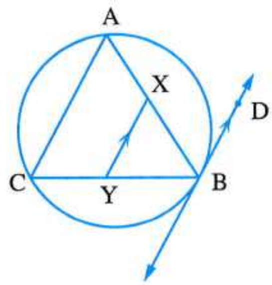
5 **[a] In the opposite figure :**

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, X ∈ $\overline{AB}$, Y ∈ $\overline{BC}$, where $\overline{XY} \parallel \overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.



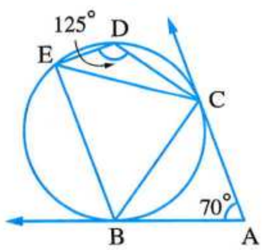
[b] In the opposite figure :

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 70°

, m (∠ D) = 125°

Prove that : 1 BC = CE   2 $\overrightarrow{AC} \parallel \overline{BE}$



22 Aswan Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer from those given :**

1 A square , its side length is 6 cm. , then its surface area is .... cm^2

(a) 12   (b) 24   (c) 36   (d) 48

2 ABCD is a cyclic quadrilateral in which m (∠ B) = 70° , then m (∠ D) = ...

(a) 50° (b) 70° (c) 100° (d) 110°

3 The measure of the exterior angle of an equilateral triangle at one of its vertices equals ....

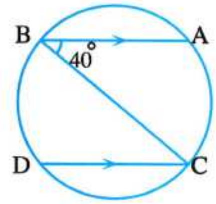
(a) 120° (b) 100° (c) 60° (d) 30°

4 Two circles M and N , the lengths of their two radii are 9 cm. and 5 cm. If MN = 6 cm. , then the two circles are ....

(a) touching externally. (b) intersecting.

(c) distant. (d) touching internally.

5 **In the opposite figure :**



$\overline{AB} // \overline{CD}$, m (∠ B) = 40°

, then m ($\overset{\frown}{BD}$) = ...

(a) 20° (b) 40°

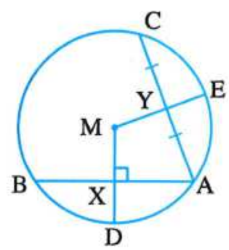
(c) 80° (d) 100°

6 The length of the side which is opposite to the angle with measure 30° in a right-angled triangle is equal to .... the length of the hypotenuse.

(a) double (b) third (c) quarter (d) half

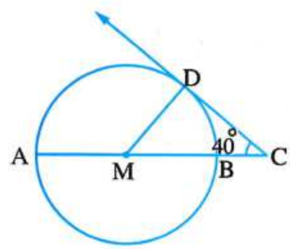
---

2 [a] **In the opposite figure :**



$\overline{AB}$, $\overline{AC}$ are two chords equal in length in the circle M , Y is the midpoint of $\overline{AC}$, $\overrightarrow{MY}$ intersects the circle M at E , $\overrightarrow{MX} \perp \overline{AB}$ intersecting it at X and intersecting the circle M at D **Prove that :** YE = XD

[b] **In the opposite figure :**



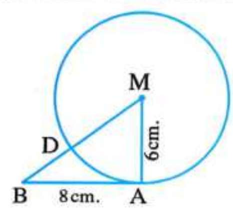
$\overrightarrow{CD}$ is a tangent to the circle M at D

, m (∠ C) = 40°

Find : m ($\overset{\frown}{AD}$) the smaller.

---

3 [a] **In the opposite figure :**



$\overline{AB}$ is a tangent-segment to the circle M at A

, MA = 6 cm. , AB = 8 cm.

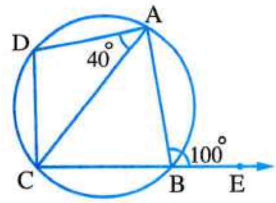
Find : The length of $\overline{DB}$

[b] In the opposite figure :

$E \in \overrightarrow{CB}$, m (∠ ABE) = 100°

, m (∠ DAC) = 40°

Prove that : m $(\overset{\frown}{DA})$ = m $(\overset{\frown}{DC})$

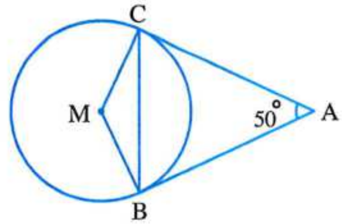


4 **[a] In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M

, m (∠ A) = 50°

Find : m (∠ ABC) , m (∠ BMC)

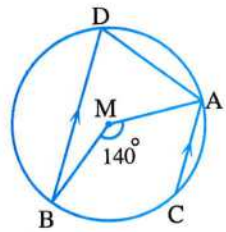


[b] In the opposite figure :

$\overline{AC}$ // $\overline{DB}$

, m (∠ AMB) = 140°

Find : m (∠ CAD)



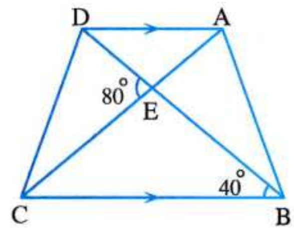
5 **[a] In the opposite figure :**

$\overline{AC} \cap \overline{BD} = \{E\}$, $\overline{AD}$ // $\overline{BC}$

, m (∠ DBC) = 40°

, m (∠ DEC) = 80°

Prove that : ABCD is a cyclic quadrilateral.

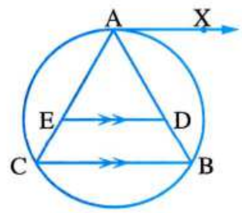


[b] In the opposite figure :

Δ ABC is drawn inscribed in the circle

, $\overrightarrow{AX}$ is a tangent to the circle , $\overline{DE}$ // $\overline{BC}$

Prove that : $\overrightarrow{AX}$ is a tangent to the circle passing through the points A , D and E



23 New Valley Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The number of axes of symmetry of an isosceles triangle equals ....

(a) zero. (b) 1 (c) 2 (d) 3

2 A tangent to a circle of diameter length 6 cm. is at a distance of .... cm. from its centre.

(a) 12 (b) 6 (c) 3 (d) 2

3 If $\tan(x + 10°) = \sqrt{3}$ where x is the measure of an acute angle , then $x =$ ....

(a) 30° (b) 45° (c) 50° (d) 60°

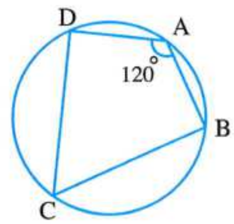
4 M and N are two intersecting circles , both their radii lengths are 3 cm. and 5 cm. , then MN ∈ ....

(a)]8 , ∞[(b)]− ∞ , 2[(c)]0 , 2[(d)]2 , 8[

5 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 45° (b) 90° (c) 120° (d) 180°

6 **In the opposite figure :**



If m (∠ A) = 120°

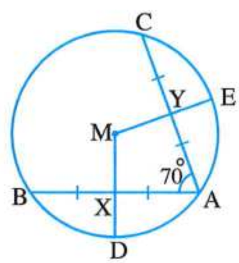
, then m (∠ C) = ...

(a) 60° (b) 90°

(c) 120° (d) 180°

---

2 [a] **In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M

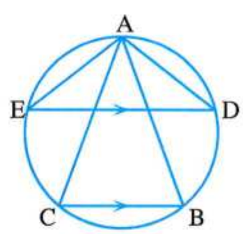
, X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$

, m (∠ CAB) = 70°

1 **Calculate :** m (∠ DME)

2 **Prove that :** XD = YE

[b] **In the opposite figure :**



ABC is an inscribed triangle inside a circle

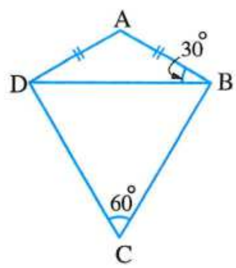
, $\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)

---

3 [a] State two cases of cyclic quadrilateral.

[b] **In the opposite figure :**

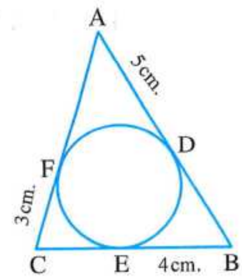


ABCD is a quadrilateral in which AB = AD

, m (∠ ABD) = 30° , m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.

4 **[a] In the opposite figure :**



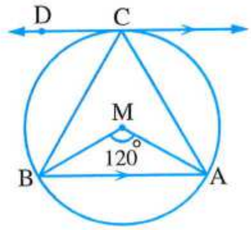
A circle is drawn touching the sides of the triangle ABC

$\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at D , E , F respectively

, AD = 5 cm. , BE = 4 cm. , CF = 3 cm.

Find : The perimeter of Δ ABC

[b] In the opposite figure :



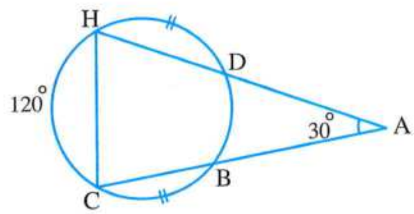
$\overleftrightarrow{CD}$ is a tangent to the circle at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$, m (∠ AMB) = 120°

Prove that : The triangle CAB is an equilateral triangle.

---

5 **[a] In the opposite figure :**



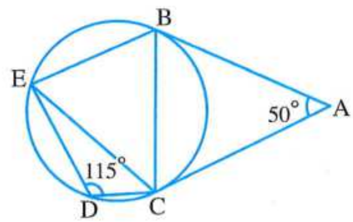
m (∠ A) = 30° , m ($\overset{\frown}{CH}$) = 120°

, m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{DH}$)

1 **Find :** m ($\overset{\frown}{DB}$) the minor arc.

2 **Prove that :** AB = AD

[b] In the opposite figure :



$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle

at B and C , m (∠ A) = 50°

, m (∠ CDE) = 115°

Prove that : **1** $\overrightarrow{BC}$ bisects ∠ ABE

2 CB = CE

24 South Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If the circumference of a circle = 8 π cm. , then the length of its diameter = ... cm.

(a) 2 (b) 4 (c) 8 (d) 16

2 If the length of the base of a triangle is 16 cm. and its corresponding height is 9 cm.

, then its area = ... cm^2.

(a) 25 (b) 72 (c) 36 (d) 144

3 The centre of the circumcircle of a triangle is the intersection point of ....

(a) the axes of symmetry of its sides. (b) its heights.

(c) the bisectors of its interior angles. (d) its medians.

4 M and N are two circles touching externally , the two radii lengths are 3 cm. and 5 cm. , then MN = ... cm.

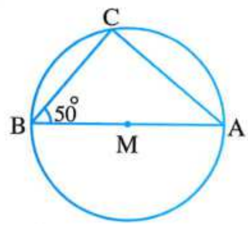
(a) 8 (b) 5 (c) 2 (d) 3

5 **In the opposite figure :**

If $\overline{AB}$ is a diameter in the circle M

, m (∠ B) = 50°

, then m (∠ A) = ...



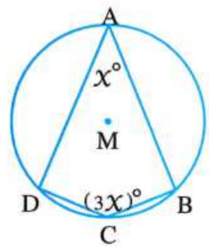
(a) 40° (b) 50° (c) 90° (d) 100°

6 **In the opposite figure :**

If m (∠ A) = x°

, m (∠ C) = (3 x)°

, then x = ...



(a) 15° (b) 45° (c) 95° (d) 135°

---

2 $\overline{AB}$ and $\overline{AC}$ are two chords equal in length a circle M , and $\overrightarrow{MX} \perp \overline{AB}$ intersecting it at D and intersecting the circle at X , $\overrightarrow{MY} \perp \overline{AC}$ intersecting it at E and intersecting the circle at Y

Prove that : DX = EY

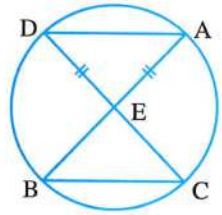
---

3 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{DC}$ are two chords intersecting at E

, AE = DE

Prove that : $\overline{AD}$ // $\overline{CB}$



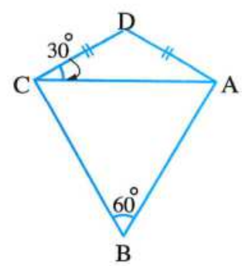
[b] **In the opposite figure :**

AD = CD

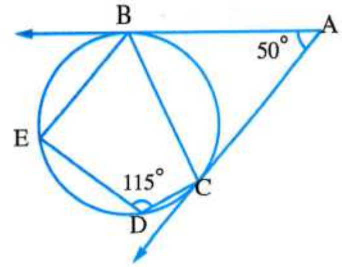
, m (∠ ACD) = 30°

, m (∠ B) = 60°

Prove that : The figure ABCD is a cyclic quadrilateral.



4 **[a] In the opposite figure :**



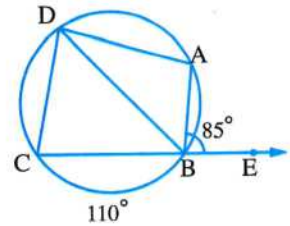
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 50°

, m (∠ CDE) = 115°

Prove that : $\overrightarrow{BC}$ bisects ∠ ABE

[b] In the opposite figure :



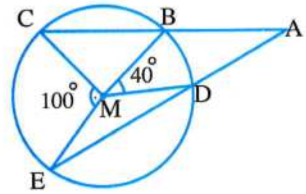
E ∈ $\overrightarrow{CB}$, such that m (∠ ABE) = 85°

, m $(\widehat{BC})$ = 110°

Find : m (∠ ADB)

---

5 **[a] In the opposite figure :**

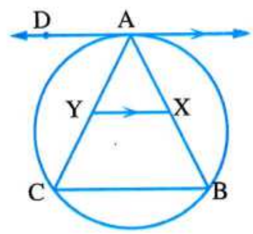


M is a circle , $\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$

, m (∠ BMD) = 40° , m (∠ EMC) = 100°

Find : m (∠ A)

[b] In the opposite figure :



$\overleftrightarrow{AD}$ is a tangent to the circle

, $\overleftrightarrow{AD}$ // $\overline{XY}$

Prove that : The figure XYCB is a cyclic quadrilateral.

25 North Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

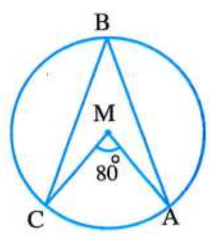
1 The measure of the inscribed angle drawn in a semicircle is ....

(a) 180° (b) 90° (c) 45° (d) 60°

2 The measure of the exterior angle of an equilateral triangle is ....

(a) 60° (b) 120° (c) 180° (d) 45°

3 **In the opposite figure :**



The measure of the central angle ∠ AMC = 80°

, then m (∠ ABC) = ...

(a) 80° (b) 160° (c) 40° (d) 20°

4 The measure of the supplementary angle of the of angle of measure 70° equals ....

(a) 70° (b) 20° (c) 110° (d) 290°

5 M , N are two circles touching externally , the radii lengths of them are 8 cm. , 5 cm. , then MN = ... cm.

(a) 3 (b) 13 (c) 8 (d) 5

6 The point of intersection of the medians of the triangle divides each of them by the ratio .... from the vertex.

(a) 1 : 2 (b) 2 : 3 (c) 3 : 1 (d) 2 : 1

---

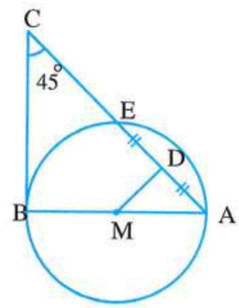
2 [a] **In the opposite figure :**

$\overline{BC}$ is a tangent-segment to the circle M at B

, D is the midpoint of $\overline{AE}$

, m (∠ ACB) = 45°

Prove that : MD = AD



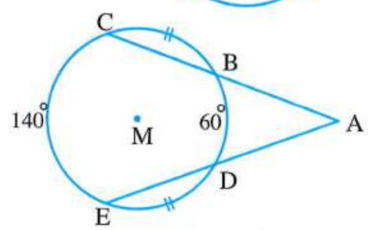
[b] **In the opposite figure :**

m $(\overset{\frown}{ED})$ = m $(\overset{\frown}{BC})$

, m $(\overset{\frown}{BD})$ = 60°

, m $(\overset{\frown}{EC})$ = 140°

Find with proof : 1 m (∠ A) 2 m $(\overset{\frown}{BC})$



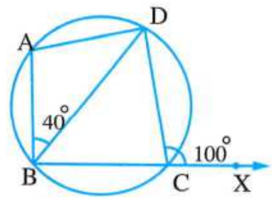
---

3 [a] **In the opposite figure :**

X ∈ $\overrightarrow{BC}$, m (∠ DCX) = 100°

, m (∠ ABD) = 40°

Prove that : AB = AD



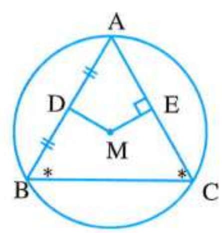
[b] **In the opposite figure :**

ABC is a triangle drawn inside a circle M

, m (∠ B) = m (∠ C) , D is the midpoint of $\overline{AB}$

, $\overline{ME} \perp \overline{AC}$

Prove that : MD = ME



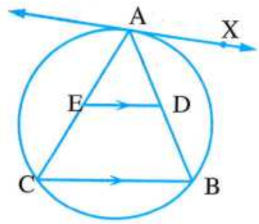
---

4 [a] **In the opposite figure :**

ABC is a triangle drawn inside a circle

, $\overleftrightarrow{AX}$ is a tangent to the circle at A , $\overline{DE}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AX}$ is a tangent to the circle passing through the vertices of the triangle ADE

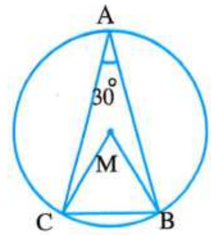


[b] In the opposite figure :

A circle M in which

m (∠ A) = 30°

Prove that : Δ MBC is an equilateral triangle.



---

5 **[a] In the opposite figure :**

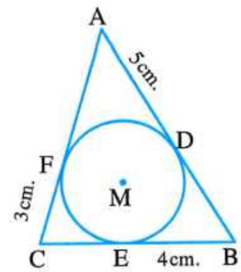
M is the inscribed circle of the triangle ABC ,

it touches its sides $\overline{AB}$, $\overline{BC}$, $\overline{AC}$

at D , E , F respectively

, AD = 5 cm. , BE = 4 cm. , CF = 3 cm.

Find : The perimeter of the triangle ABC

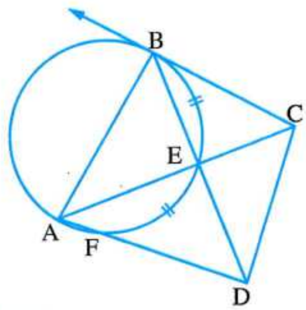


[b] In the opposite figure :

$\overrightarrow{CB}$ is a tangent to the circle at B

, E is the midpoint of $\overset{\frown}{BF}$

Prove that : ABCD is a cyclic quadrilateral.



26 Red Sea Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 ABCD is a cyclic quadrilateral in which , m (∠ A) = 40° , then m (∠ C) = ...

(a) 40° (b) 50° (c) 320° (d) 140°

2 The sum of measures of the interior angles of the triangle equals ....

(a) 60° (b) 120° (c) 180° (d) 360°

3 M and N are two intersecting circles , both their radii lengths are 4 cm. and 7 cm. , then MN ∈ ....

(a)]11 , ∞[(b)]3 , ∞[(c)]0 , 3[(d)]3 , 11[

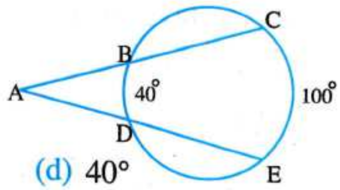
4 A circle , its radius length = 8 cm. , then its circumference = ... cm.

(a) 4 π (b) 16 π (c) 64 π (d) 36 π

5 A square , its side length = 5 cm. , then its area = ... cm^2.

(a) 25 (b) 20 (c) 10 π (d) 25 π

6 **In the opposite figure :**



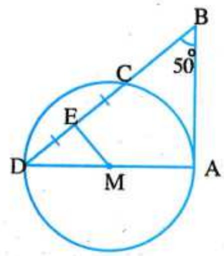
$m(\overset{\frown}{CE}) = 100°$, $m(\overset{\frown}{BD}) = 40°$

, then $m(\angle A) =$ ....

(a) 50° (b) 30° (c) 20° (d) 40°

---

2 **[a] In the opposite figure :**



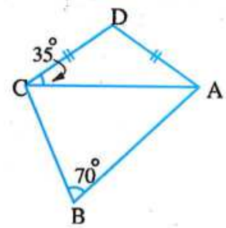
$\overline{AD}$ is a diameter in the circle M

, $\overline{AB}$ is a tangent-segment to the circle at A

, E is the midpoint of $\overline{DC}$, $m(\angle B) = 50°$

Find with proof : $m(\angle AME)$

[b] In the opposite figure :



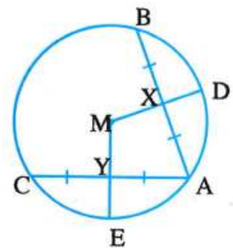
DA = DC

, $m(\angle ACD) = 35°$, $m(\angle B) = 70°$

Prove that : ABCD is a cyclic quadrilateral.

---

3 **[a] In the opposite figure :**



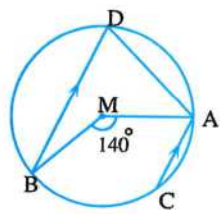
M is a circle , AB = AC

, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$

Prove that : XD = YE

[b] In the opposite figure :



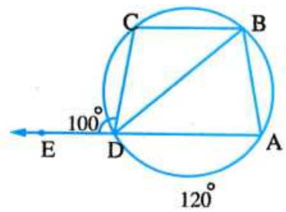
M is a circle , $m(\angle AMB) = 140°$

, $\overline{AC} \parallel \overline{DB}$

Find with proof : **1** $m(\angle D)$ **2** $m(\angle DAC)$

---

4 **[a] In the opposite figure :**



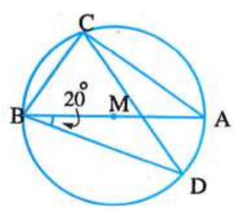
ABCD is a quadrilateral inscribed in a circle

, $E \in \overrightarrow{AD}$, $m(\angle CDE) = 100°$

, $m(\overset{\frown}{AD}) = 120°$

Find with proof : **1** $m(\angle ABC)$ **2** $m(\angle CBD)$

[b] In the opposite figure :

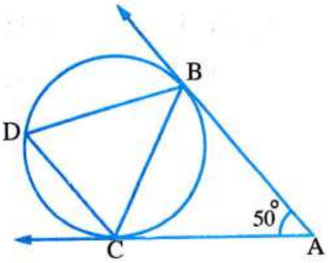


$\overline{AB}$ is a diameter in the circle M

, $m(\angle ABD) = 20°$

Find with proof : **1** $m(\angle ACB)$ **2** $m(\angle BCD)$

5 **[a] In the opposite figure :**



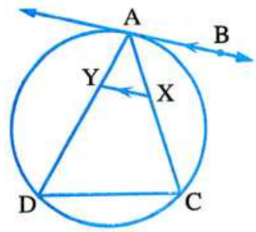
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 50°

Find with proof : 1 m (∠ ABC)

2 m (∠ D)

[b] In the opposite figure :



$\overleftrightarrow{AB}$ is a tangent to the circle at A

, $\overleftrightarrow{AB}$ // $\overline{YX}$

Prove that : XCDY is a cyclic quadrilateral.

27 Matrouh Governorate

Answer the following questions : ***(Calculators are allowed)***

1 **Choose the correct answer from those given :**

1 The measure of an inscribed angle is .... the measure of the central angle , subtended by the same arc.

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{5}$

2 The circumference of a circle equals .... length unit.

(a) πr^2 (b) πr (c) $2 \pi r$ (d) $2 \pi r^2$

3 The number of symmetry axes of a circle equals ....

(a) 1 (b) 2 (c) 4 (d) an infinite number.

4 ABCD is a cyclic quadrilateral , which has m (∠ A) = 60° , then m (∠ C) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

5 The area of a rhombus with a diagonal lengths of 6 cm. , 8 cm. equals ....

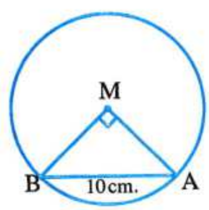
(a) 48 cm. (b) 48 cm². (c) 24 cm. (d) 24 cm².

6 If the two circles M , N are touching externally , the radius length of one of them is 5 cm. , MN = 9 cm. , then the radius length of the other circle equals ....

(a) 3 cm. (b) 4 cm. (c) 7 cm. (d) 14 cm.

---

2 **[a] In the opposite figure :**

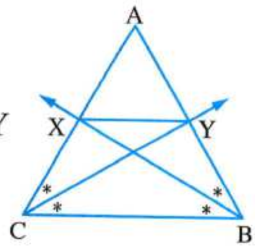


$\overline{AB}$ is a chord in the circle M with length 10 cm.

, m (∠ AMB) = 90°

Find : 1 m (∠ A) 2 The length of $\overline{MA}$

[b] In the opposite figure :



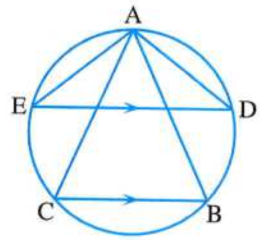
ABC is a triangle in which AB = AC , $\overrightarrow{BX}$ bisects ∠ ABC and intersects $\overline{AC}$ at X , $\overrightarrow{CY}$ bisects ∠ ACB and intersects $\overline{AB}$ at Y

Prove that : 1 BCXY is a cyclic quadrilateral.

2 $\overleftrightarrow{XY}$ // $\overleftrightarrow{BC}$

---

3 **[a] In the opposite figure :**

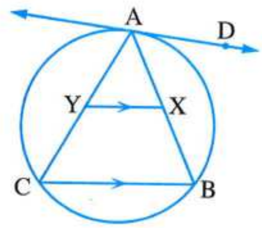


ABC is an inscribed triangle inside a circle

, $\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)

[b] In the opposite figure :



ABC is a triangle inscribed in a circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A

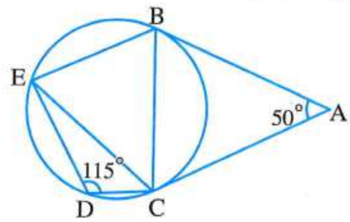
, X ∈ $\overline{AB}$

, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

---

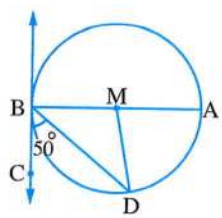
4 **[a] In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C , m (∠ A) = 50° , m (∠ CDE) = 115°

Prove that : 1 $\overrightarrow{BC}$ bisects ∠ ABE 2 CB = CE

[b] In the opposite figure :



$\overline{AB}$ is a diameter in the circle M

, $\overleftrightarrow{BC}$ is a tangent at B , m (∠ DBC) = 50°

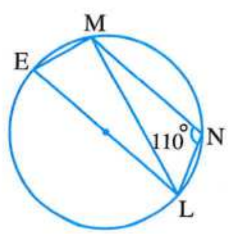
Find : m (∠ AMD)

---

5 **[a]** $\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , m (∠ MXY) = 30°

Prove that : 1 MXY is an isosceles triangle. 2 AXY is an equilateral triangle.

[b] In the opposite figure :



$\overline{LE}$ is a diameter of the circle

, m (∠ MNL) = 110°

Find : m (∠ MLE)

Answers of governorates' examinations of geometry

1 Cairo

1

1 c 2 d 3 b 4 c 5 d 6 a

2

[a] In Δ AHD : $\because$ HA = HD

$\therefore$ m ($\angle$ HAD) = m ($\angle$ HDA) = 35°

$\therefore$ m ($\angle$ H) = 180° – (35° + 35°) = 110° (First req.)

$\therefore$ m ($\angle$ H) = m ($\angle$ ABC) = 110°

$\therefore$ ABDH is a cyclic quadrilateral. (Second req.)

[b] m ($\angle$ ABD) = 180° – 140° = 40° (First req.)

, $\because$ $\overleftrightarrow{AC}$ is a tangent

$\therefore$ m ($\angle$ H) (inscribed) = m ($\angle$ ABD) (tangency)

$\therefore$ m ($\angle$ H) = 40° (Second req.)

3

[a] $\because$ $\overline{XY}$ is a diameter.

$\therefore$ m ($\angle$ XZY) = 90° (First req.)

$\because$ m ($\angle$ YZL) = $\frac{1}{2}$ m ($\angle$ YML)

(inscribed and central angles subtended by $\overset{\frown}{YL}$)

$\therefore$ m ($\angle$ YZL) = $\frac{1}{2}$ × 60° = 30° (Second req.)

[b]

4

[a] $\because$ m ($\angle$ BAC) = m ($\angle$ DAC) = 50°

$\therefore$ m ($\angle$ BAD) = 2 × 50° = 100°

, $\because$ ABCD is a cyclic quadrilateral.

$\therefore$ m ($\angle$ BCD) = 180° – 100° = 80° (The req.)

[b] $\because$ $\overline{AC}$ is the common chord.

, $\overleftrightarrow{MN}$ is the line of centres. $\therefore$ $\overleftrightarrow{MN} \perp \overline{AC}$

, $\because$ MX = MO (two radii of circle M)

, HX = DO $\therefore$ MH = MD

, $\because$ $\overrightarrow{MH} \perp \overline{AB}$, $\overline{MD} \perp \overline{AC}$

$\therefore$ AB = AC (Q.E.D.)

5

[a] m ($\angle$ AMB) = m ($\angle$ CMD) = 40° (V.O.A)

(First req.)

, $\because$ m ($\angle$ AMB) = 40° $\therefore$ m ($\overset{\frown}{AB}$) = 40°

, $\because$ $\overline{AD}$ // $\overline{BH}$

$\therefore$ m ($\overset{\frown}{DH}$) = m ($\overset{\frown}{AB}$) = 40° (Second req.)

[b] $\because$ $\overline{AX}$, $\overline{AY}$ are two tangent-segments.

$\therefore$ AY = AX = 6 cm. (First req.)

, $\because$ $\overline{AX}$ is a tangent-segment $\therefore$ $\overline{MX} \perp \overline{AX}$

$\therefore$ m ($\angle$ AXM) = 90° (Second req.)

In Δ AXM : $\therefore$ m ($\angle$ XAM) = 180° – (90° + 65°)

= 25°

, $\because$ $\overrightarrow{AM}$ bisects $\angle$ XAY

$\therefore$ m ($\angle$ XAY) = 2 × 25° = 50° (Third req.)

2 Giza

1

1 c 2 b 3 a 4 d 5 a 6 d

2

[a] In Δ AMC : $\because$ m ($\angle$ ACM) = 90°

$\therefore$ $(AC)^2 = (13)^2 - (5)^2 = 144$

$\therefore$ AC = $\sqrt{144}$ = 12 cm.

, $\because$ $\overline{MC} \perp \overline{AB}$ $\therefore$ C is the midpoint of $\overline{AB}$

$\therefore$ AB = 2 AC = 2 × 12 = 24 cm.

, $\because$ MD = MA = r = 13 cm.

$\therefore$ CD = 13 – 5 = 8 cm. (The req.)

[b] $\because$ $\overline{AC}$ // $\overline{MD}$, $\overleftrightarrow{AM}$ is a transversal.

$\therefore$ m ($\angle$ AMD) = m ($\angle$ CAB) = 40°

(alternate angles)

, $\because$ m ($\angle$ ABD) = $\frac{1}{2}$ m ($\angle$ AMD)

(inscribed and central angles subtended by $\overset{\frown}{AD}$)

$\therefore$ m ($\angle$ ABD) = $\frac{1}{2}$ × 40° = 20° (The req.)

3

[a] $\because$ m ($\angle$ BAD) = 180° – 86° = 94°

$\therefore$ m ($\angle$ BAD) = m ($\angle$ DCE) = 94°

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore$ AB = AC

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$ (First req.)

, $\because \overline{MC} \perp \overline{AC}$

$\therefore m(\angle ACM) = 90^\circ$

$\therefore m(\angle MCB) = 90^\circ - 65^\circ = 25^\circ$ (Second req.)

, $\because \overline{MB} \perp \overline{AB}$ $\therefore m(\angle ABM) = 90^\circ$

From the quadrilateral ABMC :

$\therefore m(\angle CMB) = 360^\circ - (90^\circ + 90^\circ + 50^\circ) = 130^\circ$ (Third req.)

4

[a] In ΔABC : $\because m(\angle B) = m(\angle C)$

$\therefore AB = AC$

, $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$, $\because \overline{MY} \perp \overline{AC}$

$\therefore MX = MY$ (Q.E.D.)

[b] $\because m(\angle A) = \frac{1}{2}[m(\widehat{CD}) - m(\widehat{BH})]$

$\therefore 28^\circ = \frac{1}{2}[m(\widehat{CD}) - 30^\circ]$

$\therefore 56^\circ = m(\widehat{CD}) - 30^\circ$

$\therefore m(\widehat{CD}) = 56^\circ + 30^\circ = 86^\circ$ (The req.)

5

[a] $\because$ ABCD is a cyclic quadrilateral.

$\therefore m(\angle BAD) = 180^\circ - 70^\circ = 110^\circ$

In ΔABD : $\therefore m(\angle ABD) = 180^\circ - (110^\circ + 30^\circ)$
$= 40^\circ$ (The req.)

[b] $\because \overrightarrow{AC}$, $\overrightarrow{AB}$ are two tangents $\therefore AC = AB$

In ΔABC :

$\therefore m(\angle ACB) = m(\angle ABC) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$

$\therefore m(\angle CHB)$ (inscribed)
$= m(\angle ACB)$ (tangency) $= 70^\circ$ (First req.)

, $\because \overrightarrow{AC} \parallel \overline{BH}$, $\overleftrightarrow{BC}$ is a transversal

$\therefore m(\angle CBH) = m(\angle ACB) = 70^\circ$ (alternate angles)

In ΔBCH : $\therefore m(\angle BCH) = 180^\circ - (70^\circ + 70^\circ)$
$= 40^\circ$

$\therefore m(\widehat{BH}) = 2\, m(\angle BCH) = 2 \times 40^\circ = 80^\circ$ (Second req.)

3 Alexandria

1

1 d 2 d 3 b 4 c 5 a 6 c

2

[a] $\because \overline{AB}$ is a tangent-segment to the circle M

$\therefore \overline{MA} \perp \overline{AB}$ $\therefore m(\angle MAB) = 90^\circ$

, $\because$ H is the midpoint of $\overline{CD}$

$\therefore \overline{MH} \perp \overline{CD}$ $\therefore m(\angle MHB) = 90^\circ$

From the quadrilateral ABHM :

$\therefore m(\angle AMH) = 360^\circ - (90^\circ + 90^\circ + 50^\circ) = 130^\circ$ (The req.)

[b] $m(\angle A) = \frac{1}{2}[m(\widehat{CH}) - m(\widehat{BD})]$

$= \frac{1}{2}(100^\circ - 30^\circ) = 35^\circ$ (The req.)

3

[a] In ΔABM :

$\because MA = MB = r$

$\therefore m(\angle MAB) = m(\angle MBA) = 50^\circ$

$\therefore m(\angle AMB) = 180^\circ - 2 \times 50^\circ = 80^\circ$

, $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended by $\widehat{AB}$)

$\therefore m(\angle ACB) = \frac{1}{2} \times 80^\circ = 40^\circ$

, $\because m(\widehat{AC}) = m(\widehat{BC})$ $\therefore AC = BC$

In ΔABC :

$\therefore m(\angle BAC) = m(\angle ABC) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$

$\therefore m(\angle CAM) = 70^\circ - 50^\circ = 20^\circ$ (The req.)

[b] $\because AB = CD$, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

$\therefore MX = MY$, $\because MH = ME = r$

$\therefore XH = EY$ (Q.E.D.)

4

[a] $\because \overline{AB} \parallel \overline{CD}$ $\therefore m(\widehat{AC}) = m(\widehat{BD})$

$\therefore m(\angle AEC) = m(\angle DEB)$

Adding $m(\angle CED)$ to both sides :

$\therefore m(\angle AED) = m(\angle CEB)$ (Q.E.D.)

[b] In ΔLYZ : $\because ZL = ZY$

$\therefore m(\angle ZYL) = m(\angle ZLY) = 40^\circ$

$\therefore m(\angle LZY) = 180^\circ - 2 \times 40^\circ = 100^\circ$

, $\because m(\angle LZY) + m(\angle LXY) = 100^\circ + 80^\circ = 180^\circ$

$\therefore$ XYZL is a cyclic quadrilateral. (Q.E.D.)

5

[a] $\because \overline{AD}, \overline{AE}$ are two tangent-segments to the circle

$\therefore AD = AE = 5$ cm.

, $\because \overline{BD}, \overline{BH}$ are two tangent-segments to the circle

$\therefore BD = BH = 4$ cm.

, $\because \overline{CH}, \overline{CE}$ are two tangent-segments to the circle

$\therefore CH = CE = 3$ cm.

$\therefore$ The perimeter of $\Delta ABC = 5 + 5 + 4 + 4 + 3 + 3$

$= 24$ cm. (The req.)

[b] $\because \overrightarrow{AB}, \overrightarrow{AC}$ are two tangents. $\therefore AB = AC$

In ΔABC :

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180° - 50°}{2} = 65°$

, $\because$ BCDH is a cyclic quadrilateral

$\therefore m(\angle CBH) = 180° - 115° = 65°$

$\therefore m(\angle ABC) = m(\angle CBH) = 65°$

$\therefore \overrightarrow{BC}$ bisects $\angle ABH$ (Q.E.D.)

4 El-Kalyoubia

1

1 d 2 c 3 b 4 c 5 d 6 d

2

[a] $\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$ $\therefore m(\angle ADM) = 90°$

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore \overline{ME} \perp \overline{AC}$ $\therefore m(\angle AEM) = 90°$

From the quadrilateral MDAE :

$\therefore m(\angle DME) = 360° - (90° + 90° + 120°) = 60°$

, $\because m(\angle YMX) = m(\angle DME) = 60°$ (V.O.A)

, $MY = MX = r$

$\therefore \Delta MXY$ is an equilateral triangle. (Q.E.D.)

[b] In ΔABM :

$\because MA = MB = r$

$\therefore m(\angle MAB) = m(\angle MBA) = 50°$

$\therefore m(\angle AMB) = 180° - 2 \times 50° = 80°$

, $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

$\therefore m(\angle ACB) = \frac{1}{2} \times 80° = 40°$ (The req.)

3

[a] In the greater circle :

$\because \overline{ME} \perp \overline{AB}$ $\therefore$ E is the midpoint of $\overline{AB}$

$\therefore AE = BE$ (1)

In the smaller circle

, $\because \overline{ME} \perp \overline{CD}$ $\therefore$ E is the midpoint of $\overline{CD}$

$\therefore CE = DE$ (2)

Subtracting (2) from (1) :

$\therefore AC = BD$ (Q.E.D.)

[b] $\because m(\angle A) = \frac{1}{2} [m(\overset{\frown}{EC}) - m(\overset{\frown}{BD})]$

$\therefore 30° = \frac{1}{2} [m(\overset{\frown}{EC}) - 44°]$

$\therefore 60° = m(\overset{\frown}{EC}) - 44°$

$\therefore m(\overset{\frown}{EC}) = 60° + 44° = 104°$ (The req.)

4

[a] In ΔABC : $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

$\therefore \frac{1}{2} m(\angle ABC) = \frac{1}{2} m(\angle ACB)$

$\therefore m(\angle ABX) = m(\angle ACY)$

and they are drawn on $\overline{XY}$ and on one side of it

$\therefore$ BCXY is a cyclic quadrilateral. (Q.E.D.)

[b] $\because \overrightarrow{AB}, \overrightarrow{AC}$ are two tangent to the smaller circle

$\therefore AB = AC = 10$ cm.

, $\because \overrightarrow{AB}, \overrightarrow{AD}$ are two tangents to the greater circle

$\therefore AB = AD = 10$ cm.

$\therefore x + 7 = 10$ $\therefore x = 3$ cm. (The req.)

5

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle BAD) = 180° - 70° = 110°$

In ΔABD : $\therefore m(\angle ABD) = 180° - (110° + 30°)$

$= 40°$ (The req.)

[b] In ΔABC : $\because AC = BC$

$\therefore m(\angle BAC) = m(\angle ABC) = 65°$

$\therefore m(\angle CAD) = 130° - 65° = 65°$

$\therefore m(\angle ABC) = m(\angle CAD) = 65°$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of ΔABC (Q.E.D.)

5 El-Sharkia

1

1 b	2 a	3 d	4 a	5 d	6 b

2

[a] ∵ X is the midpoint of $\overline{AB}$

∴ $\overline{MX} \perp \overline{AB}$, ∵ $\overline{MY} \perp \overline{CD}$

, AB = CD ∴ MX = MY

, ∵ ME = MF = r

By subtracting : ∴ XE = YF (Q.E.D.)

[b] ∵ $m(\angle BDC) = \frac{1}{2} m(\angle BMC)$

(inscribed and central angles subtended by $\overset{\frown}{BC}$)

∴ $m(\angle BDC) = \frac{1}{2} \times 120° = 60°$

, ∵ $\overline{AB} // \overline{DC}$, $\overleftrightarrow{BD}$ is a transversal

∴ $m(\angle ABD) = m(\angle BDC) = 60°$

(alternate angles) (1)

, ∵ $\overline{AB}$, $\overline{AD}$ are two tangent-segments.

∴ AB = AD (2)

From (1) and (2) : ∴ Δ ABD is an equilateral triangle (Q.E.D.)

3

[a] ∵ $\overline{AB}$ is a diameter. ∴ $m(\angle ACB) = 90°$

, ∵ $m(\angle ACE) = m(\angle ADE) = 90°$

and they are drawn on $\overline{AE}$ and on one side of it

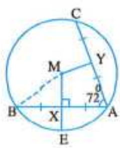
∴ ACDE is a cyclic quadrilateral (First req.)

∴ $m(\angle ACD) + m(\angle AED) = 180°$

∴ $m(\angle ACD) = 180° - 70° = 110°$

∴ $m(\angle DCE) = 110° - 90° = 20°$ (Second req.)

[b] ∵ Y is the midpoint of $\overline{AC}$



∴ $\overline{MY} \perp \overline{AC}$

∴ $m(\angle AYM) = 90°$

, ∵ $\overrightarrow{MX} \perp \overline{AB}$

∴ $m(\angle AXM) = 90°$

∴ From the quadrilateral AXMY :

$m(\angle XMY) = 360° - (90° + 90° + 72°) = 108°$ (First req.)

, ∵ $\overrightarrow{MX} \perp \overline{AB}$ ∴ X is the midpoint of $\overline{AB}$

∵ $BX = \frac{1}{2} AB = 8$ cm.

In Δ BXM : ∵ $m(\angle BXM) = 90°$

∴ $(MX)^2 = (BM)^2 - (BX)^2 = (10)^2 - (8)^2 = 36$

∴ MX = 6 cm.

∴ XE = 10 – 6 = 4 cm. (Second req.)

4

[a] ∵ $m(\angle AEC) = \frac{1}{2} [m(\overset{\frown}{AC}) - m(\overset{\frown}{BD})]$

∴ $40° = \frac{1}{2} [100° - m(\overset{\frown}{BD})]$

∴ $80° = 100° - m(\overset{\frown}{BD})$

∴ $m(\overset{\frown}{BD}) = 100° - 80° = 20°$

, ∵ $\overline{AB}$ is a dimaeter ∴ $m(\overset{\frown}{AB}) = 180°$

∴ $m(\overset{\frown}{CD}) = 180° - (100° + 20°) = 60°$ (The req.)

[b] ∵ $\overline{AB}$ is a diameter ∴ $m(\overset{\frown}{AB}) = 180°$

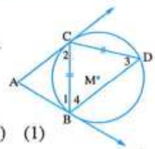
∴ $m(\overset{\frown}{CAB}) = 180° + 50° = 230°$

∴ $m(\angle CDB) = \frac{1}{2} m(\overset{\frown}{CAB}) = \frac{1}{2} \times 230° = 115°$

(The req.)

5

[a] ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents



∴ AB = AC

∴ $m(\angle 1) = m(\angle 2)$

∴ $m(\angle A) = 180° - 2\, m(\angle 1)$ (1)

In Δ BCD : ∵ CB = CD

∴ $m(\angle 4) = m(\angle 3)$

∴ $m(\angle BCD) = 180° - 2\, m(\angle 3)$ (2)

, ∵ $m(\angle 3)$ (inscribed) = $m(\angle 1)$ (tangency) (3)

From (1) , (2) and (3) :

∴ $m(\angle A) = m(\angle BCD)$

∴ $\overrightarrow{CD}$ is a tangent to the circle passing through the vertices of Δ ABC (Q.E.D.)

[b] ∵ E is the midpoint of $\overline{AC}$

∴ $\overline{ME} \perp \overline{AC}$ ∴ $m(\angle MED) = 90°$

, ∵ $\overline{BD}$ is a tangent-segment ∴ $\overline{MB} \perp \overline{BD}$

∴ $m(\angle MBD) = 90°$

, ∵ $m(\angle MED) + m(\angle MBD) = 90° + 90° = 180°$

∴ MEDB is a cyclic quadrilateral. (Q.E.D.)

6 El-Monofia

1

1 c	2 d	3 d	4 d	5 b	6 d

2

[a] $\because$ C is the midpoint of $\overline{AB}$

$\therefore AC = \frac{1}{2} AB = 12$ cm.

, $\overline{MC} \perp \overline{AB}$ $\therefore m(\angle ACM) = 90°$

$\therefore$ In Δ ACM : $(MC)^2 = (MA)^2 - (AC)^2$

$= (13)^2 - (12)^2 = 25$

$\therefore MC = \sqrt{25} = 5$ cm.

$\therefore CD = 13 - 5 = 8$ cm. (The req.)

[b] In Δ ABD : $\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = 30°$

$\therefore m(\angle BAD) = 180° - (30° + 30°) = 120°$

, $\because m(\angle BAD) + m(\angle C) = 120° + 60° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

3

[a] In Δ ABC : $\because m(\angle B) = m(\angle C) = 50°$

$\therefore AB = AC$, $\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

$\therefore MX = MY$ (First req.)

, $\because \overline{MX} \perp \overline{AB}$ $\therefore$ X is the midpoint of $\overline{AB}$

$\therefore AC = AB = 2 \times 3 = 6$ cm. (Second req.)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle A) = 180° - 120° = 60°$ (First req.)

, $\because \overline{AB}$ is a diameter. $\therefore m(\angle ADB) = 90°$

$\therefore$ In Δ ADB : $m(\angle ABD) = 180° - (90° + 60°)$

$= 30°$ (Second req.)

4

[a] $\because \overrightarrow{AB}$ is a tangent to the circle

$\therefore \overline{MB} \perp \overrightarrow{AB}$ $\therefore m(\angle ABM) = 90°$

$\therefore$ In Δ ABM : $m(\angle AMB) = 180° - (90° + 40°)$

$= 50°$

, $\because m(\angle BDC) = \frac{1}{2} m(\angle BMC)$

(inscribed and central angles subtended by $\overset{\frown}{BC}$)

$\therefore m(\angle BDC) = \frac{1}{2} \times 50° = 25°$ (The req.)

[b] $\because \overleftrightarrow{AX}$ is a common tangent for two circles

$\therefore m(\angle BDA)$ (inscribed) $= m(\angle BAX)$ (tangency)

, $m(\angle CEA)$ (inscribed) $= m(\angle CAX)$ (tangency)

$\therefore m(\angle BDA) = m(\angle CEA)$

and they are corresponding angles

$\therefore \overline{BD} \parallel \overline{CE}$ (Q.E.D.)

5

[a] $m(\overset{\frown}{AD}) = 2\, m(\angle ACD) = 2 \times 26° = 52°$

(First req)

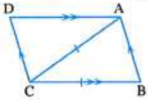
, $\because m(\angle BEC) = \frac{1}{2} [m(\overset{\frown}{BC}) + m(\overset{\frown}{AD})]$

$\therefore 92° = \frac{1}{2} [m(\overset{\frown}{BC}) + 52°]$

$\therefore 184° = m(\overset{\frown}{BC}) + 52°$

$\therefore m(\overset{\frown}{BC}) = 184° - 52° = 132°$ (Second req.)

[b] In Δ ABC :



$\because AC = BC$

$\therefore m(\angle B) = m(\angle BAC)$ (1)

$\because \overline{AB} \parallel \overline{CD}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore m(\angle DCA) = m(\angle BAC)$ (alternate angles) (2)

From (1) and (2) : $\therefore m(\angle B) = m(\angle DCA)$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circle passing thourgh the vertices of Δ ABC (Q.E.D.)

7 El-Gharbia

1

1 b 2 c 3 a 4 d 5 c 6 c

2

[a] $\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

, $MX = MY$ $\therefore AB = CD$

, $\because \overline{MY} \perp \overline{CD}$ $\therefore$ Y is the midpoint of CD

$\therefore AB = CD = 2 \times 7 = 14$ cm. (The req.)

[b] $\because$ X is the midpoint of $\overline{AC}$

$\therefore \overline{MX} \perp \overline{AC}$ $\therefore m(\angle AXM) = 90°$

, $\because \overline{BY}$ is a tangent-segment

$\therefore \overline{MB} \perp \overline{BY}$ $\therefore m(\angle MBY) = 90°$

$\therefore m(\angle AXY) = m(\angle ABY) = 90°$

and they are drawn on $\overline{AY}$ and on one side of it

$\therefore$ AXBY is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\because \overline{AB} \parallel \overline{CD}$

$\therefore m(\overset{\frown}{BD}) = m(\overset{\frown}{AC}) = 30°$

$\therefore m(\angle BED) = \frac{1}{2} m(\overset{\frown}{BD}) = \frac{1}{2} \times 30° = 15°$

(The req.)

[b] $\because \overleftrightarrow{DY}$ is a tangent to the circle

$\therefore$ m ($\angle$ ACB) (inscribed) = m ($\angle$ BAD) (tangency)

, $\because$ m ($\angle$ BAC) + m ($\angle$ BAD) = 130°

$\therefore$ m ($\angle$ BAC) + m ($\angle$ ACB) = 130°

In Δ ABC :

$\therefore$ m ($\angle$ ABC) = 180° – 130° = 50° (The req.)

4

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ ADC) = m ($\angle$ CBE) = 85°

$\therefore$ m ($\angle$ CDB) = 85° – 55° = 30° (The req.)

[b] $\because$ m ($\angle$ ADB) = $\frac{1}{2}$ m ($\angle$ AMB)

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

$\therefore$ m ($\angle$ ADB) = $\frac{1}{2} \times 140° = 70°$

, $\because \overline{AC} \parallel \overline{BD}$, $\overleftrightarrow{AD}$ is a transversal.

$\therefore$ m ($\angle$ CAD) + m ($\angle$ ADB) = 180°

(two interior angles in the same side of the transversal)

$\therefore$ m ($\angle$ CAD) = 180° – 70° = 110° (The req.)

5

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$, $\because \overline{AB} \parallel \overline{CD}$

$\therefore \overline{MY} \perp \overline{CD}$

$\therefore$ Y is the midpoint of $\overline{CD}$ (Q.E.D.)

[b] $\because \overrightarrow{EA}$, $\overrightarrow{EC}$ are two tangents to the circle M

$\therefore$ EA = CE (1)

, $\because \overrightarrow{EB} = \overrightarrow{ED}$ are two tangents to the circle N

$\therefore$ EB = ED (2)

Adding (1) and (2) : $\therefore$ AB = CD (Q.E.D.)

8 El-Dakahlia

1

[a] 1 c   2 b   3 a

[b] $\because$ m ($\angle$ BMD) = 2 m ($\angle$ A)

(central and inscribed angles subtended by $\overset{\frown}{BD}$)

, m ($\angle$ BMD) = m ($\angle$ BCD)

$\therefore$ m ($\angle$ BCD) = 2 m ($\angle$ A)

, $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ A) + m ($\angle$ BCD) = 180°

$\therefore$ m ($\angle$ A) + 2 m ($\angle$ A) = 180°

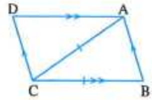
$\therefore$ 3 m ($\angle$ A) = 180°

$\therefore$ m ($\angle$ A) = 60° (The req.)

2

[a] 1 a   2 c   3 b

[b] In Δ ABC :



$\because$ AC = BC

$\therefore$ m ($\angle$ B) = m ($\angle$ BAC) (1)

$\because \overline{AB} \parallel \overline{CD}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ DCA) = m ($\angle$ BAC) (alternate angles) (2)

From (1) and (2) : $\therefore$ m ($\angle$ B) = m ($\angle$ DCA)

$\therefore \overleftrightarrow{CD}$ is a tangent to the circumcircle of Δ ABC

(Q.E.D.)

3

[a] $\because \overline{AB}$ is a diameter $\therefore$ m ($\overset{\frown}{AB}$) = 180°

, $\because$ m ($\overset{\frown}{BD}$) = m ($\overset{\frown}{AD}$)

$\therefore$ m ($\overset{\frown}{BD}$) = 180° ÷ 2 = 90°

, $\because$ m ($\overset{\frown}{BD}$) = 3 m ($\overset{\frown}{AC}$)

$\therefore$ m ($\overset{\frown}{AC}$) = 90° ÷ 3 = 30°

$\therefore$ m ($\angle$ AEC) = $\frac{1}{2}$ [m ($\overset{\frown}{BD}$) + m ($\overset{\frown}{AC}$)]

= $\frac{1}{2}$ (90° + 30°) = 60° (The req.)

[b] Const. : Draw $\overline{MX}$, $\overline{MY}$, $\overline{MZ}$

Proof : $\because \overline{AB}$, $\overline{BC}$, $\overline{AC}$ are three tangents to the smaller circle

$\therefore \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{BC}$

, $\overline{MZ} \perp \overline{AC}$

, $\because$ MX = MY = MZ = r (radii of the smaller circle)

$\therefore$ AB = BC = AC

$\therefore \Delta$ ABC is an equilateral triangle (Q.E.D.)

4

[a] $\because$ The two circles are touching internally.

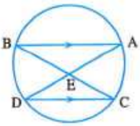
$\therefore$ MN = 10 – 6 = 4 cm.

, $\because \overleftrightarrow{MN} \perp \overline{AB}$

, the area of Δ BMN = $\frac{1}{2}$ × MN × AB

$\therefore 24 = \frac{1}{2} \times 4 \times$ AB $\therefore$ AB = 12 cm. (The req.)

[b] $\because \overline{AB} \parallel \overline{CD}$



$\therefore m(\widehat{AC}) = m(\widehat{BD})$

$\therefore m(\angle ABC) = m(\angle BAD)$

$\therefore$ In Δ EAB : AE = EB

$\therefore \Delta$ EAB is an isosceles triangle. (Q.E.D.)

5

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle

$\therefore$ AB = AC (1)

, $\because \overrightarrow{AM}$ bisects $\angle$ A $\therefore m(\angle A) = 60°$ (2)

From (1) and (2) :

$\therefore \Delta$ ABC is an equilateral triangle

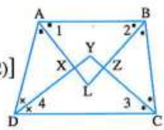
$\therefore$ The perimeter of $\Delta ABC = 3 \times 8 = 24$ cm. (First req.)

, $\because m(\angle ABC) = 60°$

$\therefore m(\angle E)$ (inscribed) $= m(\angle ABC)$ (tangency).

$\therefore m(\angle E) = 60°$ (Second req.)

[b] In Δ ABL :



$\because m(\angle L)$

$= 180° - [m(\angle 1) + m(\angle 2)]$

, $\because m(\angle 1) = \frac{1}{2} m(\angle A)$

, $m(\angle 2) = \frac{1}{2} m(\angle B)$

$\therefore m(\angle L) = 180° - \left[\frac{1}{2} m(\angle A) + \frac{1}{2} m(\angle B)\right]$ (1)

In Δ CDY :

$\because m(\angle Y) = 180° - [m(\angle 3) + m(\angle 4)]$

, $\because m(\angle 3) = \frac{1}{2} m(\angle C)$, $m(\angle 4) = \frac{1}{2} m(\angle D)$

$\therefore m(\angle Y) = 180° - \left[\frac{1}{2} m(\angle C) + \frac{1}{2} m(\angle D)\right]$ (2)

Adding (1) and (2) :

$\therefore m(\angle L) + m(\angle Y)$

$= 180° - \left[\frac{1}{2} m(\angle A) + m(\angle B)\right]$

$+ 180° - \left[\frac{1}{2} m(\angle C) + \frac{1}{2} m(\angle D)\right] = 360°$

$- \frac{1}{2} [m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D)]$

, $\because m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360°$

$\therefore m(\angle L) + m(\angle Y) = 360° - \frac{1}{2} \times 360° = 180°$

$\therefore$ XYZL is a cyclic quadrilateral. (Q.E.D.)

9 Ismailia

1

1 c   2 d   3 a   4 b   5 d   6 c

2

[a] $\because m(\angle AED) = \frac{1}{2} [m(\widehat{AD}) + m(\widehat{BC})]$

$\therefore 115° = \frac{1}{2} [130° + m(\widehat{BC})]$

$\therefore 230° = 130° + m(\widehat{BC})$

$\therefore m(\widehat{BC}) = 230° - 130° = 100°$ (The req.)

[b] $\because$ ABXY is a cyclic quadrilateral

$\therefore m(\angle A) = m(\angle CXY) = 90°$

$\therefore \overline{CB}$ is a diameter of the given circle. (Q.E.D.)

3

[a] $\because m(\angle ADB) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended by $\widehat{AB}$)

$\therefore m(\angle ADB) = \frac{1}{2} \times 140° = 70°$

, $\because \overline{AC} \parallel \overline{BD}$, $\overleftrightarrow{AD}$ is a transversal

$\therefore m(\angle CAD) + m(\angle ADB) = 180°$

(two interior angles in the same side of the transversal)

$\therefore m(\angle CAD) = 180° - 70° = 110°$ (The req.)

[b] In Δ ABD : $\because$ AB = AD

$\therefore m(\angle ABD) = m(\angle ADB) = \frac{180° - 80°}{2} = 50°$

$\therefore m(\angle CBD) = 130° - 50° = 80°$

$\therefore m(\angle A) = m(\angle CBD) = 80°$

$\therefore \overline{BC}$ is a tangent-segment to the circle which passes through the points A , B and D (Q.E.D.)

4

[a] $\because$ X is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$, $\because$ AB = AC

$\therefore$ MX = MY , $\because$ MD = ME = r

$\therefore$ DX = EY (Q.E.D.)

[b] $\because \overline{XY}$ is a diameter.

$\therefore m(\widehat{LX}) = 180° - (70° + 60°) = 50°$

$\therefore m(\angle XYZ) = \frac{1}{2} m(\widehat{XZ}) = \frac{1}{2} (60° + 50°) = 55°$

, $m(\angle LXY) = \frac{1}{2} m(\widehat{LY}) = \frac{1}{2} (60° + 70°) = 65°$

, $\because$ XYZL is a cyclic quadrilateral

$\therefore m(\angle YZL) = 180° - 65° = 115°$

, $m(\angle XLZ) = 180° - 55° = 125°$ (The req.)

5

[a] $\because$ BCDE is a cyclic quadrilateral.

$\therefore$ m (∠ EBC) + m (∠ CDE) = 180°

$\therefore x + 2x = 180°$ $\therefore 3x = 180°$, $\therefore x = 60°$

, $\because \overrightarrow{BC}$ bisects ∠ ABE

$\therefore$ m (∠ ABC) = m (∠ EBC) = 60° (1)

, $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore$ AB = AC (2)

From (1) and (2) :

$\therefore$ Δ ABC is an equilateral triangle. (Q.E.D.)

[b] $\because$ The two circles are touching internally

$\therefore$ MN = 10 – 6 = 4 cm.

, $\because \overleftrightarrow{MN} \perp \overline{XY}$ $\therefore$ m (∠ MXY) = 90°

, $\because$ the area of Δ YMN = $\frac{1}{2}$ × MN × XY

$\therefore 24 = \frac{1}{2} \times 4 \times XY$ $\therefore$ XY = 12 cm.

In Δ MXY : $\because$ m (∠ MXY) = 90°

$\therefore (MY)^2 = (MX)^2 + (XY)^2 = (10)^2 + (12)^2 = 244$

$\therefore MY = \sqrt{244} \approx 15.6$ cm. (The req.)

10 Suez

1

1 b	2 b	3 d	4 c	5 a	6 c

2

[a] $\because \overrightarrow{AD}$ is a tangent to the circle.

$\therefore \overline{MD} \perp \overrightarrow{AD}$ $\therefore$ m (∠ MDA) = 90°

, $\because$ E is the midpoint of $\overline{BC}$

$\therefore \overline{ME} \perp \overline{BC}$ $\therefore$ m (∠ MEA) = 90°

From the quadrilateral ADME :

$\therefore$ m (∠ DME) = 360° – (90° + 90° + 60°)
= 120° (The req.)

[b] m (∠ AEC) = $\frac{1}{2}$ [m ($\widehat{BD}$) + m ($\widehat{AC}$)]

= $\frac{1}{2}$ (100° + 50°) = 75° (The req.)

3

[a] $\because$ AB = AC , $\overrightarrow{MD} \perp \overline{AB}$, $\overrightarrow{ME} \perp \overline{AC}$

$\therefore$ MD = ME , $\because$ MX = MY = r

$\therefore$ XD = YE (Q.E.D.)

[b] $\because \overleftrightarrow{BC}$ is a tangent to the circle.

$\therefore$ m (∠ ABC) (tangency) = m (∠ ADB) (inscribed)
= 70° (First req.)

, m ($\widehat{AB}$) = 2 m (∠ ADB) = 2 × 70° = 140°
(Second req.)

4

[a] State by yourself.

[b] In Δ ABD : $\because$ AB = AD

$\therefore$ m (∠ ABD) = m (∠ ADB) = 30°

$\therefore$ m (∠ BAD) = 180° – 2 × 30° = 120°

$\therefore$ m (∠ BAD) = m (∠ DCE) = 120°

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] $\because$ m ($\widehat{BD}$) = 2 m (∠ BCD) = 2 × 25° = 50°

$\because$ D is the midpoint of $\widehat{AB}$

$\therefore$ m ($\widehat{AB}$) = 2 m ($\widehat{BD}$) = 2 × 50° = 100°

$\therefore$ m (∠ AMB) = m ($\widehat{AB}$) = 100° (The req.)

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore$ AB = AC

$\therefore$ In Δ ABC :

m (∠ ABC) = m (∠ ACB) = $\frac{180° - 50°}{2}$ = 65°

, $\because$ EBCD is a cyclic quadrilateral

$\therefore$ m (∠ EBC) + m (∠ EDC) = 180°

$\therefore$ m (∠ EBC) = 180° – 115° = 65°

$\therefore$ m (∠ ABC) = m (∠ EBC) = 65°

$\therefore \overrightarrow{BC}$ bisects ∠ ABE (Q.E.D.)

11 Port Said

1 d	2 b	3 c	4 c	5 b	6 b
7 a	8 a	9 b	10 b	11 c	12 b
13 c	14 c	15 c	16 d	17 c	18 a
19 d	20 a	21 d			

22 $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

, $\because \overline{MY} \perp \overline{AC}$, AB = AC

$\therefore$ MX = MY (Q.E.D.)

23 $\because$ ABDC is a cyclic quadrilateral

$\therefore$ m (∠ ABD) + m (∠ ACD) = 180°

$\therefore$ m (∠ ABD) = 180° – 115° = 65°

, $\because \overline{AB}$ is a diameter $\therefore$ m (∠ ADB) = 90°

$\therefore$ In Δ ABD : m (∠ DAB) = 180° – (90° + 65°)
= 25° (The req.)

24 $\because \overrightarrow{AB}$ is a tangent to the circle

$\therefore$ m (∠ BDC) (inscribed) = m (∠ ABC) (tangency) $= 70°$

, $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore$ AB = AC

In Δ ABC : $\therefore$ m (∠ ABC) = m (∠ ACB) = 70°

$\therefore$ m (∠ A) = 180° – (70° + 70°) = 40° (The req.)

12 Damietta

1

1 d 2 b 3 a 4 c 5 b 6 a

2

[a] $\because$ m (∠ EBC) = $\frac{1}{2}$ m (∠ EMC)

(inscribed and central angles subtended by $\overset{\frown}{EC}$)

$\therefore$ m (∠ EBC) = $\frac{1}{2} \times 120° = 60°$

$\therefore$ m (∠ ABE) = 180° – 60° = 120°

In Δ ABE : $\because$ AB = BE

$\therefore$ m (∠ BAE) = m (∠ BEA) = $\frac{180° - 120°}{2} = 30°$ (The req.)

[b] $\because \overrightarrow{YB}$ is a tangent , $\overline{AB}$ is a diameter

$\therefore \overline{AB} \perp \overrightarrow{YB}$ $\therefore$ m (∠ ABY) = 90°

, $\because$ X is the midpoint of $\overline{AC}$

$\therefore \overline{MX} \perp \overline{AC}$ $\therefore$ m (∠ MXA) = 90°

$\therefore$ m (∠ ABY) = m (∠ AXY) = 90°

and they are drawn on $\overline{AY}$ and on one side of it

$\therefore$ AXBY is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

, MX = MY $\therefore$ AB = AC

In Δ ABC :

$\therefore$ m (∠ B) = m (∠ C) = $\frac{180° - 50°}{2} = 65°$ (The req.)

[b] $\because \overline{ED} \parallel \overline{CB}$

$\therefore$ m $(\overset{\frown}{BD})$ = m $(\overset{\frown}{EC})$

$\therefore$ m (∠ BAD) = m (∠ CAE)

Adding m (∠ BAC) to both sides :

$\therefore$ m (∠ DAC) = m (∠ EAB) (Q.E.D.)

4

[a] In Δ ABC : $\because$ AB = BC = CA

$\therefore$ m (∠ ABC) = m (∠ ACB) = m (∠ BAC) = 60°

, $\because \overrightarrow{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m (∠ CAD) = m (∠ BCA) = 60° (alternate angles)

$\therefore$ m (∠ CAD) = m (∠ ABC)

$\therefore \overrightarrow{AD}$ is a tangent to the circumcircle of Δ ABC (Q.E.D.)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m (∠ BAD) = 180° – 75° = 105°

, $\because$ ABFE is a cyclic quadrilateral and ∠ BAD is exterior of it.

$\therefore$ m (∠ F) = m (∠ BAD) = 105° (First req.)

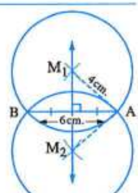
$\therefore$ m (∠ F) + m (∠ BCD) = 105° + 75° = 180°

and they are interior angles in the same side of $\overleftrightarrow{FC}$

$\therefore \overline{CD} \parallel \overline{EF}$ (Second req.)

5

[a]



We can draw two circles.

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore$ AB = AC

In Δ ABC :

$\therefore$ m (∠ ABC) = m (∠ ACB) = $\frac{180° - 40°}{2} = 70°$

, $\because$ BCED is a cyclic quadrilateral

$\therefore$ m (∠ CBD) = 180° – 110° = 70°

$\therefore$ m (∠ ABC) = m (∠ CBD) = 70°

$\therefore \overrightarrow{BC}$ bisects ∠ ABD (Q.E.D.)

13 Kafr El-Sheikh

1

1 b 2 c 3 d 4 b 5 a 6 b

2

[a] $\because \overleftrightarrow{MN}$ is the line of centres

, $\overline{AB}$ is the common chord

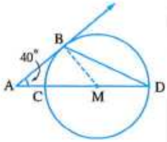
$\therefore \overleftrightarrow{MN} \perp \overline{AB}$ $\therefore$ m (∠ ACN) = 90°

From the quadrilateral XCNY :

$\therefore$ m (∠ XYN) = 360° – (90° + 135° + 45°) = 90°

$\therefore \overleftrightarrow{XY}$ is a tangent to the circle N at Y (Q.E.D.)

[b] Const. : Draw $\overline{MB}$



Proof : $\because \overrightarrow{AB}$ is a tangent

$\therefore \overline{MB} \perp \overrightarrow{AB}$

$\therefore m(\angle ABM) = 90°$

In Δ AMB :

$\therefore m(\angle BMA) = 180° - (90° + 40°) = 50°$

$\therefore m(\angle BDC) = \frac{1}{2} m(\angle BMC) = \frac{1}{2} \times 50° = 25°$

(inscribed and central angles subtended by $\widehat{BC}$)

(The req.)

3

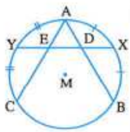
[a] $\because$ D is the midpoint of $\overline{AB}$ $\therefore \overline{MD} \perp \overline{AB}$

, $\because \overline{ME} \perp \overline{AC}$, MD = ME $\therefore$ AB = AC

In Δ ABC :

$\therefore m(\angle B) = m(\angle C) = \frac{180° - 40°}{2} = 70°$ (The req.)

[b] $\because m(\angle ADE)$



$= \frac{1}{2}[m(\widehat{AY}) + m(\widehat{XB})]$ (1)

, $m(\angle AED)$

$= \frac{1}{2}[m(\widehat{AX}) + m(\widehat{CY})]$ (2)

$\because$ X is the midpoint of $\widehat{AB}$

, Y is the midpoint of $\widehat{AC}$

$\therefore m(\widehat{AX}) = m(\widehat{BX})$ (3)

, $m(\widehat{AY}) = m(\widehat{CY})$ (4)

From (1) , (2) , (3) and (4) :

$\therefore m(\angle ADE) = m(\angle AED)$

In Δ ADE : $\therefore$ AD = AE (Q.E.D.)

4

[a] In Δ XYL : $\because$ XY = YL

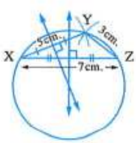
$\therefore m(\angle X) = m(\angle XLY) = \frac{180° - 100°}{2} = 40°$

$\therefore m(\angle X) = m(\angle Z) = 40°$

and they are drawn on $\overline{YL}$ and on one side of it.

$\therefore$ The points X , Y , L and Z have only one circle passing through them. (Q.E.D.)

[b]



r = 4.1 cm.

5

[a] In Δ ABC : $\because$ BA = BC

$\therefore m(\angle BAC) = m(\angle BCA) = 50°$

$\therefore m(\angle B) = 180° - 2 \times 50° = 80°$

, $\because m(\angle B) + m(\angle D) = 80° + 100° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b] $\because$ EDBC is a cyclic quadrilateral

$\therefore m(\angle CBD) = 180° - 130° = 50°$

, $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents

$\therefore$ AB = AC

In Δ ABC :

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180° - 80°}{2} = 50°$

$\therefore m(\angle ABC) = m(\angle CBD) = 50°$

$\therefore \overrightarrow{BC}$ bisects $\angle$ ABD (Q.E.D. 1)

, $\because m(\angle ACB) = m(\angle CBD) = 50°$

and they are alternate angles

$\therefore \overleftrightarrow{BD} // \overleftrightarrow{AC}$ (Q.E.D. 2)

14 El-Beheira

1

1 a 2 d 3 b 4 d 5 d 6 b

2

[a] $\because$ E is the midpoint of $\overline{BC}$

$\therefore \overline{ME} \perp \overline{BC}$ $\therefore m(\angle MEA) = 90°$

, $\because \overrightarrow{AD}$ is a tangent $\therefore \overline{MD} \perp \overrightarrow{AD}$

$\therefore m(\angle MDA) = 90°$

$\therefore$ From the quadrilateral ADME :

$m(\angle DME) = 360° - (90° + 90° + 56°) = 124°$

(The req.)

[b] $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended by $\widehat{AB}$)

$\therefore m(\angle ACB) = \frac{1}{2} \times 120° = 60°$ (1)

, $\because \overleftrightarrow{CD} // \overline{AB}$ $\therefore m(\widehat{CA}) = m(\widehat{CB})$

$\therefore$ CA = CB (2)

From (1) and (2) :

$\therefore \Delta$ CAB is an equilateral triangle. (Q.E.D.)

3

[a] $\because FX = EY , MF = ME = r$ $\quad \therefore MX = MY$

$, \because \overrightarrow{MX} \perp \overline{AB} \quad , \quad \overrightarrow{MY} \perp \overline{CD}$

$\therefore AB = CD$ (Q.E.D.)

[b] $\because$ ABCD is cyclic quadrilateral

$\therefore m(\angle ADC) = m(\angle CBE) = 85°$

$, \because m(\angle ADB) = \frac{1}{2} m(\widehat{AB}) = \frac{1}{2} \times 100° = 50°$

$\therefore m(\angle BDC) = 85° - 50° = 35°$ (The req.)

4

[a] In ΔADE : $\because AE = DE$

$\therefore m(\angle A) = m(\angle D) \quad \therefore m(\widehat{BD}) = m(\widehat{AC})$

$, \because m(\angle C) = \frac{1}{2} m(\widehat{BD})$

$, m(\angle B) = \frac{1}{2} m(\widehat{AC})$

$\therefore m(\angle C) = m(\angle B)$

In ΔEBC : $\therefore EB = EC$ (Q.E.D.)

[b] $\because \overline{AB}$ is a diameter $\quad \therefore m(\angle ACB) = 90°$

$, \because \overrightarrow{DE} \perp \overrightarrow{AB} \quad \therefore m(\angle ADE) = 90°$

$\therefore m(\angle ACE) = m(\angle ADE) = 90°$

and they are drawn on $\overline{AE}$ and on one side of it

$\therefore$ ACDE is a cyclic quadrilateral. (Q.E.D.)

5

[a] $\because \overrightarrow{XA} , \overrightarrow{XB}$ are two tangents

$\therefore XA = XB$

In ΔXAB :

$\therefore m(\angle XAB) = m(\angle XBA) = \frac{180° - 70°}{2} = 55°$

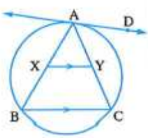
$, \because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle BAD) = 180° - 125° = 55°$

$\therefore m(\angle XAB) = m(\angle BAD) = 55°$

$\therefore \overrightarrow{AB}$ bisects $\angle DAX$ (Q.E.D.)

[b] $\because \overline{XY} \parallel \overline{BC}$



$, \overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle AXY) = m(\angle ABC)$

(corresponding angles)

$, \because m(\angle ABC)$ (inscribed)

$= m(\angle CAD)$ (tangency)

$\therefore m(\angle AXY) = m(\angle YAD)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

15 El-Fayoum

1

1 b 2 b 3 a 4 d 5 d 6 b

2

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB} \quad \therefore m(\angle MXA) = 90°$

$, \because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC} \quad \therefore m(\angle MYA) = 90°$

From the quadrilateral AXMY :

$\therefore m(\angle EMD) = 360° - (90° + 90° + 70°) = 110°$ (First req.)

$, \because AB = AC \quad \therefore MX = MY$

$, \because MD = ME = r \quad \therefore XD = YE$ (Second req.)

[b] $\because \overline{AB}$ is a diameter $\quad \therefore m(\widehat{AB}) = 180°$

$, \because \overline{AB} \parallel \overline{CD}$

$\therefore m(\widehat{AC}) = m(\widehat{BD}) = \frac{180° - 100°}{2} = 40°$ (First req.)

$, \because m(\angle AEC) = \frac{1}{2} m(\widehat{AC}) = 20°$

$\therefore 3x - 25° = 20° \quad \therefore 3x = 45°$

$\therefore x = 15°$ (Second req.)

3

[a] The measure of the arc $= \frac{1}{4} \times 360° = 90°$

The length of the arc $= \frac{90°}{360°} \times 2 \times \frac{22}{7} \times 14$

$= 22$ cm. (The req.)

[b] $\because \overrightarrow{AD}$ is a tangent to the circle

$\therefore m(\angle ACB)$ (inscribed) $= m(BAD)$ (tangency)

$\therefore m(\angle ACB) + m(\angle CAB) = 130°$

In ΔABC :

$\therefore m(\angle B) = 180° - 130° = 50°$ (The req.)

4

[a] $\because$ E is the midpoint of $\overline{AC}$

$\therefore \overline{ME} \perp \overline{AC}$

$\therefore m(\angle MED) = 90°$

$, \because \overline{BD}$ is a tangent-segment to the circle

$\therefore \overline{MB} \perp \overline{BD}$

$\therefore m(\angle MBD) = 90°$

$\therefore m(\angle MED) + m(\angle MBD) = 90° + 90° = 180°$

$\therefore$ EMBD is a cyclic quadrilateral. (Q.E.D.)

[b] $\because \overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

$\therefore XA = XB$

$\therefore$ In ΔABX :

$m(\angle XAB) = m(\angle XBA) = \frac{180^\circ - 70^\circ}{2} = 55^\circ$ (1)

, $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle BAD) = 180^\circ - 125^\circ = 55^\circ$ (2)

From (1) and (2) :

$\therefore m(\angle XAB) = m(\angle BAD) = 55^\circ$

$\therefore \overrightarrow{AB}$ bisects $\angle DAX$ (Q.E.D.)

5

[a] $\because m(\angle A) = \frac{1}{2}[m(\widehat{EC}) - m(\widehat{BD})]$

$\therefore 30^\circ = \frac{1}{2}[m(\widehat{EC}) - 44^\circ]$

$\therefore 60^\circ = m(\widehat{EC}) - 44^\circ$

$\therefore m(\widehat{EC}) = 60^\circ + 44^\circ = 104^\circ$

, $\because m(\widehat{ED}) = 2\, m(\angle ECD) = 2 \times 48^\circ = 96^\circ$

$\therefore m(\widehat{BC}) = 360^\circ - (104^\circ + 96^\circ + 44^\circ)$

$= 116^\circ$ (The req.)

[b] $\because \overrightarrow{AF} \parallel \overline{DE}$, $\overleftrightarrow{AE}$ is a transversal

$\therefore m(\angle AED) = m(\angle EAF)$ (alternate angles)

, $\because m(\angle ACB)$ (inscribed) $= m(\angle BAF)$ (tangency)

$\therefore m(\angle ACB) = m(\angle AED)$

$\therefore$ BCDE is a cyclic quadrilateral. (Q.E.D.)

16 Beni Suef

1

1 c 2 a 3 b 4 a 5 a 6 b

2

[a] $\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$

, $\because \overline{ME} \perp \overline{AC}$, $MD = ME$

$\therefore AB = AC$

In ΔABC : $\therefore m(\angle B) = m(\angle C) = 70^\circ$

$\therefore m(\angle A) = 180^\circ - 2 \times 70^\circ = 40^\circ$ (The req.)

[b] In ΔBMC : $\because MB = MC = r$

$\therefore m(\angle MCB) = m(\angle MBC) = \frac{180^\circ - 100^\circ}{2} = 40^\circ$

, $\because m(\angle BDC) = \frac{1}{2} m(\angle BMC) = \frac{1}{2} \times 100^\circ = 50^\circ$

(inscribed and central angles subtended by $\widehat{BC}$)

, $\because \angle ABD$ is an exterior angle of ΔBCD

$\therefore m(\angle BCD) = 120^\circ - 50^\circ = 70^\circ$

$\therefore m(\angle DCM) = 70^\circ - 40^\circ = 30^\circ$ (The req.)

3

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle ADC) = m(\angle CBE) = 100^\circ$

, $\because m(\angle ADB) = \frac{1}{2} m(\widehat{AB}) = \frac{1}{2} \times 110^\circ = 55^\circ$

$\therefore m(\angle BDC) = 100^\circ - 55^\circ = 45^\circ$ (The req.)

[b] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore AB = AC$

In ΔABC :

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 50}{2} = 65^\circ$

, $\because$ EBCD is a cyclic quadrilateral

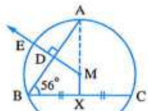
$\therefore m(\angle EBC) = 180^\circ - 115^\circ = 65^\circ$

$\therefore m(\angle ABC) = m(\angle EBC) = 65^\circ$

$\therefore \overrightarrow{BC}$ bisects $\angle ABE$ (Q.E.D.)

4

[a] Const : Draw $\overline{AM}$



Proof :

X is the midpoint of $\overline{CB}$

$\therefore \overline{MX} \perp \overline{BC}$

$\therefore m(\angle MXB) = 90^\circ$

, $\because \overline{MD} \perp \overline{AB}$

$\therefore m(\angle MDB) = 90^\circ$

From the quadrilateral BDMX :

$\therefore m(\angle DMX)$

$= 360^\circ - (90^\circ + 90^\circ + 56^\circ) = 124^\circ$ (First req.)

, $\because \overline{MD} \perp \overline{AB}$

$\therefore$ D is the midpoint of $\overline{AB}$

$\therefore AD = \frac{1}{2} AB = 4$ cm.

In ΔADM : $\because m(\angle ADM) = 90^\circ$, $AM = r = 5$ cm.

$\therefore MD = \sqrt{(AM)^2 - (AD)^2} = \sqrt{25 - 16} = \sqrt{9} = 3$ cm.

$\therefore DE = 5 - 3 = 2$ cm. (Second req.)

[b] $\because m(\angle AEC) = \frac{1}{2}[m(\widehat{AC}) - m(\widehat{BD})]$

$\therefore 30^\circ = \frac{1}{2}[80^\circ - m(\widehat{BD})]$

$\therefore 60^\circ = 80^\circ - m(\widehat{BD})$

$\therefore m(\widehat{BD}) = 80^\circ - 60^\circ = 20^\circ$

, $\because \overline{AB}$ is a diameter $\therefore m(\widehat{AB}) = 180^\circ$

$\therefore m(\widehat{CD}) = 180^\circ - (80^\circ + 20^\circ) = 80^\circ$ (The req.)

5

[a] $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m (∠ AYX) = m (∠ ACB) (corresponding angles)

, $\because$ m (∠ ACB) (inscribed)

= m (∠ BAD) (tangency)

$\therefore$ m (∠ AYX) = m (∠ XAD)

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] $\because \overrightarrow{YB}$ is a tangent , $\overline{AB}$ is a diameter

$\therefore \overline{AB} \perp \overrightarrow{YB}$ $\therefore$ m (∠ ABY) = 90°

, $\because$ X is the midpoint of $\overline{AC}$

$\therefore \overline{MX} \perp \overline{AC}$ $\therefore$ m (∠ MXA) = 90°

$\therefore$ m (∠ ABY) = m (∠ AXY) = 90°

and they are drawn on $\overline{AY}$ and on one side of it

$\therefore$ AXBY is a cyclic quadrilateral (Q.E.D. 1)

, $\because$ m (∠ ABY) = 90°

$\therefore$ The centre of the circle passing through the vertices of the quadrilateral AXBY is the midpoint of $\overline{AY}$ (Q.E.D. 2)

17 El-Menia

1

[1] d [2] a [3] a [4] c [5] c [6] c

2

[a] $\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$ $\therefore$ m (∠ MDA) = 90°

, $\because$ H is the midpoint of $\overline{AC}$

$\therefore \overline{MH} \perp \overline{AC}$ $\therefore$ m (∠ MHA) = 90°

$\therefore$ From the quadrilateral ADMH :

m (∠ DMH) = 360° – (90° + 90° + 60°) = 120° (The req.)

[b] $\because \overline{XY} \parallel \overleftrightarrow{BD}$, $\overleftrightarrow{BC}$ is a transversal

$\therefore$ m (∠ DBY) = m (∠ BYX) (alternate angles) (1)

, $\because$ m (∠ A) (inscribed)

= m (∠ DBC) (tangency) (2)

From (1) and (2) :

$\therefore$ m (∠ A) = m (∠ BYX)

$\therefore$ AXYC is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\because$ X is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$ $\therefore \overline{MY} \perp \overline{AC}$

, $\because$ AB = AC $\therefore$ MX = MY

, $\because$ MD = ME = r

$\therefore$ XD = YE (Q.E.D.)

[b] $\because$ m (∠ ABC) = $\frac{1}{2}$ m (∠ AMC) = $\frac{1}{2} \times 150° = 75°$

(inscribed and central angles subtended by $\widehat{AC}$)

, $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m (∠ CDA) = 180° – 75° = 105° (The req.)

4

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore$ AB = AC

In Δ ABC :

$\therefore$ m (∠ ABC) = m (∠ ACB) = $\frac{180° - 40°}{2} = 70°$

$\therefore$ m (∠ D) (inscribed) = m (∠ ABC) (tangency) = 70° (The req.)

[b] $\because \overline{BC}$ is a diameter $\therefore$ m (∠ BAC) = 90°

, $\because \overline{ED} \perp \overline{BC}$ $\therefore$ m (∠ EDB) = 90°

, $\because$ m (∠ BAE) + m (∠ EDB) = 90° + 90° = 180°

$\therefore$ ABDE is a cyclic quadrilateral (Q.E.D.)

5

[a] $\because$ m $(\widehat{AB})$ = m $(\widehat{BC})$ = m $(\widehat{AC})$ = 360° ÷ 3 = 120°

$\therefore$ m (∠ A) = $\frac{1}{2}$ m $(\widehat{BC})$ = $\frac{1}{2} \times 120° = 60°$ (The req.)

[b] $\because$ m (∠ BMC) = 2 m (∠ BAC) = 2 × 30° = 60° (1)

(central and inscribed angles subtended by $\widehat{BC}$)

, $\because$ MB = MC = r (2)

From (1) and (2) :

$\therefore$ Δ MBC is an equilateral triangle. (Q.E.D.)

18 Assiut

1

[1] a [2] b [3] a [4] c [5] c [6] d

2

[a] $\because$ D is the midpoint of $\overline{XY}$

$\therefore \overline{MD} \perp \overline{XY}$ $\therefore$ m (∠ MDX) = 90°

, $\because$ E is the midpoint of $\overline{XZ}$

$\therefore \overline{ME} \perp \overline{XZ}$ $\therefore m(\angle MEX) = 90^\circ$

, $\because MD = ME$ $\therefore XY = XZ$ (1)

From the quadrilateral XDME :

$\therefore m(\angle X) = 360^\circ - (90^\circ + 90^\circ + 120^\circ) = 60^\circ$ (2)

From (1) and (2) :

$\therefore \Delta XYZ$ is an equilateral triangle. (Q.E.D.)

[b] $\because \overline{AB} \parallel \overline{CD}$ $\therefore m(\widehat{AC}) = m(\widehat{BD})$

, $\because \overline{AB}$ is a diameter $\therefore m(\widehat{AB}) = 180^\circ$

$\therefore m(\widehat{AC}) = \frac{180^\circ - 100^\circ}{2} = 40^\circ$

$\therefore m(\angle AEC) = \frac{1}{2} m(\widehat{AC}) = \frac{1}{2} \times 40^\circ = 20^\circ$

(The req.)

3

[a] $\because m(\angle BAD) = \frac{1}{2} m(\angle BMD) = \frac{1}{2} \times 150^\circ = 75^\circ$

(inscribed and central angles subtended by $\widehat{BD}$)

, $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle BCD) = 180^\circ - 75^\circ = 105^\circ$ (The req.)

[b] $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle AYX) = m(\angle ACB)$ (corresponding angles)

, $\because m(\angle ACB)$ (inscribed) $= m(\angle BAD)$ (tangency)

$\therefore m(\angle AYX) = m(\angle XAD)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

4

[a] 1 $MN = 8 + 6 = 14$ cm.

2 $MN = 8 - 6 = 2$ cm.

3 MN = zero

[b] $\because m(\angle BCD) = \frac{1}{2} m(\angle M)$

(inscribed and central angles subtended by $\widehat{BD}$)

$\therefore m(\angle BCD) = \frac{1}{2} \times 130^\circ = 65^\circ$

$\because \overline{AB} \parallel \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal

$\therefore m(\angle ABC) = m(\angle BCD) = 65^\circ$

(alternate angles)

$\because \overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

$\therefore AB = AC$

In ΔABC : $\therefore m(\angle ACB) = m(\angle ABC) = 65^\circ$

$\therefore m(\angle A) = 180^\circ - 2 \times 65^\circ = 50^\circ$ (The req.)

5

[a] $\because \overline{BC}$ is a tangent-segment to the circle

$\therefore \overline{MB} \perp \overline{BC}$ $\therefore m(\angle MBC) = 90^\circ$

, $\because \overline{ME} \perp \overline{AD}$ $\therefore m(\angle MEC) = 90^\circ$

, $\because m(\angle MBC) + m(\angle MEC) = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ EMBC is a cyclic quadrilateral (First req.)

, $\because AB = 2\,AM = 2 \times 4 = 8$ cm.

In ΔABC : $\because m(\angle ABC) = 90^\circ$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (8)^2 + (6)^2 = 100$

$\therefore AC = \sqrt{100} = 10$ cm. (Second req.)

[b] $\because m(\widehat{AD}) = m(\widehat{BE})$

Adding $m(\widehat{ED})$ to both sides :

$\therefore m(\widehat{AE}) = m(\widehat{BD})$

$\therefore m(\angle EBA) = m(\angle DAB)$

In ΔABC :

$\therefore AC = BC$ (Q.E.D.)

19 Souhag

1

1 a 2 c 3 c 4 a 5 b 6 c

2

[a] $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle AXY) = m(\angle ABC)$ (corresponding angles)

, $\because m(\angle ABC)$ (inscribed) $= m(\angle CAD)$ (tangency)

$\therefore m(\angle AXY) = m(\angle YAD)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] In ΔMAB : $\because MA = MB = r$

$\therefore m(\angle MAB) = m(\angle MBA) = 50^\circ$

$\therefore m(\angle AMB) = 180^\circ - 2 \times 50^\circ = 80^\circ$

$\therefore m(\widehat{AB}) = m(\angle AMB) = 80^\circ$

$\therefore m(\angle ACB) = \frac{1}{2} m(\widehat{AB}) = \frac{1}{2} \times 80^\circ$

$= 40^\circ$ (First req.)

, $m(\widehat{ACB}) = 360^\circ - 80^\circ = 280^\circ$ (Second req.)

3

[a] $\because \overleftrightarrow{MN}$ is the line of centres

, $\overline{AB}$ is a common chord

$\therefore \overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$

, ∵ $D \in \overleftrightarrow{MN}$ ∴ AD = BD

, ∵ $\overline{MX} \perp \overline{AD}$, $\overline{MY} \perp \overline{BD}$

∴ MX = MY (Q.E.D.)

[b] ∵ $\overline{AB} \,//\, \overline{CD}$ ∴ $m(\widehat{AC}) = m(\widehat{BD})$

, ∵ $\overline{AB}$ is a diameter ∴ $m(\widehat{AB}) = 180°$

∴ $m(\widehat{BD}) = \frac{180° - 80°}{2} = 50°$

∴ $m(\angle DEB) = \frac{1}{2} m(\widehat{BD}) = \frac{1}{2} \times 50° = 25°$

, $m(\angle AWE) = \frac{1}{2}[m(\widehat{AE}) + m(\widehat{BD})]$

$= \frac{1}{2}(100° + 50°) = 75°$ (The req.)

4

[a] ∵ XYZD is a cyclic quadrilateral

∴ $m(\angle Z) = m(\angle WXD) = 80°$ (First req.)

, ∵ $m(\angle Y) + m(\angle D) = 180°$

, $m(\angle Y) = \frac{1}{2} m(\angle D)$

∴ $\frac{1}{2} m(\angle D) + m(\angle D) = 180°$

∴ $\frac{3}{2} m(\angle D) = 180°$

∴ $m(\angle D) = 120°$ (Second req.)

[b] ∵ D is the midpoint of $\overline{AB}$

∴ $\overline{MD} \perp \overline{AB}$ ∴ $m(\angle MDA) = 90°$

, ∵ E is the midpoint of $\overline{AC}$

∴ $\overline{ME} \perp \overline{AC}$ ∴ $m(\angle MEA) = 90°$

From the quadrilateral ADME :

∴ $m(\angle XMY) = 360° - (90° + 90° + 60°)$

$= 120°$ (First req.)

, ∵ AB = AC , $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

∴ MD = ME

, ∵ MX = MY = r

∴ XD = YE (Second req.)

5

[a] ∵ $\overline{AB}$, $\overline{AC}$ are two tangent-segments

∴ AB = AC

In Δ ABC : ∴ $m(\angle ACB) = m(\angle ABC)$

, $\overrightarrow{AM}$ bisects ∠ BAC

∴ $m(\angle BAC) = 2 \times 20° = 40°$

∴ $m(\angle ACB) = \frac{180° - 40°}{2} = 70°$ (First req.)

∴ $m(\angle BEC)$ (inscribed) $= m(\angle ACB)$ (tangency)

$= 70°$ (Second req.)

[b] In Δ ADC : ∵ AD = DC

∴ $m(\angle DAC) = m(\angle DCA) = 30°$

∴ $m(\angle ADC) = 180° - 2 \times 30° = 120°$

, ∵ Δ ABC is an equilateral triangle

∴ $m(\angle ABC) = 60°$

∴ $m(\angle ADC) + m(\angle ABC) = 120° + 60° = 180°$

∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

20 Qena

1

1 b	2 c	3 c	4 b	5 a	6 d

2

[a] ∵ The arc is opposite to an inscribed angle of measure 45°

∴ The measure of the arc = 2 × 45° = 90°

, the length of the arc $= \frac{90°}{360°} \times 2 \times \frac{22}{7} \times 7$

= 11cm. (The req.)

[b] ∵ X is the midpoint of $\overline{AB}$

∴ $\overline{MX} \perp \overline{AB}$ ∴ $m(\angle MXA) = 90°$

, ∵ Y is the midpoint of $\overline{AC}$

∴ $\overline{MY} \perp \overline{AC}$ ∴ $m(\angle MYA) = 90°$

From the quadrilateral AXMY :

∴ $m(\angle DME) = 360° - (90° + 90° + 70°) = 110°$ (First req.)

, ∵ AB = AC ∴ MX = MY

, ∵ MD = ME = r ∴ XD = YE (Second req.)

3

[a] ∵ $\overline{DE} \,//\, \overline{BC}$

∴ $m(\widehat{BD}) = m(\widehat{CE})$

∴ $m(\angle DAB) = m(\angle EAC)$

Adding $m(\angle BAC)$ to both sides :

∴ $m(\angle DAC) = m(\angle BAE)$ (Q.E.D.)

[b] ∵ $m(\widehat{AC}) = 2\, m(\angle ABC) = 2 \times 40° = 80°$

, ∵ D is the midpoint of $\widehat{AC}$

∴ $m(\widehat{AD}) = m(\widehat{DC}) = 40°$

, ∵ $\overline{AB}$ is a diameter ∴ $m(\widehat{AB}) = 180°$

∴ $m(\widehat{DCB}) = 180° - 40° = 140°$

∴ $m(\angle DAB) = \frac{1}{2} m(\widehat{DCB}) = \frac{1}{2} \times 140° = 70°$

(The req.)

4

[a] $\because \overline{AB}$ is a diameter $\quad \therefore m(\angle ACB) = 90^\circ$

$, \because \overrightarrow{DE} \perp \overline{AB} \quad \therefore m(\angle ADE) = 90^\circ$

$\therefore m(\angle ACE) = m(\angle ADE) = 90^\circ$

and they are drawn on $\overline{AE}$ and on one side of it

$\therefore$ ACDE is a cyclic quadrilateral. (Q.E.D.)

[b] In ΔABC :

$\because m(\angle BAC) = 90^\circ , AC = \frac{1}{2} BC$

$\therefore m(\angle B) = 30^\circ$

$\therefore m(\angle C) = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$

$\therefore m(\angle C) = m(\angle BAD) = 60^\circ$

$\therefore \overrightarrow{AD}$ is a tangent to the circle passing through the vertices of ΔABC (Q.E.D.)

5

[a] $\because m(\angle BCD) = \frac{1}{2} m(\angle M)$

(inscribed and central angles subtended by $\widehat{BD}$)

$\therefore m(\angle BCD) = \frac{1}{2} \times 130^\circ = 65^\circ$

$, \because \overline{AB} // \overline{CD} , \overleftrightarrow{BC}$ is a transversal

$\therefore m(\angle ABC) = m(\angle BCD) = 65^\circ$

(alternate angles)

$, \because \overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

$\therefore AB = AC$

In ΔABC : $\therefore m(\angle ACB) = m(\angle ABC) = 65^\circ$

$\therefore m(\angle ACB) = m(\angle BCD) = 65^\circ$

$\therefore \overrightarrow{CB}$ bisects $\angle ACD$ (First req.)

$, m(\angle A) = 180^\circ - 2 \times 65^\circ = 50^\circ$ (Second req.)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle D) = m(\angle ABE) = 100^\circ$

In ΔACD :

$\therefore m(\angle ACD) = 180^\circ - (100^\circ + 40^\circ) = 40^\circ$

$\therefore m(\angle CAD) = m(\angle ACD)$

$\therefore m(\widehat{CD}) = m(\widehat{AD})$ (Q.E.D.)

21 Luxor

1

1 b	2 d	3 b	4 c	5 a	6 d

2

[a] $\because \overline{AB}$ is a tangent-segment to the circle

$\therefore \overline{MA} \perp \overline{AB} \quad \therefore m(\angle MAB) = 90^\circ$

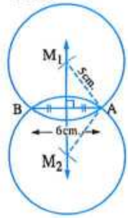
$\therefore$ In ΔMAB which is right at A :

$(MB)^2 = (MA)^2 + (AB)^2 = (5)^2 + (12)^2 = 169$

$\therefore MB = \sqrt{169} = 13$ cm.

$\therefore BD = 13 - 5 = 8$ cm. (The req.)

[b]



$\therefore$ We can draw two circles.

3

[a] $\because$ The two circles are touching internally

$\therefore MN = 10 - 6 = 4$ cm.

$, \because \overrightarrow{MN} \perp \overline{AB}$

$\therefore$ The area of $\Delta BMN = \frac{1}{2} \times MN \times AB$

$\therefore 24 = \frac{1}{2} \times 4 \times AB$

$\therefore AB = 12$ cm. (The req.)

[b] $\because \overline{AB} // \overline{CD} \quad \therefore m(\widehat{AC}) = m(\widehat{BD})$

$\therefore m(\angle AEC) = m(\angle BED)$

Adding $m(\angle CED)$ to both sides :

$\therefore m(\angle AED) = m(\angle BEC)$ (Q.E.D.)

4

[a] $\because$ X is the midpoint of $\overline{AB} \quad \therefore \overline{MX} \perp \overline{AB}$

$, \because \overrightarrow{MY} \perp \overline{AC} , AB = AC \quad \therefore MX = MY$

$, \because MD = ME = r \quad \therefore XD = YE$

(Q.E.D. 1)

In ΔXMY : $\because MX = MY$

$\therefore m(\angle MXY) = m(\angle MYX)$

$, \because m(\angle MXB) = m(\angle MYC) = 90^\circ$

By adding : $\therefore m(\angle YXB) = m(\angle XYC)$

(Q.E.D. 2)

[b] In ΔACD : $\because AD = CD$

$\therefore m(\angle DAC) = m(\angle DCA) = 40^\circ$

$\therefore m(\angle D) = 180^\circ - 2 \times 40^\circ = 100^\circ$

$, \because m(\angle B) + m(\angle D) = 80^\circ + 100^\circ = 180^\circ$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] ∵ $\overline{XY} \parallel \overleftrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

∴ m (∠ DBX) = m (∠ YXB) (alternate angles) (1)

, ∵ m (∠ C) (inscribed)

= m (∠ ABD) (tangency) (2)

From (1) and (2) :

∴ m (∠ C) = m (∠ YXB)

∴ AXYC is a cyclic quadrilateral. (Q.E.D.)

[b] ∵ BCDE is a cyclic quadrilateral

∴ m (∠ CBE) + m (∠ D) = 180°

∴ m (∠ CBE) = 180° – 125° = 55°

, ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

∴ AB = AC

∴ In Δ ABC :

m (∠ ACB) = m (∠ ABC) = $\frac{180° - 70°}{2}$ = 55°

, ∵ m (∠ BEC) (inscribed)

= m (∠ ACB) (tangency) = 55°

∴ In Δ CBE : m (∠ CBE) = m (∠ BEC) = 55°

∴ BC = CE (Q.E.D. 1)

, ∵ m (∠ CBE) = m (∠ ACB) = 55°

and they are alternate angles

∴ $\overrightarrow{AC} \parallel \overline{BE}$ (Q.E.D. 2)

22 Aswan

1

1 c 2 d 3 a 4 b 5 c 6 d

2

[a] ∵ Y is the midpoint of $\overline{AC}$

∴ $\overline{MY} \perp \overline{AC}$

, ∵ $\overline{MX} \perp \overline{AB}$, AB = AC

∴ MY = MX

, ∵ ME = MD = r

∴ YE = XD (Q.E.D.)

[b] ∵ $\overrightarrow{CD}$ is a tangent to the circle

∴ $\overline{MD} \perp \overrightarrow{CD}$ ∴ m (∠ MDC) = 90°

In Δ MCD :

∴ m (∠ CMD) = 180° – (90° + 40°) = 50°

∴ m $(\overset{\frown}{AD})$ = m (∠ AMD) = 180° – 50° = 130°

(The req.)

3

[a] ∵ $\overline{AB}$ is a tangent-segment to the circle

∴ $\overline{MA} \perp \overline{AB}$ ∴ m (∠ MAB) = 90°

∴ In Δ MAB which is right at A :

$(MB)^2 = (MA)^2 + (AB)^2 = 6^2 + 8^2 = 100$

∴ MB = $\sqrt{100}$ = 10 cm.

∴ DB = 10 – 6 = 4 cm. (The req.)

[b] ∵ ABCD is a cyclic quadrilateral

∴ m (∠ D) = m (∠ ABE) = 100°

In Δ ACD :

∴ m (∠ DCA) = 180° – (100° + 40°) = 40°

∴ m (∠ DCA) = m (∠ DAC)

∴ m $(\overset{\frown}{DA})$ = m $(\overset{\frown}{DC})$ (Q.E.D.)

4

[a] ∵ $\overline{AB}$, $\overline{AC}$ are two tangent-segments

∴ AB = AC

In Δ ABC :

∴ m (∠ ABC) = m (∠ ACB) = $\frac{180° - 50°}{2}$ = 65°

(First req.)

∴ m (∠ BMC) (central) = 2 m (∠ ABC) (tangency)

= 2 × 65° = 130°

(Second req.)

[b] ∵ m (∠ D) = $\frac{1}{2}$ m (∠ AMB)

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

∴ m (∠ D) = $\frac{1}{2}$ × 140° = 70°

, ∵ $\overline{AC} \parallel \overline{DB}$, $\overleftrightarrow{AD}$ is a transversal

∴ m (∠ DAC) + m (∠ D) = 180°

(two interior angles in the same side of the transversal)

∴ m (∠ CAD) = 180° – 70° = 110° (The req.)

5

[a] ∵ ∠ DEC is an exterior angle of Δ BEC

∴ m (∠ ECB) = 80° – 40° = 40°

, ∵ $\overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

∴ m (∠ DAC) = m (∠ ACB) = 40°

(alternate angles)

∴ m (∠ DBC) = m (∠ DAC) = 40°

and they are drawn on $\overline{DC}$ and on one side of it

∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b] $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m (∠ AED) = m (∠ ACB)

(corresponding angles)

, $\because$ m (∠ ACB) (inscribed) = m (∠ XAB)

(tangency)

$\therefore$ m (∠ AED) = m (∠ XAB)

$\therefore \overrightarrow{AX}$ is a tangent to the circle passing through the points A , D and E (Q.E.D.)

23 New Valley

1

1 b   2 c   3 c   4 d   5 b   6 a

2

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore$ m (∠ MXA) = 90°

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\therefore$ m (∠ MYA) = 90°

From the quadrilateral AXMY :

$\therefore$ m (∠ DME) = 360° – (90° + 90° + 70°) = 110°

(First req.)

, $\because$ AB = AC , $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

$\therefore$ MX = MY   , $\because$ MD = ME = r

$\therefore$ XD = YE (Second req.)

[b] $\because \overline{DE} \parallel \overline{BC}$

$\therefore$ m $(\widehat{BD})$ = m $(\widehat{EC})$

$\therefore$ m (∠ DAB) = m (∠ EAC)

Adding m (∠ BAC) to both sides :

$\therefore$ m (∠ DAC) = m (∠ BAE) (Q.E.D.)

3

[a] State by yourself.

[b] In Δ ABD : $\because$ AB = AD

$\therefore$ m (∠ ABD) = m (∠ ADB) = 30°

$\therefore$ m (∠ BAD) = 180° – 2 × 30° = 120°

, $\because$ m (∠ BAD) + m (∠ C) = 120° + 60° = 180°

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

4

[a] $\because \overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle

$\therefore$ AD = AF = 5 cm.

, $\because \overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle

$\therefore$ BD = BE = 4 cm.

, $\because \overline{CE}$, $\overline{CF}$ are two tangent-segments to the circle

$\therefore$ CE = CF = 3 cm.

$\therefore$ The perimeter of Δ ABC = 5 + 5 + 4 + 4 + 3 + 3

= 24 cm. (The req.)

[b] $\because$ m (∠ ACB) = $\frac{1}{2}$ m (∠ AMB)

(inscribed and central angles subtended by $\widehat{AB}$)

$\therefore$ m (∠ ACB) = $\frac{1}{2}$ × 120° = 60° (1)

, $\because \overleftrightarrow{CD} \parallel \overline{AB}$ $\therefore$ m $(\widehat{AC})$ = m $(\widehat{BC})$

$\therefore$ AC = BC (2)

From (1) and (2) :

$\therefore$ Δ CAB is an equilateral triangle. (Q.E.D.)

5

[a] $\because$ m (∠ A) = $\frac{1}{2}$ [m $(\widehat{HC})$ – m $(\widehat{BD})$]

$\therefore$ 30° = $\frac{1}{2}$ [120° – m $(\widehat{BD})$]

$\therefore$ 60° = 120° – m $(\widehat{BD})$

$\therefore$ m $(\widehat{BD})$ = 120° – 60° = 60° (First req.)

, $\because$ m $(\widehat{BC})$ = m $(\widehat{DH})$

Adding m $(\widehat{BD})$ to both sides :

$\therefore$ m $(\widehat{CD})$ = m $(\widehat{HB})$ $\therefore$ m (∠ C) = m (∠ H)

In Δ ACH : $\therefore$ AC = AH (1)

, $\because$ m $(\widehat{BC})$ = m $(\widehat{DH})$ $\therefore$ BC = DH (2)

Subtracting (2) from (1) :

$\therefore$ AB = AD (Second req.)

[b] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle

$\therefore$ AB = AC

$\therefore$ In Δ ABC :

m (∠ ABC) = m (∠ ACB) = $\frac{180° - 50°}{2}$ = 65°

, $\because$ EBCD is a cyclic quadrilateral

$\therefore$ m (∠ EBC) = 180° – 115° = 65°

$\therefore$ m (∠ EBC) = m (∠ ABC) = 65°

$\therefore \overrightarrow{BC}$ bisects ∠ ABE (Q.E.D. 1)

, $\because$ m (∠ BEC) (inscribed)

= m (∠ ABC) (tangency) = 65°

$\therefore$ m (∠ EBC) = m (∠ BEC) = 65°

In Δ BCE : $\therefore$ CB = CE (Q.E.D. 2)

24 South Sinai

1

1 c 2 b 3 a 4 a 5 a 6 b

2

$\because AB = AC$, $\overline{MD} \perp \overline{AB}$

, $\overline{ME} \perp \overline{AC}$

$\therefore MD = ME$

, $\because MX = MY = r$

$\therefore DX = EY$ (Q.E.D.)

3

[a] In Δ ADE : $\because AE = DE$

$\therefore m(\angle BAD) = m(\angle ADC)$

$\therefore m(\widehat{BD}) = m(\widehat{AC})$

$\therefore m(\angle BAD) = m(\angle ABC)$

and they are alternate angles

$\therefore \overline{AD} \parallel \overline{CB}$ (Q.E.D.)

[b] In Δ ACD : $\because AD = CD$

$\therefore m(\angle DCA) = m(\angle DAC) = 30^\circ$

$\therefore m(\angle D) = 180^\circ - 2 \times 30^\circ = 120^\circ$

, $\because m(\angle B) + m(\angle D) = 60^\circ + 120^\circ = 180^\circ$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

4

[a] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore AB = AC$

$\therefore$ In Δ ABC :

$m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$

, $\because$ EBCD is a cyclic quadrilateral

$\therefore m(\angle EBC) = 180^\circ - 115^\circ = 65^\circ$

$\therefore m(\angle EBC) = m(\angle ABC) = 65^\circ$

$\therefore \overrightarrow{BC}$ bisects $\angle$ ABE (Q.E.D.)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle ADC) = m(\angle ABE) = 85^\circ$

, $\because m(\angle BDC) = \frac{1}{2} m(\widehat{BC}) = \frac{1}{2} \times 110^\circ = 55^\circ$

$\therefore m(\angle ADB) = 85^\circ - 55^\circ = 30^\circ$ (The req.)

5

[a] $\because m(\widehat{BD}) = m(\angle BMD) = 40^\circ$

, $m(\widehat{CE}) = m(\angle CME) = 100^\circ$

$\therefore m(\angle A) = \frac{1}{2}\left[m(\widehat{CE}) - m(\widehat{BD})\right]$

$= \frac{1}{2}(100^\circ - 40^\circ) = 30^\circ$ (The req.)

[b] $\because \overline{XY} \parallel \overleftrightarrow{AD}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle DAY) = m(\angle AYX)$ (alternate angles) (1)

, $\because m(\angle B)$ (inscribed)

$= m(\angle DAC)$ (tangency) (2)

From (1) and (2) :

$\therefore m(\angle B) = m(\angle AYX)$

$\therefore$ XYCB is a cyclic quadrilateral. (Q.E.D.)

25 North Sinai

1

1 b 2 b 3 c 4 c 5 b 6 d

2

[a] $\because$ D is the midpoint of $\overline{AE}$

$\therefore \overline{MD} \perp \overline{AE}$ $\therefore m(\angle CDM) = 90^\circ$

, $\because \overline{BC}$ is a tangent-segment

$\therefore \overline{MB} \perp \overline{BC}$ $\therefore m(\angle CBM) = 90^\circ$

$\therefore m(\angle CDM) + m(\angle CBM) = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ BCDM is a cyclic quadrilateral

$\therefore m(\angle DMA) = m(\angle C) = 45^\circ$

In Δ ADM : $\therefore m(\angle A) = 180^\circ - (90^\circ + 45^\circ) = 45^\circ$

$\therefore m(\angle DMA) = m(\angle A)$

$\therefore MD = AD$ (Q.E.D.)

[b] $m(\angle A) = \frac{1}{2}\left[m(\widehat{CE}) - m(\widehat{BD})\right]$

$= \frac{1}{2}(140^\circ - 60^\circ) = 40^\circ$ (First req.)

, $m(\widehat{BC}) = m(\widehat{ED}) = \frac{360^\circ - (140^\circ + 60^\circ)}{2} = 80^\circ$ (Second req.)

3

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle BAD) = m(\angle DCX) = 100^\circ$

In Δ ABD :

$\therefore m(\angle ADB) = 180^\circ - (100^\circ + 40^\circ) = 40^\circ$

$\therefore m(\angle ABD) = m(\angle ADB)$

$\therefore AB = AD$ (Q.E.D.)

[b] In ΔABC : $\because m(\angle B) = m(\angle C)$

$\therefore AB = AC$

, $\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$, $\because \overline{ME} \perp \overline{AC}$

$\therefore MD = ME$ (Q.E.D.)

4

[a] $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle AED) = m(\angle C)$ (corresponding angles)

, $\because m(\angle C)$ (inscribed) $= m(\angle BAX)$ (tangency)

$\therefore m(\angle AED) = m(\angle DAX)$

$\therefore \overleftrightarrow{AX}$ is a tangent to the circle passing through the vertices of ΔADE (Q.E.D.)

[b] $\because m(\angle BMC) = 2\, m(\angle BAC) = 2 \times 30° = 60°$ (1)

(central and inscribed angles subtended by $\overset{\frown}{BC}$)

, $\because MB = MC = r$ (2)

From (1) and (2) :

$\therefore \Delta MBC$ is an equilateral triangle. (Q.E.D.)

5

[a] $\because \overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle

$\therefore AD = AF = 5$ cm.

, $\because \overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle

$\therefore BD = BE = 4$ cm.

, $\because \overline{CE}$, $\overline{CF}$ are two tangent-segments to the circle

$\therefore CE = CF = 3$ cm.

$\therefore$ The perimeter of $\Delta ABC = 5 + 5 + 4 + 4 + 3 + 3$

$= 24$ cm. (The req.)

[b] $\because \overrightarrow{CB}$ is a tangent

$\therefore m(\angle CBE) = \frac{1}{2}\, m(\overset{\frown}{BE})$

, $\because m(\angle EAF) = \frac{1}{2}\, m(\overset{\frown}{EF})$

, $\because$ E is the midpoint of $\overset{\frown}{BF}$

$\therefore m(\overset{\frown}{BE}) = m(\overset{\frown}{EF})$

$\therefore m(\angle CBD) = m(\angle CAD)$

and they are drawn on $\overline{CD}$ and on one side of it

$\therefore ABCD$ is a cyclic quadrilateral. (Q.E.D.)

26 Red Sea

1

1 d 2 c 3 d 4 b 5 a 6 b

2

[a] $\because$ E is the midpoint of $\overline{CD}$ $\therefore \overline{ME} \perp \overline{CD}$

$\therefore m(\angle BEM) = 90°$

, $\because \overline{AB}$ is a tangent-segment to the circle

$\therefore \overline{MA} \perp \overline{AB}$ $\therefore m(\angle MAB) = 90°$

From the quadrilateral ABEM :

$\therefore m(\angle AME) = 360° - (90° + 90° + 50°)$

$= 130°$ (The req.)

[b] In ΔADC : $\because DA = DC$

$\therefore m(\angle DAC) = m(\angle DCA) = 35°$

$\therefore m(\angle D) = 180° - 2 \times 35° = 110°$

, $\because m(\angle B) + m(\angle D) = 70° + 110° = 180°$

$\therefore ABCD$ is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\because$ X is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$

, Y is the midpoint of $\overline{AC}$ $\therefore \overline{MY} \perp \overline{AC}$

, $\because AB = AC$

$\therefore MX = MY$

, $\because MD = ME = r$

$\therefore XD = YE$ (Q.E.D.)

[b] $\because m(\angle D) = \frac{1}{2}\, m(\angle AMB)$

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

$\therefore m(\angle D) = \frac{1}{2} \times 140° = 70°$ (First req.)

, $\because \overline{AC} \parallel \overline{BD}$, $\overleftrightarrow{AD}$ is a transversal

$\therefore m(\angle D) + m(\angle DAC) = 180°$

(interior angles on the same side of the transversal)

$\therefore m(\angle DAC) = 180° - 70° = 110°$ (Second req.)

4

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ ABC) = m ($\angle$ CDE) = 100° (First req.)

, $\because$ m ($\angle$ ABD) = $\frac{1}{2}$ m ($\widehat{AD}$) = $\frac{1}{2}$ × 120° = 60°

$\therefore$ m ($\angle$ CBD) = 100° – 60° = 40° (Second req.)

[b] $\because$ $\overline{AB}$ is a diameter

$\therefore$ m ($\angle$ ACB) = 90° (First req.)

, $\because$ m ($\angle$ ACD) = m ($\angle$ ABD) = 20°

(two inscribed angles subtended by $\widehat{AD}$)

$\therefore$ m ($\angle$ BCD) = 90° – 20° = 70° (Second req.)

5

[a] $\because$ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore$ AB = AC

In Δ ABC :

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = $\frac{180° - 50°}{2}$ = 65°

(First req.)

, m ($\angle$ D) (inscribed)

= m ($\angle$ ABC) (tangency) = 65° (Second req.)

[b] $\because$ $\overleftrightarrow{AB}$ // $\overline{XY}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ AXY) = m ($\angle$ BAX) (alternate angles)

, $\because$ m ($\angle$ D) (inscribed) = m ($\angle$ BAC) (tangency)

$\therefore$ m ($\angle$ D) = m ($\angle$ AXY)

$\therefore$ XCDY is a cyclic quadrilateral. (Q.E.D.)

27 Matrouh

1

1 b   2 c   3 d   4 d   5 d   6 b

2

[a] In Δ AMB : $\because$ MA = MB = r

$\therefore$ m ($\angle$ A) = m ($\angle$ B) = $\frac{180° - 90°}{2}$ = 45°

(First req.)

, $\because$ cos ($\angle$ A) = $\frac{MA}{AB}$

$\therefore$ cos 45° = $\frac{MA}{10}$

$\therefore$ MA = 10 × $\frac{1}{\sqrt{2}}$ = $5\sqrt{2}$ cm. (Second req.)

[b] In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)

$\therefore$ $\frac{1}{2}$ m ($\angle$ ABC) = $\frac{1}{2}$ m ($\angle$ ACB)

$\therefore$ m ($\angle$ YBX) = m ($\angle$ YCX) and they are drawn on $\overline{YX}$ and on one side of it

$\therefore$ The figure BCXY is a cyclic quadrilateral.

(Q.E.D. 1)

$\therefore$ m ($\angle$ BXY) = m ($\angle$ BCY)

(they are drawn on $\overline{BY}$ and on one side of it)

$\because$ m ($\angle$ CBX) = m ($\angle$ BCY)

$\therefore$ m ($\angle$ CBX) = m ($\angle$ BXY)

and they are alternate angles

$\therefore$ $\overleftrightarrow{XY}$ // $\overleftrightarrow{BC}$ (Q.E.D. 2)

3

[a] $\because$ $\overline{DE}$ // $\overline{BC}$

$\therefore$ m ($\widehat{BD}$) = m ($\widehat{EC}$)

$\therefore$ m ($\angle$ BAD) = m ($\angle$ CAE)

Adding m ($\angle$ BAC) to both sides :

$\therefore$ m ($\angle$ DAC) = m ($\angle$ BAE) (Q.E.D.)

[b] $\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ AYX) = m ($\angle$ ACB) (corresponding angles)

, $\because$ m ($\angle$ ACB) (inscribed) = m ($\angle$ BAD) (tangency)

$\therefore$ m ($\angle$ AYX) = m ($\angle$ XAD)

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

4

[a] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle

$\therefore$ AB = AC

In Δ ABC :

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = $\frac{180° - 50°}{2}$ = 65°

, $\because$ EBCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ EBC) = 180° – 115° = 65°

$\therefore$ m ($\angle$ EBC) = m ($\angle$ ABC) = 65°

$\therefore$ $\overrightarrow{BC}$ bisects $\angle$ ABE (Q.E.D. 1)

, ∵ m (∠ BEC) (inscribed)

= m (∠ ABC) (tangency) = 65°

In Δ BCE : ∴ m (∠ EBC) = m (∠ BEC) = 65°

∴ CB = CE (Q.E.D. 2)

[b] ∵ $\overleftrightarrow{BC}$ is a tangent

∴ m (∠ BMD) (central)

= 2 m (∠ CBD) (tangency) = 2 × 50° = 100°

∴ m (∠ AMD) = 180° – 100° = 80° (The req.)

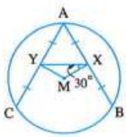
5

[a] ∵ X is the midpoint of $\overline{AB}$

∴ $\overline{MX} \perp \overline{AB}$

∵ Y is the midpoint of $\overline{AC}$

∴ $\overline{MY} \perp \overline{AC}$



∵ AB = AC ∴ MX = MY

∴ Δ MXY is an isosceles triangle (Q.E.D. 1)

∵ m (∠ AXM) = 90° , m (∠ MXY) = 30°

∴ m (∠ AXY) = 90° – 30° = 60° (1)

∵ X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$,

AB = AC

∴ AX = AY (2)

From (1) and (2) :

∴ Δ AXY is an equilateral triangle. (Q.E.D. 2)

[b] ∵ MNLE is a cyclic quadrilateral

∴ m (∠ E) = 180° – 110° = 70°

, ∵ $\overline{LE}$ is a diameter

∴ m (∠ LME) = 90°

In Δ LME : ∴ m (∠ MLE) = 180° – (90° + 70°)

= 20° (The req.)

Governorates' Examinations on Geometry

1 Cairo Governorate

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 The measure of the reflex angle of the angle whose measure is 100° equals ....°

(a) 80 (b) 90 (c) 200 (d) 260

2 If A lies on the circle M of diameter length 8 cm. , then MA = ... cm.

(a) 2 (b) 4 (c) 6 (d) 8

3 The number of axes of symmetry of the parallelogram equals ....

(a) 0 (b) 1 (c) 2 (d) 3

4 If ABCD is a cyclic quadrilateral , where m (∠ B) = 50° , then m (∠ D) = ...°

(a) 25 (b) 50 (c) 100 (d) 130

5 If the measure of one of the two base angles of an isosceles triangle is 40° , then the measure of the vertex angle is ....°

(a) 40 (b) 80 (c) 100 (d) 140

6 The inscribed angle drawn in a semicircle is .... angle.

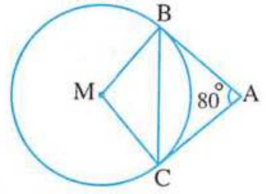
(a) an acute (b) a right (c) an obtuse (d) a straight

2 [a] Find the measure of the arc which represents $\frac{1}{4}$ the measure of the circle , then calculate the length of this arc if the radius length of the circle is 14 cm. $\left(\text{Where } \pi = \frac{22}{7}\right)$

[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M at B and C , m (∠ A) = 80°

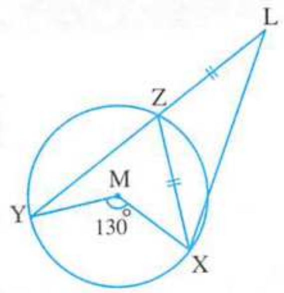
Find with proof : m (∠ BCM)



3 [a] Using your geometric tools , draw $\overline{AB}$ with length 5 cm. , then draw a circle passing through the two points A and B whose radius length is 3 cm. How many circles can be drawn ?

[b] In the opposite figure :

M is a circle , m (∠ XMY) = 130° and ZX = ZL



Find with proof :

1 m $(\overset{\frown}{XY})$

2 m (∠ XZY)

3 m (∠ L)

---

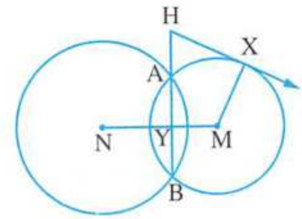
4 [a] **In the opposite figure :**

M and N are two intersecting circles at A and B

, $\overrightarrow{HX}$ is a tangent to the circle M at X

, $\overline{MN} \cap \overline{AB} = \{Y\}$

Prove that : HXMY is a cyclic quadrilateral.



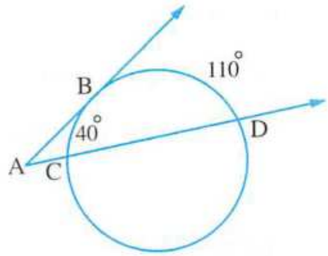
[b] **In the opposite figure :**

If $\overrightarrow{AB}$ is a tangent to the circle at B

, $\overrightarrow{AC}$ intersects the circle at C , D

, m $(\overset{\frown}{BD})$ = 110° , m $(\overset{\frown}{BC})$ = 40°

, **find with proof :** m (∠ A)



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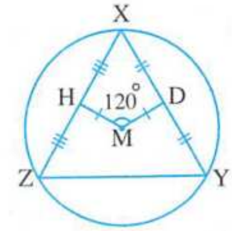
5 [a] **In the opposite figure :**

XYZ is an inscribed triangle in the circle M

, D , H are the midpoints of $\overline{XY}$ and $\overline{XZ}$ respectively

If MD = MH and m (∠ DMH) = 120°

, **prove that :** The triangle XYZ is an equilateral triangle.

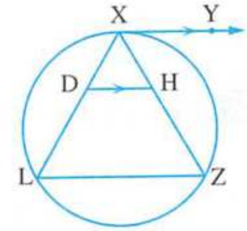


[b] **In the opposite figure :**

If $\overrightarrow{XY}$ is a tangent to the circle at X

, $\overrightarrow{XY} \mathbin{/\!/} \overline{DH}$

, **prove that :** DHZL is a cyclic quadrilateral.



2 Giza Governorate

Answer the following questions :

1 Choose the correct answer :

1 The point of concurrence of the medians of the triangle divides the median by the ratio …………… from the base.

(a) 3 : 9 (b) 3 : 1 (c) 4 : 2 (d) 2 : 4

2 If the straight line L is a tangent to the circle M whose diameter length is 8 cm.
, then the distance between L and the centre of the circle equals …………… cm.

(a) 3 (b) 4 (c) 6 (d) 8

3 The measure of the exterior angle of the equilateral triangle at any vertex equals ……………°

(a) 60 (b) 108 (c) 120 (d) 135

4 The measure of the arc which represents half the measure of the circle equals ……………°

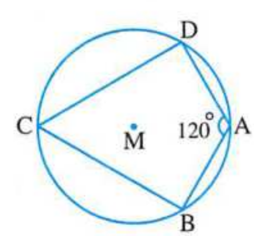
(a) 180 (b) 90 (c) 120 (d) 240

5 In Δ ABC , if $(BC)^2 = (AB)^2 + (AC)^2$, m (∠ B) = 50° , then m (∠ C) = …………… °

(a) 90 (b) 50 (c) 40 (d) 130

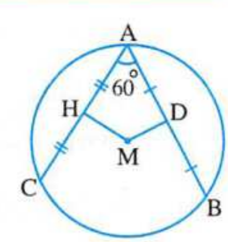
6 **In the opposite figure :**

M is a circle , m (∠ A) = 120°
, then m (∠ C) = ……………°



(a) 110 (b) 60
(c) 55 (d) 180

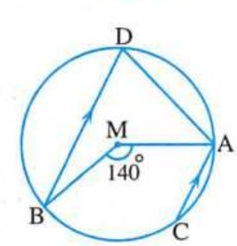
2 [a] **In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two chords in a circle M
, D is the midpoint of $\overline{AB}$
, H is the midpoint of $\overline{AC}$
, m (∠ BAC) = 60°

Find with proof : m (∠ DMH)

[b] **In the opposite figure :**



$\overline{AC}$ // $\overline{DB}$, m (∠ AMB) = 140°

Find : m (∠ CAD) with proof.

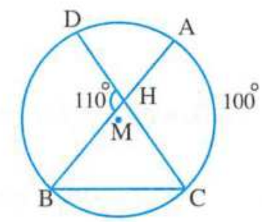
3 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{DC}$ are two chords in a circle

, $\overline{AB} \cap \overline{CD} = \{H\}$, m (∠ DHB) = 110°

, m ($\overset{\frown}{AC}$) = 100°

Find : m (∠ DCB)

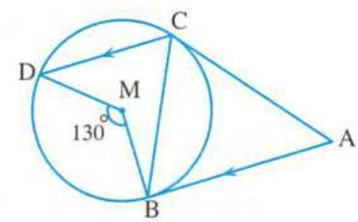


[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments

to the circle M , $\overline{AB}$ // $\overline{CD}$

, m (∠ BMD) = 130°



1 **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD

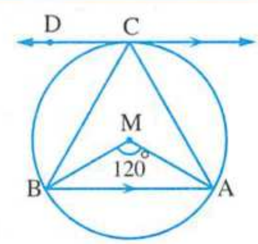
2 **Find :** m (∠ A)

4 [a] **In the opposite figure :**

$\overleftrightarrow{CD}$ is a tangent to the circle at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$, m (∠ AMB) = 120°

Prove that : Δ CAB is an equilateral triangle.



[b] **In the opposite figure :**

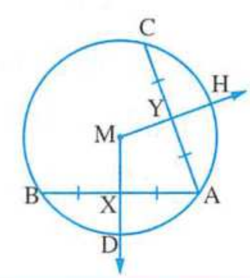
$\overline{AB}$ and $\overline{AC}$ are two chords equal

in length in the circle M

, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$

Prove that : XD = YH

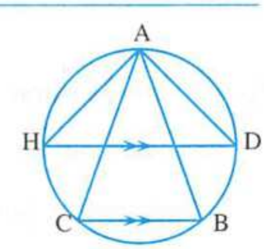


5 [a] **In the opposite figure :**

ABC is a triangle inside the circle

, $\overline{DH}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAH)



[b] **In the opposite figure :**

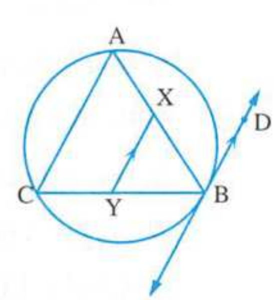
ABC is a triangle inside the circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, X ∈ $\overline{AB}$, Y ∈ $\overline{BC}$, $\overline{XY}$ // $\overleftrightarrow{BD}$

Prove that :

AXYC is a cyclic quadrilateral.



3 Alexandria Governorate

Answer the following questions : ***(Calculators are permitted)***

1 Choose the correct answer from those given :

1 If the straight line L is a tangent to the circle of diameter length 8 cm.
, then the distance between L and the centre of the circle equals .... cm.

(a) 3 (b) 4 (c) 6 (d) 8

2 The square whose side length is 5 cm. , then its surface area equals .... cm^2.

(a) 20 (b) 50 (c) 25 (d) 100

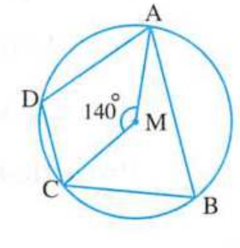
3 The inscribed angle drawn in a semicircle is ....

(a) acute. (b) obtuse. (c) straight. (d) right.

4 The intersection point of the medians of the triangle divides each median by the ratio .... from the base.

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

5 **In the opposite figure :**



A circle M , m (∠ CMA) = 140°

, then m (∠ CDA) = ...°

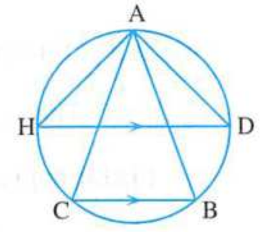
(a) 70 (b) 110

(c) 40 (d) 140

6 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) 2 (b) $\sqrt{2}$ (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

2 [a] In the opposite figure :



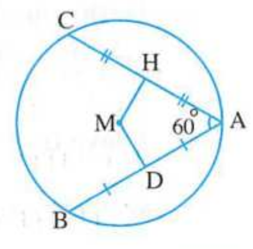
ABC is an inscribed triangle in a circle

, $\overline{DH}$ // $\overline{BC}$

Prove that :

m (∠ DAB) = m (∠ CAH)

[b] In the opposite figure :



$\overline{AB}$ and $\overline{AC}$ are two chords

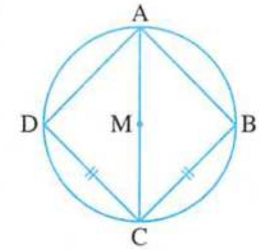
in the circle M , D is the midpoint of $\overline{AB}$

, H is the midpoint of $\overline{AC}$, m (∠ A) = 60°

Find with proof : m (∠ DMH)

3 [a] **In the opposite figure :**

ABCD is a quadrilateral inscribed in a circle M , $\overline{AC}$ is a diameter in the circle , CB = CD



Prove that : m $(\widehat{AB})$ = m $(\widehat{AD})$

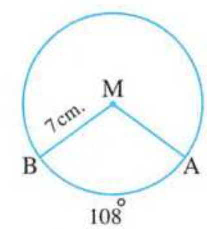
[b] ABC is an inscribed triangle in a circle , X $\in \widehat{AB}$, Y $\in \widehat{AC}$ where m $(\widehat{AX})$ = m $(\widehat{AY})$, $\overline{CX} \cap \overline{AB} = \{D\}$, $\overline{BY} \cap \overline{AC} = \{H\}$ **Prove that :** BCHD is a cyclic quadrilateral.

4 [a] **In the opposite figure :**

M is a circle with radius length 7 cm.

, m $(\widehat{AB})$ = 108°

Find : the length of $\widehat{AB}$ $\left(\pi = \frac{22}{7}\right)$

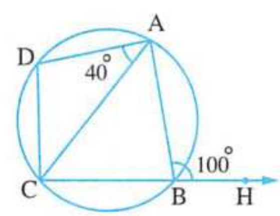


[b] **In the opposite figure :**

m (∠ ABH) = 100°

, m (∠ CAD) = 40°

Prove that : CD = AD



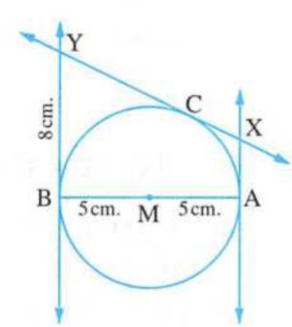
5 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M , C $\in$ the circle M

A tangent was drawn to the circle at C to intersect the two drawn tangents for it at A , B at X , Y respectively where AB = 10 cm.

, XC = 5 cm. , YB = 8 cm.

Find : the perimeter of AXYB

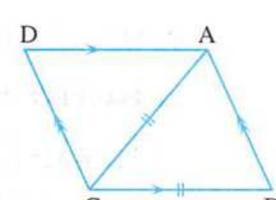


[b] **In the opposite figure :**

ABCD is a parallelogram in which AC = BC

Prove that :

$\overleftrightarrow{CD}$ is a tangent to the circle circumscribed about the triangle ABC



4 El-Kalyoubia Governorate

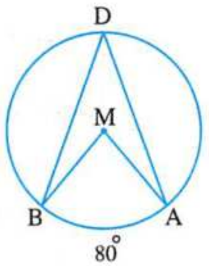
Answer the following questions :

1 Choose the correct answer :

1 The measure of the inscribed angle drawn in a semicircle equals ....°

(a) 360 (b) 180 (c) 120 (d) 90

2 **In the opposite figure :**



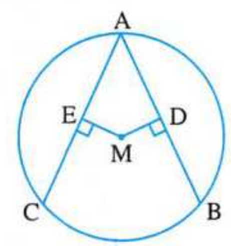
A circle of centre M , m $(\widehat{AB}) = 80°$

, then m (∠ ADB) = ...°

(a) 40 (b) 60

(c) 120 (d) 160

3 **In the opposite figure :**



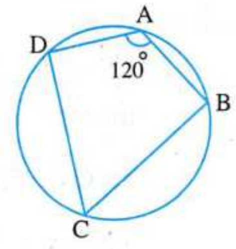
AB = AC , $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

, MD = 6 cm. , then ME = ... cm.

(a) 12 (b) 8

(c) 6 (d) 3

4 **In the opposite figure :**



If m (∠ A) = 120°

, then m (∠ C) = ...°

(a) 150 (b) 120

(c) 90 (d) 60

5 If the surface of circle M ∩ the surface of circle N = {A} , then the two circles M and N are ....

(a) touching internally. (b) touching externally.

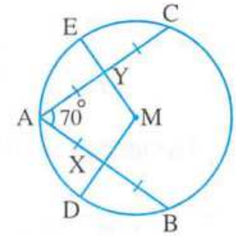
(c) intersecting. (d) concentric.

6 The number of the common tangents of two circles touching externally is ....

(a) 0 (b) 1 (c) 2 (d) 3

2 **[a] In the opposite figure :**

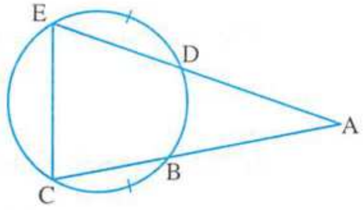
$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$, m (∠ CAB) = 70°



1 **Calculate :** m (∠ DME)

2 **Prove that :** XD = YE

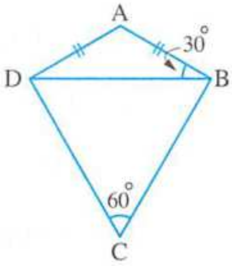
[b] In the opposite figure :



m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{DE}$)

Prove that : AB = AD

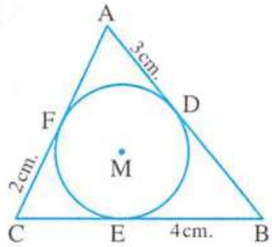
3 **[a] In the opposite figure :**



ABCD is a quadrilateral in which AB = AD , m (∠ ABD) = 30° , m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.

[b] In the opposite figure :

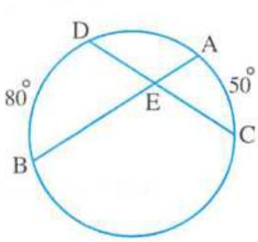


A circle is drawn touching the sides of the triangle ABC , $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at D , E , F , AD = 3 cm. , BE = 4 cm. , CF = 2 cm.

Find : the perimeter of Δ ABC

4 **[a]** ABC is a triangle inscribed in a circle , $\overleftrightarrow{AD}$ is a tangent to the circle at A , X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$, **prove that :** $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

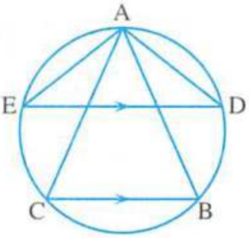
[b] In the opposite figure :



$\overline{AB}$ and $\overline{CD}$ are two chords in the circle , $\overline{AB}$ ∩ $\overline{CD}$ = {E} , m ($\overset{\frown}{DB}$) = 80° , m ($\overset{\frown}{AC}$) = 50°

Find : m (∠ AEC)

5 **[a] In the opposite figure :**



ABC is an inscribed triangle inside a circle , $\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)

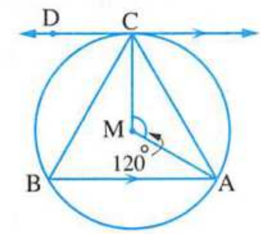
[b] In the opposite figure :

$\overleftrightarrow{CD}$ is a tangent to the circle at C

, $\overleftrightarrow{CD} \parallel \overline{AB}$, m (∠ AMC) = 120°

Prove that :

The triangle CAB is an equilateral triangle.



5 El-Sharkia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 The number of circles passing through three collinear points is ....

(a) zero. (b) 1 (c) 2 (d) 3

2 M and N are two circles touching internally. If the radius length of the circle M is 3 cm. and the radius length of the circle N is 1 cm. , then MN = ... cm.

(a) 1 (b) 4 (c) 3 (d) 2

3 If ABCD is a cyclic quadrilateral and m (∠ A) = 70° , then m (∠ C) = ...°

(a) 140 (b) 110 (c) 100 (d) 70

4 A circle of centre M and the length of its diameter is 6 cm. , A is a point in the plane of the circle M , if MA = 3 cm. , then A lies ....

(a) inside the circle. (b) outside the circle.

(c) on the circle. (d) on the centre of the circle.

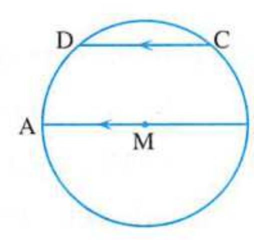
5 **In the opposite figure :**

M is a circle , m $(\overset{\frown}{BC})$ = 50°

, $\overline{AB} \parallel \overline{DC}$, then m $(\overset{\frown}{CD})$ = ...°

(a) 100 (b) 60

(c) 120 (d) 80



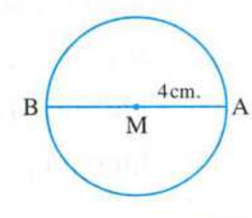
6 **In the opposite figure :**

M is a circle , $\overline{AB}$ is a diameter of the circle

, MA = 4 cm. , then the length of $\overset{\frown}{AB}$ = ... cm.

(a) 2 π (b) 4 π

(c) 8 π (d) 6 π



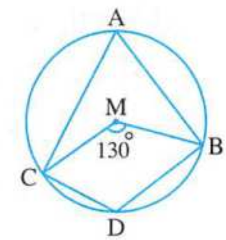
2 [a] **In the opposite figure :**

A circle of centre M

, in which m (∠ BMC) = 130°

Find : 1 m (∠ A)

2 m (∠ D)



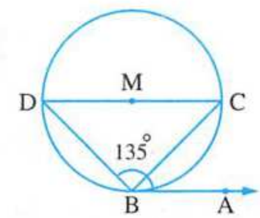
[b] **In the opposite figure :**

$\overline{DC}$ is a diameter of the circle M

, $\overrightarrow{BA}$ is a tangent to the circle M at B

, m (∠ ABD) = 135°

Prove that : $\overline{DC}$ // $\overrightarrow{BA}$



3 [a] **In the opposite figure :**

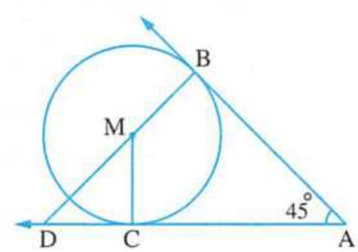
$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle M at B

, C respectively , m (∠ A) = 45° , $\overrightarrow{BM} \cap \overrightarrow{AC} = \{D\}$

Prove that :

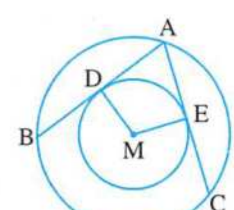
1 The figure ABMC is a cyclic quadrilateral.

2 CD = CM



[b] **In the opposite figure :**

Two concentric circles at M , $\overline{AC}$ and $\overline{AB}$ are two tangent-segments to the smaller circle at E and D and intersect the greater circle at C and B respectively. **Prove that :** AC = AB



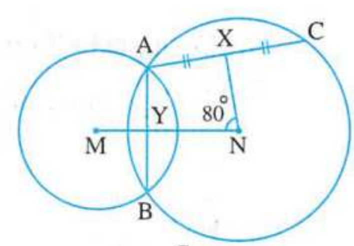
4 [a] **In the opposite figure :**

M and N are two intersecting circles at A and B

, $\overline{MN} \cap \overline{AB} = \{Y\}$, m (∠ YNX) = 80°

, X is the midpoint of $\overline{AC}$

Find : m (∠ BAC)

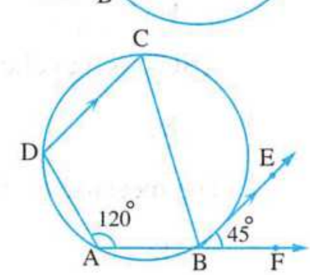


[b] **In the opposite figure :**

$\overrightarrow{BE}$ // $\overline{DC}$, m (∠ DAB) = 120°

, m (∠ FBE) = 45°

Find : m (∠ CDA)

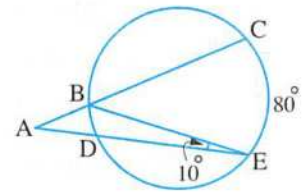


5 [a] **In the opposite figure :**

$\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$, m (∠ BED) = 10°

, m ($\overset{\frown}{EC}$) = 80°

Find : m (∠ A)

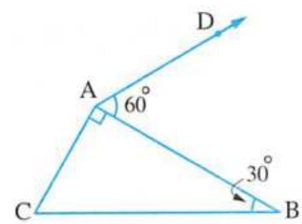


[b] **In the opposite figure :**

ABC is a right-angled triangle at A

, m (∠ DAB) = 60° , m (∠ B) = 30°

Prove that : $\overrightarrow{AD}$ is a tangent to the circle passing through the points A , B and C



6 El-Monofia Governorate

Answer the following questions : ***(Calculators are permitted)***

1 **Choose the correct answer from those given :**

1 The area of a square is 50 cm^2 , then the length of its diagonal is .... cm.

(a) 5 (b) 10 (c) 15 (d) 25

2 ∠ A , ∠ B are two complementary angles , m (∠ A) = $\frac{1}{2}$ m (∠ B) , then m (∠ A) = ...°

(a) 30 (b) 45 (c) 60 (d) 90

3 Δ ABC is right-angled at B , m (∠ C) = 30° , AC = 6 cm. , then AB = ... cm.

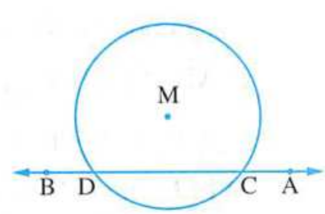
(a) 12 (b) 6 (c) 3 (d) $3\sqrt{3}$

4 **In the opposite figure :**

$\overleftrightarrow{AB} \cap$ the surface of the circle M = ...

(a) ∅ (b) {C , D}

(c) $\overline{CD}$ (d) $\overleftrightarrow{CD}$



5 ABCD is a cyclic quadrilateral , then [m (∠ A) + m (∠ C) – 100°] = ...°

(a) 80 (b) 100 (c) 180 (d) 280

6 The measure of the inscribed angle in a semicircle equals ....°

(a) 45 (b) 135 (c) 90 (d) 150

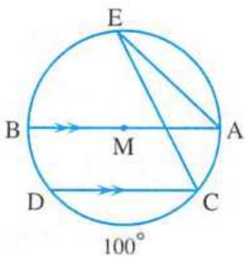
2 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M , $\overline{AB} // \overline{CD}$

If m ($\overset{\frown}{CD}$) = 100° , m (∠ AEC) = 2 x – 10°

1 **Calculate :** m ($\overset{\frown}{BD}$)

2 **Find :** the value of x



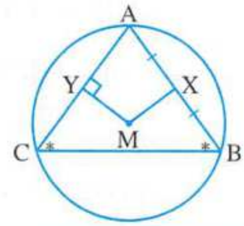
[b] In the opposite figure :

Δ ABC is inscribed in the circle M

, m (∠ B) = m (∠ C)

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY



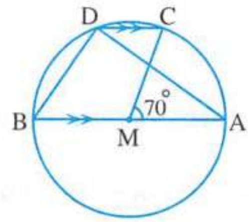
3 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M , $\overline{CD} // \overline{AB}$

, m (∠ AMC) = 70°

Calculate :

1 m (∠ ADC) **2** m (∠ ABD)

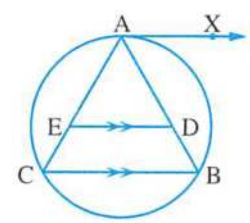


[b] In the opposite figure :

Δ ABC is inscribed in a circle

, $\overrightarrow{AX}$ is a tangent to the circle , $\overline{DE} // \overline{BC}$

Prove that : $\overrightarrow{AX}$ is a tangent to the circle passing through the points A , D and E



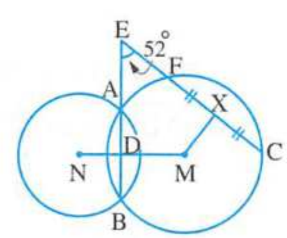
4 **[a] In the opposite figure :**

Two circles M and N are intersecting at A and B

, E ∈ $\overrightarrow{BA}$, $\overline{EC}$ intersects the circle M at C , F

, X is the midpoint of $\overline{CF}$, m (∠ E) = 52°

Calculate : m (∠ XMD)



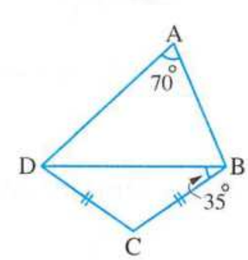
[b] In the opposite figure :

m (∠ A) = 70°

, m (∠ DBC) = 35° , CB = CD

Prove that :

ABCD is a cyclic quadrilateral.

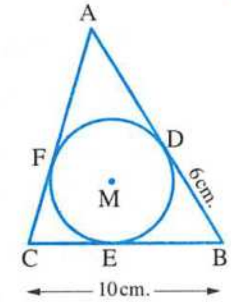


5 **[a] In the opposite figure :**

The circle M touches the sides of Δ ABC at D , E and F

If BC = 10 cm. , DB = 6 cm.

, **calculate :** the length of $\overline{CE}$

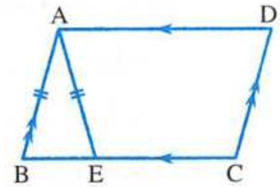


[b] In the opposite figure :

ABCD is a parallelogram

, AB = AE

Prove that : AECD is a cyclic quadrilateral.



7 El-Gharbia Governorate

Answer the following questions :

1 **Choose the correct answer :**

1 The measure of the inscribed angle which is drawn in $\frac{1}{3}$ a circle equals .... °

(a) 240 (b) 120 (c) 60 (d) 30

2 If the surface of the circle M $\cap$ the surface of the circle N = $\{A\}$, then the two circles M and N are ....

(a) distant. (b) one is inside the other.

(c) intersecting. (d) touching externally.

3 ABC is an equilateral triangle , then the number of symmetry axes of the side $\overline{BC}$ equals ....

(a) 3 (b) 2 (c) 1 (d) 0

4 ABC is a triangle in which : $(AB)^2 + (BC)^2 < (AC)^2$, then $\angle$ C is ....

(a) right. (b) acute. (c) straight. (d) obtuse.

5 The .... is a cyclic quadrilateral.

(a) trapezium (b) rhombus (c) rectangle (d) parallelogram

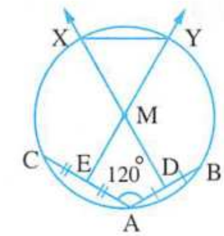
6 A rhombus whose diagonals lengths are 6 cm. and 10 cm. , then its area is .... cm^2.

(a) 60 (b) 15 (c) 30 (d) 10

2 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords including an angle of measure 120° , D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , $\overrightarrow{DM}$ and $\overrightarrow{EM}$ intersect the circle at X and Y respectively.

Prove that : Δ XYM is an equilateral triangle.

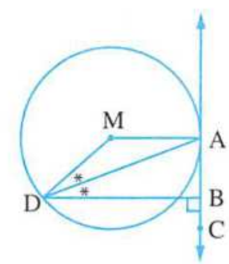


[b] **In the opposite figure :**

$\overrightarrow{DA}$ bisects ∠ BDM and cuts the circle at A , $\overline{DB} \perp \overleftrightarrow{AB}$

Prove that :

$\overleftrightarrow{AB}$ is a tangent to the circle M at A

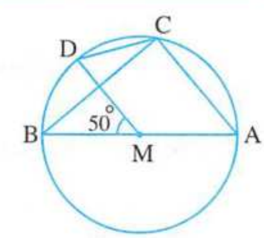


3 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m (∠ BMD) = 50°

Find : m (∠ ACD)

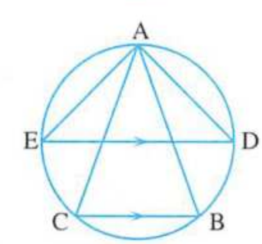


[b] **In the opposite figure :**

ABC is a triangle inscribed in a circle , $\overline{DE}$ // $\overline{BC}$

Prove that :

m (∠ DAC) = m (∠ BAE)



4 [a] **In the opposite figure :**

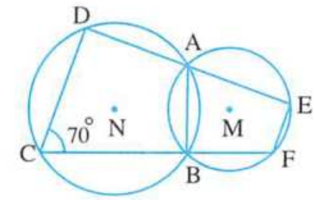
M and N are two intersecting circles at A and B

, $\overleftrightarrow{AD}$ is drawn to intersect circle M at E and circle N at D

, $\overleftrightarrow{BC}$ is drawn to intersect circle M at F and circle N at C

and m (∠ C) = 70°

1 **Find :** m (∠ F) 2 **Prove that :** $\overline{CD}$ // $\overline{EF}$

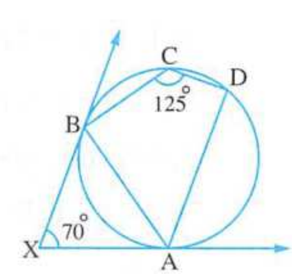


[b] **In the opposite figure :**

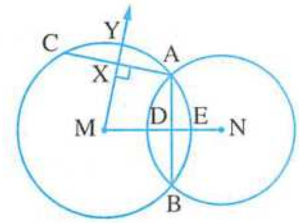
$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B , m (∠ AXB) = 70°

and m (∠ DCB) = 125°

Prove that : $\overrightarrow{AB}$ bisects ∠ DAX



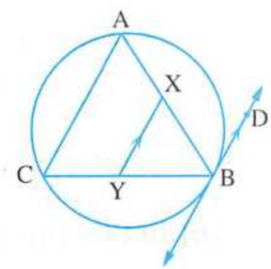
5 [a] **In the opposite figure :**



M and N are two circles intersecting at A and B , $\overrightarrow{MX} \perp \overline{AC}$ and intersects $\overline{AC}$ at X and intersects the circle M at Y , $\overline{MN}$ is drawn to intersect $\overline{AB}$ at D and intersect the circle M at E , if AC = AB

, **prove that :** XY = DE

[b] **In the opposite figure :**



ABC is a triangle inscribed in a circle ,

$\overleftrightarrow{BD}$ is a tangent to the circle at B , $X \in \overline{AB}$ and $Y \in \overline{BC}$, where $\overline{XY} \mathbin{/\!/} \overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.

8 El-Dakahlia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 [a] **Choose the correct answer :**

1 The two tangents which are drawn from the two endpoints of a diameter of a circle are ....

(a) parallel. (b) intersecting. (c) perpendicular. (d) equal.

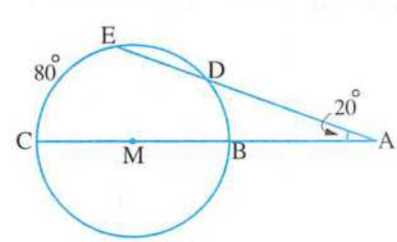
2 A chord is of length 8 cm. , in a circle of radius length 5 cm. , then the chord is at .... cm. from the centre of the circle.

(a) 1 (b) 2 (c) 3 (d) 4

3 The measure of the central angle which is opposite to an arc of length $\frac{1}{3}$ π r equals ....°

(a) 30 (b) 60 (c) 120 (d) 240

[b] **In the opposite figure :**



$\overline{BC}$ is a diameter of the circle M

, m (∠ A) = 20° , m $(\overset{\frown}{CE})$ = 80°

Find : m $(\overset{\frown}{DE})$

2 **[a] Choose the correct answer :**

1 The number of symmetry axes of two circles touching externally is ....

(a) 0 (b) 1 (c) 2 (d) ∞

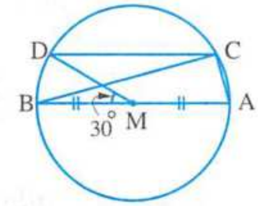
2 If the point A lies on the surface of the circle M and the length of its diameter is 6 cm. , then MA $\in$ ....

(a) $]-\infty , 6]$ (b) $]-\infty , 3]$ (c) $[0 , 3]$ (d) $]3 , \infty[$

3 ABCD is a quadrilateral inscribed in a circle , m ($\angle$ A) = 70° , then m ($\widehat{BAD}$) = ...°

(a) 35 (b) 55 (c) 140 (d) 220

[b] In the opposite figure :

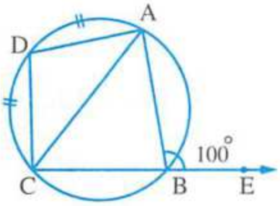


$\overline{AB}$ is a diameter of a circle M

, m ($\angle$ BMD) = 30°

Find : 1 m ($\angle$ BCD) 2 m ($\angle$ ACD)

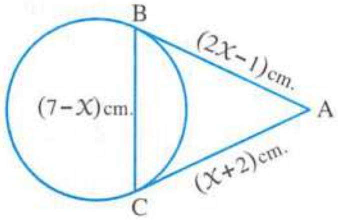
3 **[a] In the opposite figure :**



ABCD is a quadrilateral inscribed in a circle , E $\in \overrightarrow{CB}$, m ($\angle$ ABE) = 100° , D is the midpoint of $\widehat{AC}$

Find : m ($\angle$ DAC)

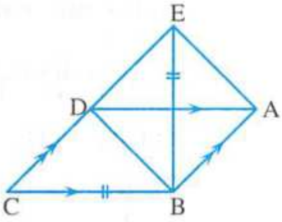
[b] In the opposite figure :



$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C , AB = (2 X − 1) cm. , AC = (X + 2) cm. , BC = (7 − X) cm.

Find : 1 The value of X 2 The perimeter of Δ ABC

4 **[a] In the opposite figure :**



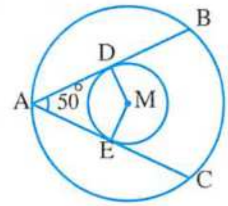
ABCD is a parallelogram ,

E $\in \overrightarrow{CD}$, BE = BC

Prove that : 1 ABDE is a cyclic quadrilateral.

2 m ($\angle$ AEB) = m ($\angle$ DBC)

[b] In the opposite figure :



Two concentric circles at M

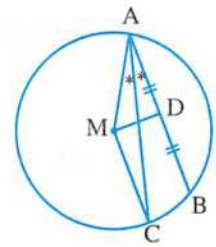
, $\overline{AB}$ and $\overline{AC}$ are two chords in the greater circle and two tangent-segments to the smaller circle at D , E respectively , m (∠ A) = 50°

1 **Find :** m (∠ EMD)

2 **Prove that :** AB = AC

---

5 **[a] In the opposite figure :**

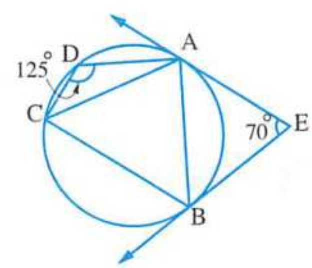


$\overline{AB}$ is a chord in a circle M

, D is the midpoint of $\overline{AB}$ and $\overrightarrow{AC}$ bisects ∠ BAM

Prove that : $\overline{DM} \perp \overline{CM}$

[b] In the opposite figure :



$\overrightarrow{EA}$ and $\overrightarrow{EB}$ are two tangents to the circle at A and B , m (∠ E) = 70° , m (∠ D) = 125°

Prove that :

1 AB = AC

2 $\overrightarrow{AC}$ is a tangent to the circle passing through the vertices of Δ ABE

9 Ismailia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The longest chord in the circle is called ....

(a) a tangent. (b) a secant. (c) a diameter. (d) an arc.

2 If the two circles M , N are touching internally , their radii lengths are 7 cm. , 10 cm. , then , MN = ... cm.

(a) 1 (b) 3 (c) 7 (d) 17

3 The inscribed angle drawn in a semicircle is ....

(a) acute. (b) obtuse. (c) straight. (d) right.

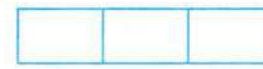
4 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

5 ABCD is a cyclic quadrilateral in which m (∠ A) = 70° , then m (∠ C) = ……………°

(a) 20 (b) 25 (c) 10 (d) 110

6 The number of rectangles in the opposite figure is ……………

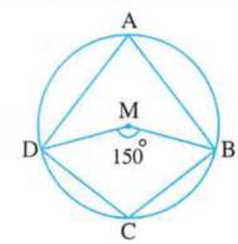


(a) 4 (b) 5 (c) 6 (d) 7

2 [a] **In the opposite figure :**

A circle M , m (∠ BMD) = 150°

Find with proof : m (∠ C)



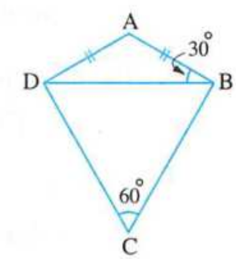
[b] **In the opposite figure :**

ABCD is a quadrilateral in which AB = AD

, m (∠ ABD) = 30° , m (∠ C) = 60°

Prove that :

ABCD is a cyclic quadrilateral.



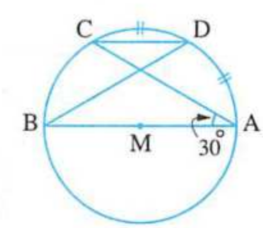
3 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M , m ($\widehat{AD}$) = m ($\widehat{DC}$)

, m (∠ CAB) = 30°

1 **Find with proof :** m (∠ CDB)

2 **Prove that :** $\overline{CD}$ // $\overline{BA}$



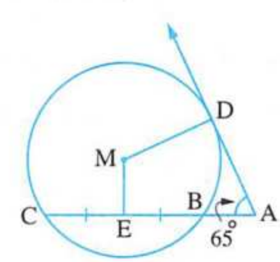
[b] **In the opposite figure :**

$\overrightarrow{AD}$ is a tangent to the circle M

, $\overline{AC}$ intersects the circle at B , C and E

is the midpoint of $\overline{BC}$, m (∠ A) = 65°

Find with proof : m (∠ DME)



4 [a] **In the opposite figure :**

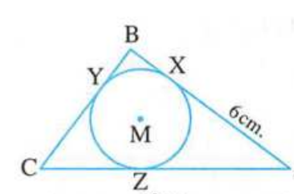
$\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ are tangent-segments to the circle M

at X , Y and Z respectively.

If AC = 10 cm. , AX = 6 cm.

and the perimeter of Δ ABC = 24 cm.

, **find :** The length of $\overline{AB}$



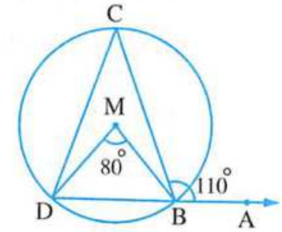
[b] In the opposite figure :

A circle of centre M , m (∠ BMD) = 80°

, m (∠ ABC) = 110°

1 **Find with proof :** m (∠ CDB)

2 **Prove that :** CB = CD



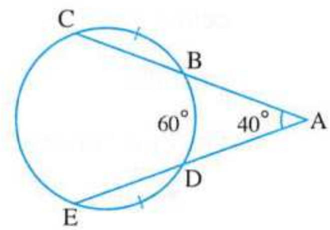
5 **[a] In the opposite figure :**

m (∠ A) = 40° , m $(\overset{\frown}{BD})$ = 60°

, m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{DE})$

Find : 1 m $(\overset{\frown}{EC})$

2 m $(\overset{\frown}{BC})$



[b] In the opposite figure :

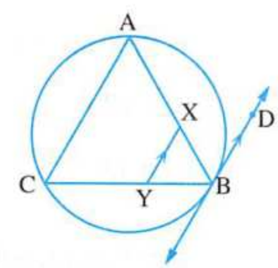
ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, X ∈ $\overline{AB}$, Y ∈ $\overline{BC}$

, where $\overline{XY}$ // $\overleftrightarrow{DB}$

Prove that : AXYC is a cyclic quadrilateral.



10 Suez Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

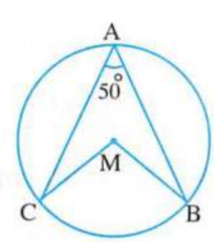
1 **In the opposite figure :**

If M is a circle , m (∠ BAC) = 50°

, then m (∠ BMC) = ...°

(a) 50 (b) 90

(c) 25 (d) 100



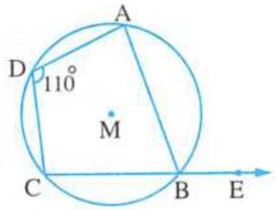
2 The number of circles which pass through three non-collinear points equals ....

(a) 0 (b) 1 (c) 2 (d) 3

3 **In the opposite figure :**

If M is a circle , $E \in \overrightarrow{CB}$, m (∠ ADC) = 110°

, then m (∠ ABE) = …………… °



(a) 70 (b) 55

(c) 110 (d) 80

4 The tangent to a circle of diameter length 6 cm. is at a distance of …………… from its centre.

(a) 6 cm. (b) 12 cm. (c) 3 cm. (d) 2 cm.

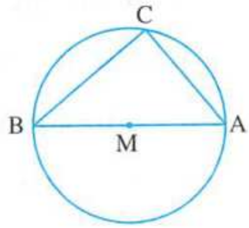
5 The circumference of the circle equals ……………

(a) $2 \pi r$ (b) πr^2 (c) $2 \pi r^2$ (d) πr

6 **In the opposite figure :**

If $\overline{AB}$ is a diameter of the circle M

, then m (∠ C) = …………… °



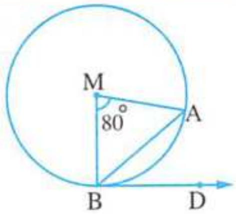
(a) 180 (b) 90

(c) 45 (d) 60

2 [a] **In the opposite figure :**

$\overrightarrow{BD}$ is a tangent to the circle M at B

, m (∠ BMA) = 80°



Find : 1 m (∠ ABD) 2 m $(\overset{\frown}{AB})$

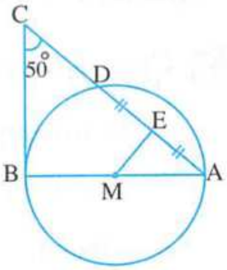
[b] **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M

, $\overline{BC}$ is a tangent-segment touching it at B

, E is the midpoint of $\overline{AD}$, m (∠ C) = 50°

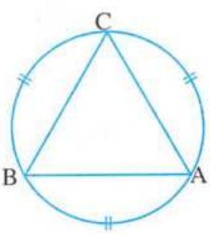
Find : m (∠ EMB)



3 [a] **In the opposite figure :**

m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{AC})$

Find : m (∠ C)

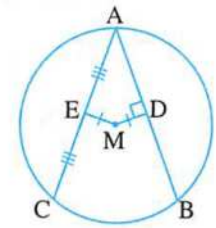


[b] **In the opposite figure :**

M is a circle , $\overline{MD} \perp \overline{AB}$

, E is the midpoint of $\overline{AC}$, MD = ME

Prove that : AB = AC

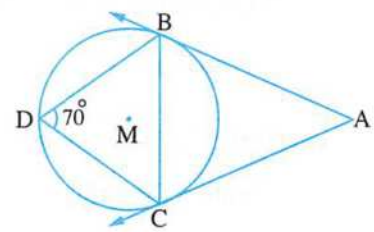


4 [a] **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle

at B and C , m (∠ BDC) = 70°

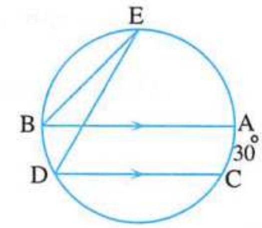
Find : 1 m (∠ ABC) 2 m (∠ BAC)



[b] **In the opposite figure :**

$\overline{AB}$ // $\overline{CD}$, m $(\widehat{AC})$ = 30°

Find : m (∠ BED)



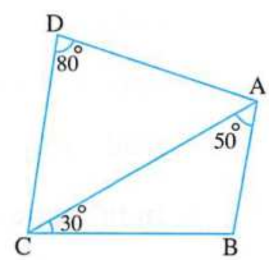
5 [a] State two cases of the cyclic quadrilateral.

[b] **In the opposite figure :**

m (∠ BAC) = 50° , m (∠ BCA) = 30°

, m (∠ ADC) = 80°

Prove that : ABCD is a cyclic quadrilateral.



11 Port Said Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The circumference of a circle of radius length 7 cm. is .... cm.

(a) 7 π (b) 8 π (c) 14 π (d) 49 π

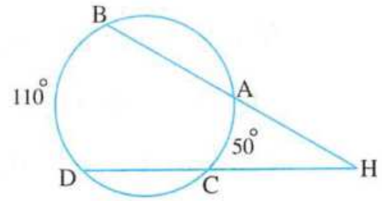
2 A circle can be drawn passing through the vertices of a ....

(a) rectangle. (b) rhombus. (c) trapezium. (d) parallelogram.

3 **In the opposite figure :**

m $(\overset{\frown}{AC}) = 50°$, m $(\overset{\frown}{BD}) = 110°$

, then m (∠ H) = ... °



(a) 60 (b) 50

(c) 40 (d) 30

4 The inscribed angle drawn in a semicircle is .... angle.

(a) an acute (b) a right (c) an obtuse (d) a straight

5 If the diameter length of a circle = 8 cm. and the line L is at a distance of 4 cm. from its centre , then the line L is .... the circle.

(a) a secant to (b) outside (c) a tangent to (d) a symmetry axis of

6 The number of common tangents of two distant circles is ....

(a) 4 (b) 3 (c) 2 (d) 1

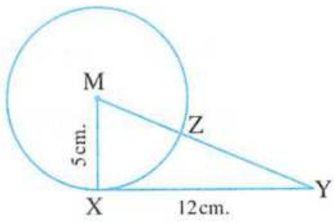
2 [a] **In the opposite figure :**

$\overline{XY}$ is a tangent-segment to the circle

, $\overline{MX}$ is a radius

, MX = 5 cm. , XY = 12 cm.

Find : YZ

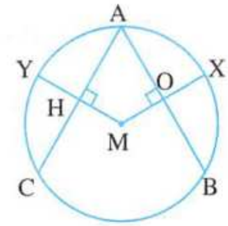


[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle

, $\overline{MO} \perp \overline{AB}$, $\overline{MH} \perp \overline{AC}$

Prove that : OX = HY



3 [a] Mention two cases in which the quadrilateral is cyclic.

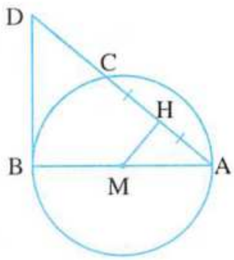
[b] **In the opposite figure :**

$\overline{AB}$ is a diameter in a circle M

, $\overline{BD}$ is a tangent-segment

and H is the midpoint of $\overline{AC}$

Prove that : DBMH is a cyclic quadrilateral.



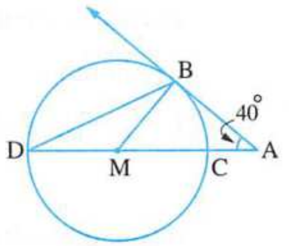
4 [a] **In the opposite figure :**

$\overrightarrow{AB}$ is a tangent

, $\overline{DC}$ is a diameter in a circle M

, m (∠ A) = 40°

Find : m (∠ BDC) with proof.

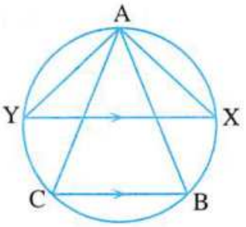


[b] **In the opposite figure :**

Δ ABC is inscribed in a circle

, $\overline{XY}$ // $\overline{BC}$

Prove that : m (∠ XAC) = m (∠ YAB)

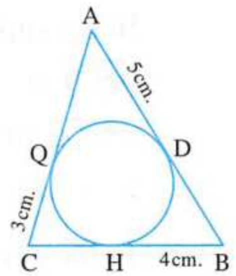


5 [a] **In the opposite figure :**

The sides of Δ ABC touches the circle externally at D , H and Q

, AD = 5 cm. , BH = 4 cm. , CQ = 3 cm.

Find : The perimeter of Δ ABC



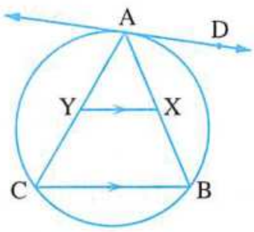
[b] **In the opposite figure :**

$\overleftrightarrow{AD}$ is a tangent to the circle at A

, $\overline{YX}$ // $\overline{CB}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , Y , X



12 Damietta Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from the given answers :**

1 The angle of measure 20° is the complementary angle of the angle of measure .... °

(a) 20 (b) 40 (c) 70 (d) 160

2 If the two circles M , N are touching externally , their radii lengths are 3 cm. , 7 cm. , then MN = ... cm.

(a) 3 (b) 4 (c) 6 (d) 10

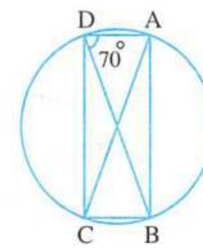
3 The two diagonals are perpendicular and not equal in length in the ....

(a) rhombus. (b) trapezium. (c) square. (d) parallelogram.

4 The measure of the inscribed angle in a semicircle is equal to .... °

(a) 30 (b) 60 (c) 90 (d) 180

5 **In the opposite figure :**



If m (∠ ADB) = 70° , then m (∠ ACB) = ... °

(a) 35 (b) 70

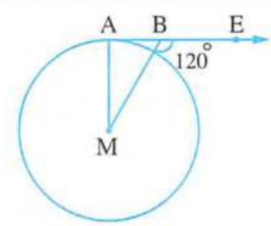
(c) 90 (d) 140

6 In Δ ABC , if $(AB)^2 = (AC)^2 + (BC)^2 + 3$, then ∠ C is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

---

2 [a] **In the opposite figure :**

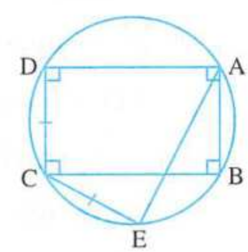


$\overrightarrow{AB}$ is a tangent to the circle at A

, m (∠ MBE) = 120°

Find with proof : m (∠ AMB)

[b] **In the opposite figure :**



ABCD is a rectangle inscribed in a circle

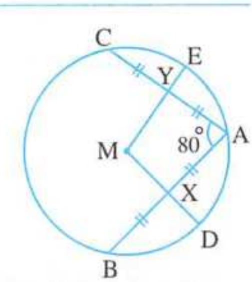
, the chord $\overline{CE}$ is drawn where CE = CD

Prove that :

1 m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{CE})$ 2 AE = BC

---

3 [a] **In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two chords equal in length

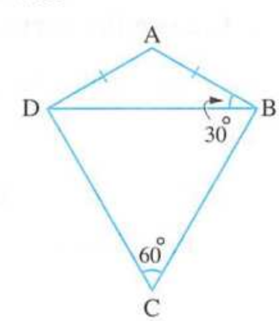
in the circle M , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$

, m (∠ BAC) = 80°

1 **Find :** m (∠ EMD) 2 **Prove that :** YE = XD

[b] **In the opposite figure :**



ABCD is a quadrilateral , AB = AD , m (∠ ABD) = 30°

, m (∠ C) = 60°

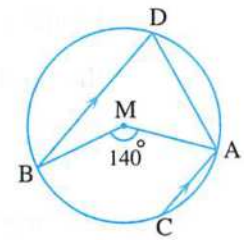
Prove that : ABCD is a cyclic quadrilateral.

4 [a] **In the opposite figure :**

$\overline{CA}$ and $\overline{BD}$ are two parallel chords

in the circle M , m ($\angle$ AMB) = 140°

Find with proof : m ($\angle$ CAD)



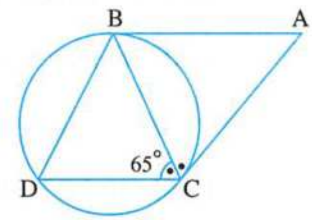
[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments

to the circle at B and C

, $\overrightarrow{CB}$ bisects $\angle$ ACD , m ($\angle$ BCD) = 65°

Find with proof : m ($\angle$ A) and m ($\angle$ D)



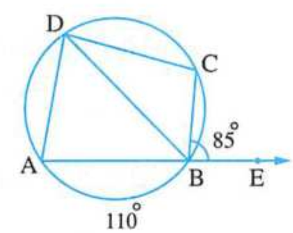
5 [a] **In the opposite figure :**

E $\in \overrightarrow{AB}$, E $\notin \overline{AB}$

, m ($\overparen{AB}$) = 110°

, m ($\angle$ CBE) = 85°

Find with proof : 1 m ($\angle$ ADB) 2 m ($\angle$ BDC)



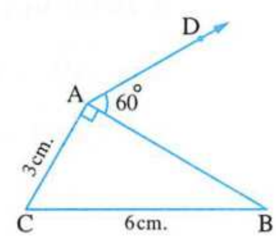
[b] **In the opposite figure :**

Δ ABC is right-angled at A

, AC = 3 cm. , BC = 6 cm. , m ($\angle$ DAB) = 60°

Prove that : $\overrightarrow{AD}$ is a tangent to the circle

passing through the vertices of Δ ABC



13 Kafr El-Sheikh Governorate

Answer the following questions : ***(Calculator is allowed)***

1 [a] **Choose the correct answer from those given :**

1 The measure of the arc which equals half the measure of the circle is .... °

(a) 360 (b) 180 (c) 120 (d) 90

2 ABC is a triangle in which $(AC)^2 > (AB)^2 + (BC)^2$, then the type of $\angle$ ABC is ....

(a) obtuse. (b) acute. (c) right. (d) straight.

3 M and N are two intersecting circles , their radii lengths are 3 cm. and 5 cm. , then : MN ∈ ....

(a)]8 , ∞[(b)]2 , ∞[(c)]0 , 2[(d)]2 , 8[

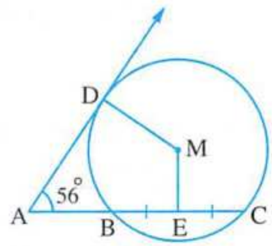
[b] **In the opposite figure :**

$\overrightarrow{AD}$ is a tangent to the circle M

, $\overrightarrow{AC}$ intersects the circle M at B and C

, E is the midpoint of $\overline{BC}$, m (∠ A) = 56°

Find : m (∠ DME)



2 [a] **Choose the correct answer from those given :**

1 The measure of the inscribed angle drawn in a semicircle equals .... °

(a) 45 (b) 120 (c) 90 (d) 180

2 The lateral area of a cube is 36 cm^2 , then its total area is .... cm^2

(a) 18 (b) 54 (c) 81 (d) 216

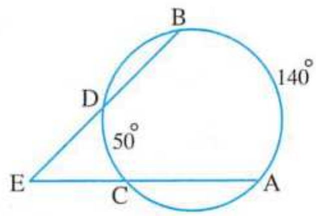
3 **In the opposite figure :**

m $(\overset{\frown}{AB})$ = 140°

, m $(\overset{\frown}{CD})$ = 50° , then m (∠ E) = ... °

(a) 45 (b) 40

(c) 95 (d) 55



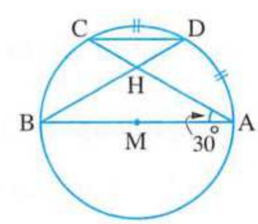
[b] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, C ∈ the circle M , m (∠ CAB) = 30°

, D is the midpoint of $\overset{\frown}{AC}$, $\overline{DB} \cap \overline{AC} = \{H\}$

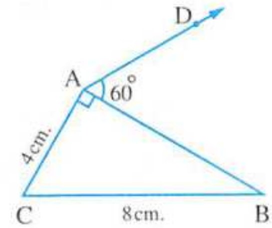
1 **Find :** m $(\overset{\frown}{AD})$ 2 **Prove that :** $\overline{AB}$ // $\overline{DC}$



3 [a] Two concentric circles at M , $\overline{AB}$ and $\overline{AC}$ are two chords in the larger circle touching the smaller circle at X and Y respectively

Prove that : AB = AC

[b] In the opposite figure :



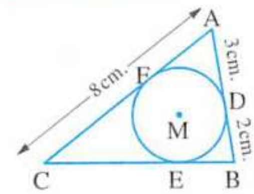
ABC is a triangle in which m (∠ BAC) = 90°

, BC = 8 cm. , AC = 4 cm.

, m (∠ BAD) = 60°

Prove that : $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC

4 **[a] In the opposite figure :**

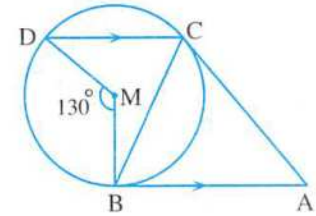


A circle is drawn touching the sides of the triangle ABC at D , E , F

, AD = 3 cm. , BD = 2 cm. , AC = 8 cm.

Find : The length of $\overline{BC}$

[b] In the opposite figure :



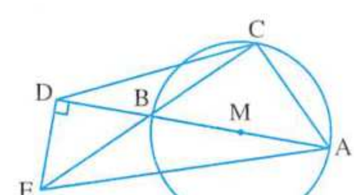
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB}$ // $\overline{CD}$, m (∠ BMD) = 130°

Find : m (∠ A)

5 **[a]** State two cases of a cyclic quadrilateral.

[b] In the opposite figure :



$\overline{AB}$ is a diameter in the circle M , D ∈ $\overleftrightarrow{AB}$, D ∉ $\overline{AB}$

, $\overleftrightarrow{DE} \perp \overleftrightarrow{AB}$, C ∈ $\overset{\frown}{AB}$, $\overleftrightarrow{CB} \cap \overleftrightarrow{DE}$ = {E}

1 **Find :** m (∠ ACB)

2 **Prove that :** ACDE is a cyclic quadrilateral.

14 El-Beheira Governorate

Answer the following questions : ***(Calculator is permitted)***

1 **Choose the correct answer from the given ones :**

1 If the origin point is the midpoint of $\overline{AB}$, A (5 , – 2) , then B is ....

(a) (5 , 2) (b) (5 , – 2) (c) (– 5 , – 2) (d) (– 5 , 2)

2 The slope of the straight line : 3 X + 2 y = 1 is ....

(a) $\frac{2}{3}$ (b) $\frac{-3}{2}$ (c) $\frac{-2}{3}$ (d) $\frac{3}{2}$

3 The measure of any interior angle of the regular pentagon is ....

(a) 90° (b) 108° (c) 120° (d) 135°

4 The ratio between the measure of the inscribed angle and the measure of the central angle subtended by the same arc equals ....

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3

5 It is possible to draw a circle passing through the vertices of a ....

(a) trapezium. (b) rhombus. (c) parallelogram. (d) rectangle.

6 If the length of a diameter of a circle is 7 cm. and the straight line L is at a distance of 3.5 cm. from its centre , then L is ....

(a) a secant to the circle at two points. (b) outside the circle.

(c) a tangent to the circle. (d) an axis of symmetry of the circle.

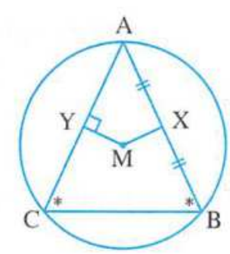
2 [a] **In the opposite figure :**

A triangle ABC is inscribed in the circle M

in which : m (∠ B) = m (∠ C)

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY



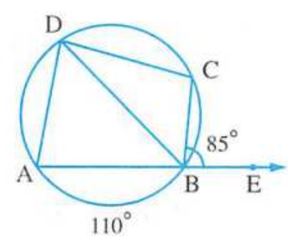
[b] **In the opposite figure :**

$E \in \overrightarrow{AB}$, $E \notin \overline{AB}$

, m $(\overset{\frown}{AB})$ = 110°

, m (∠ CBE) = 85°

Find : m (∠ BDC)



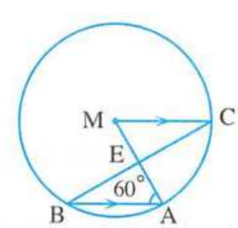
3 [a] **In the opposite figure :**

$\overline{AB}$ is a chord in the circle M

, $\overline{CM}$ // $\overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$

, m (∠ A) = 60°

Find : m (∠ B)

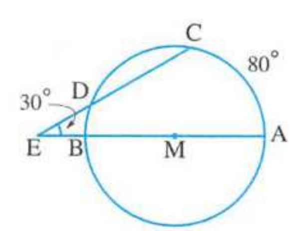


[b] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M , $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$

, m (∠ AEC) = 30° , m $(\overset{\frown}{AC})$ = 80°

Find : m $(\overset{\frown}{CD})$



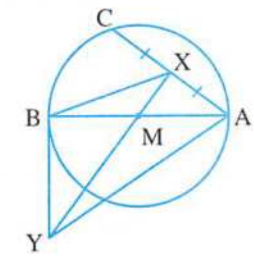
4 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, X is the midpoint of $\overline{CA}$

and $\overrightarrow{XM}$ intersects the tangent to the circle at B in Y

Prove that : The figure AXBY is a cyclic quadrilateral.



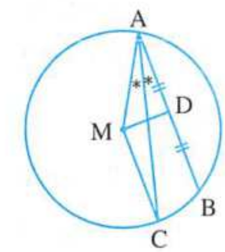
[b] **In the opposite figure :**

$\overline{AB}$ is a chord in the circle M , $\overrightarrow{AC}$ bisects ∠ BAM

and intersects the circle M at C

If D is the midpoint of $\overline{AB}$

, **prove that :** $\overline{DM} \perp \overline{CM}$

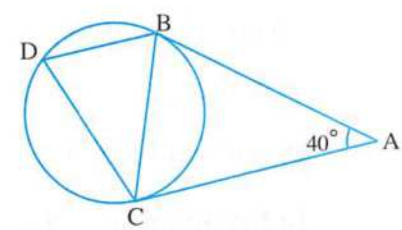


5 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments

to the circle at B and C , m (∠ A) = 40°

Find with proof : m (∠ D)



[b] **In the opposite figure :**

$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle

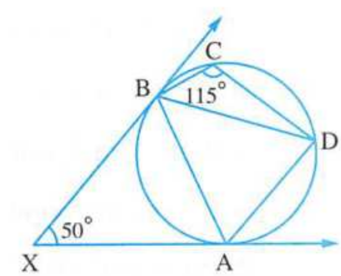
at A and B , m (∠ AXB) = 50°

, m (∠ DCB) = 115°

Prove that :

1 $\overrightarrow{AB}$ bisects ∠ DAX

2 BD = BA



15 El-Fayoum Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer :**

1 The inscribed angle in a semicircle is ·············· angle.

(a) an acute (b) an obtuse (c) a straight (d) a right

2 If ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , D is the midpoint of $\overline{AC}$, then BD = ·············· cm.

(a) 10 (b) 20 (c) 5 (d) otherwise

3 The tangent to a circle of diameter length 6 cm. is at a distance of .... cm. from its centre.

(a) 6 (b) 12 (c) 3 (d) 2

4 The number of axes of symmetry of the circle is ....

(a) 0 (b) 1 (c) 2 (d) infinite.

5 A regular polygon , the measure of one of its interior angles is 144° , then the number of its sides is .... sides.

(a) 7 (b) 8 (c) 9 (d) 10

6 In a cyclic quadrilateral , each two opposite angles are ....

(a) equal. (b) complementary. (c) supplementary. (d) alternate.

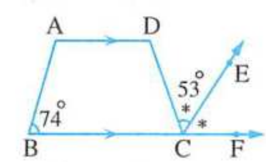
---

2 [a] **In the opposite figure :**

$\overline{AD} \parallel \overline{BC}$, m (∠ B) = 74°

, $\overrightarrow{CE}$ bisects ∠ DCF , m (∠ DCE) = 53°

Prove that : ABCD is a cyclic quadrilateral.



[b] **In the opposite figure :**

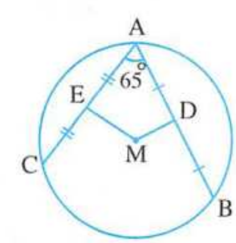
$\overline{AB}$ and $\overline{AC}$ are two chords

in the circle M , m (∠ BAC) = 65°

, D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

Find : m (∠ DME)



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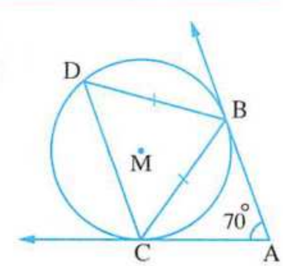
3 [a] **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents

to the circle M at B and C

, m (∠ BAC) = 70° , BD = BC

Find : m (∠ ABD)

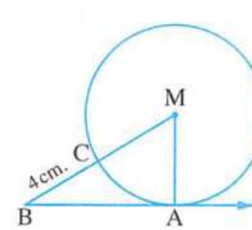


[b] **In the opposite figure :**

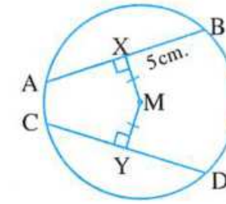
$\overrightarrow{BA}$ is a tangent to the circle M at A

, BM = 10 cm. , BC = 4 cm.

Find : the length of $\overline{AB}$



4 [a] **In the opposite figure :**

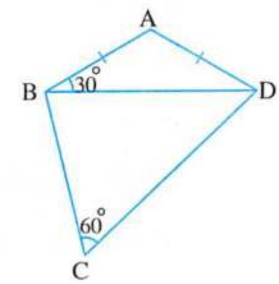


A circle M , MX = MY , XB = 5 cm.

, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

Find : the length of $\overline{CD}$

[b] **In the opposite figure :**



ABCD is a quadrilateral in which

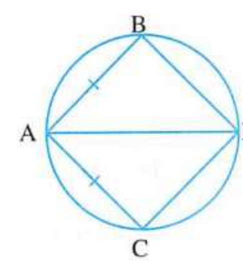
AB = AD , m (∠ ABD) = 30°

, m (∠ C) = 60°

Prove that :

ABCD is a cyclic quadrilateral.

5 [a] **In the opposite figure :**

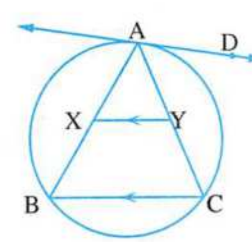


AB = AC , E ∈ $\overset{\frown}{BC}$

Prove that :

m (∠ AEB) = m (∠ AEC)

[b] **In the opposite figure :**



$\overleftrightarrow{AD}$ is a tangent to the circle at A

, $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

16 Beni Suef Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The measure of the inscribed angle drawn in a semicircle equals .... °

(a) 50 (b) 90 (c) 120 (d) 180

2 The angle whose measure is 50° complements an angle of measure .... °

(a) 310 (b) 130 (c) 50 (d) 40

3 If M and N are two circles touching externally , their radii lengths are 7 cm. and 12 cm. , then MN = ... cm.

(a) 5 (b) 7 (c) 12 (d) 19

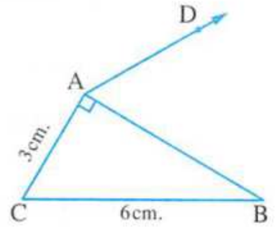
4 The number of axes of symmetry of the isosceles triangle equals ....

(a) 3 (b) 2 (c) 1 (d) zero.

5 A rhombus is of area 30 cm^2 and the length of one of its diagonals is 12 cm. , then the length of the other diagonal is .... cm.

(a) 5 (b) 12 (c) 18 (d) 21

6 **In the opposite figure :**

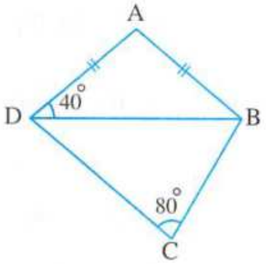


$\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC

, then m ($\angle$ DAB) = ... °

(a) 30 (b) 45 (c) 60 (d) 90

2 [a] **In the opposite figure :**

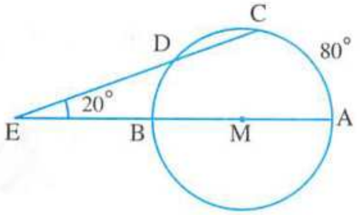


AB = AD , m ($\angle$ ADB) = 40°

, m ($\angle$ BCD) = 80°

Prove that : ABCD is a cyclic quadrilateral.

[b] **In the opposite figure :**



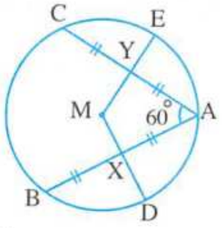
$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, m $(\overset{\frown}{AC})$ = 80°

and m ($\angle$ AEC) = 20°

Find : m $(\overset{\frown}{DC})$

3 [a] **In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M

, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$

, m ($\angle$ CAB) = 60°

1 **Find :** m ($\angle$ DME)

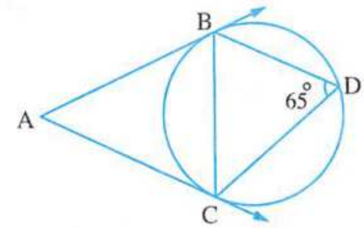
2 **Prove that :** XD = YE

[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ BDC) = 65°

Find : m (∠ BAC)



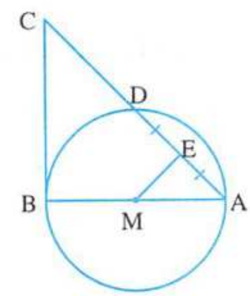
4 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circlc M

, $\overline{BC}$ is a tangent-segment to it at B

, E is the midpoint of $\overline{AD}$

Prove that : EMBC is a cyclic quadrilateral.



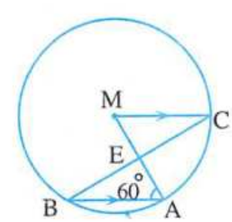
[b] In the opposite figure :

$\overline{AB}$ is a chord in the circle M

, $\overline{MC}$ // $\overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$

, m (∠ A) = 60°

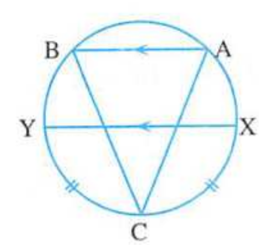
Find : m (∠ B)



5 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{XY}$ are two parallel chords in the circle , m $(\overset{\frown}{XC})$ = m $(\overset{\frown}{YC})$

Prove that : AC = BC

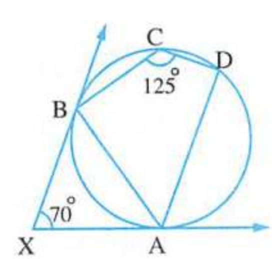


[b] In the opposite figure :

$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

, m (∠ AXB) = 70° , m (∠ DCB) = 125°

Prove that : $\overrightarrow{AB}$ bisects ∠ DAX



17 El-Menia Governorate

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer :

1. The area of a rhombus which the lengths of its diagonals are 6 cm. and 8 cm. equals .... cm^2.

   (a) 2 (b) 14 (c) 24 (d) 48

2. The measure of the inscribed angle equals .... the measure of the central angle subtended by the same arc.

   (a) half (b) twice (c) quarter (d) third

3. $\angle$ A and $\angle$ B are two complementary angles , m ($\angle$ A) = 40° , then m ($\angle$ B) = ...°

   (a) 360 (b) 140 (c) 60 (d) 50

4. M and N are two circles touching externally , their radii lengths are 3 cm. and 5 cm. , then MN = ... cm.

   (a) 3 (b) 5 (c) 8 (d) 2

5. If ABCD is a cyclic quadrilateral , then m ($\angle$ BAC) = m ($\angle$ ....)

   (a) BCA (b) DBA (c) BDC (d) ACD

6. In Δ ABC , if $(AC)^2 > (AB)^2 + (BC)^2$, then the angle B is ....

   (a) acute. (b) obtuse. (c) right. (d) straight.

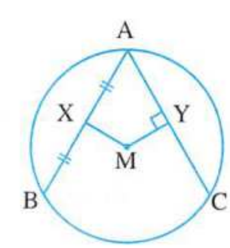
2

[a] **In the opposite figure :**

AB = AC , X is the midpoint of $\overline{AB}$

, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

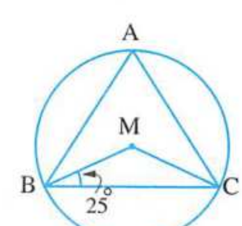


[b] **In the opposite figure :**

ABC is a triangle drawn

inside the circle M , m ($\angle$ MBC) = 25°

Find : m ($\angle$ BAC)

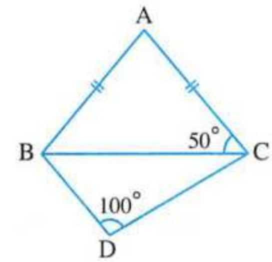


3 [a] **In the opposite figure :**

AB = AC , m (∠ D) = 100°

, m (∠ ACB) = 50°

Prove that : ABDC is a cyclic quadrilateral.

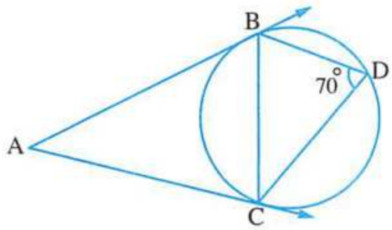


[b] **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to

the circle and m (∠ D) = 70°

Find : m (∠ A)

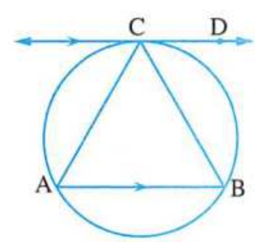


4 [a] **In the opposite figure :**

$\overleftrightarrow{CD}$ is a tangent to the

circle at C and $\overleftrightarrow{CD}$ // $\overline{AB}$

Prove that : AC = BC

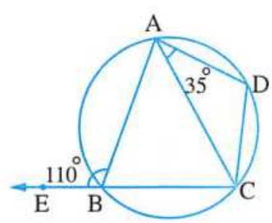


[b] **In the opposite figure :**

m (∠ ABE) = 110°

and m (∠ CAD) = 35°

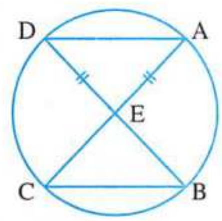
Prove that : m $(\overset{\frown}{DA})$ = m $(\overset{\frown}{DC})$



5 [a] **In the opposite figure :**

AE = DE

Prove that : EC = EB



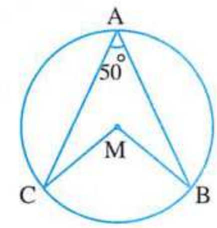
[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords

in the circle M

and m (∠ A) = 50°

Find : m (reflex ∠ CMB)



18 Assiut Governorate

Answer the following questions : (Calculator is permitted)

1 Choose the correct answer :

1 The area of the rhombus whose diagonal lengths are 3 cm. and 4 cm. is .... cm^2.

(a) 48 (b) 24 (c) 12 (d) 6

2 The inscribed angle drawn in a semicircle is ....

(a) acute. (b) obtuse. (c) right. (d) straight.

3 If Δ ABC ~ Δ XYZ , m ($\angle$ A) = 50° , m ($\angle$ B) = 60° , then m ($\angle$ Z) = ...°

(a) 110 (b) 70 (c) 60 (d) 50

4 If M and N are two circles touching internally , their radii lengths are 3 cm. and 5 cm. , then MN = ... cm.

(a) 2 (b) 3 (c) 6 (d) 8

5 If the ratio between the perimeters of two squares is 1 : 3 , then the ratio between their areas is ....

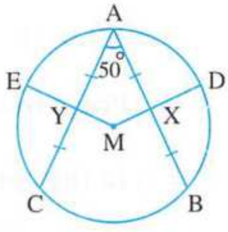
(a) 1 : 3 (b) 3 : 1 (c) 9 : 1 (d) 1 : 9

6 If ABCD is a cyclic quadrilateral , then m ($\angle$ A) + m ($\angle$ C) – 80° = ...°

(a) 60 (b) 80 (c) 100 (d) 180

2 [a] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$, m ($\angle$ CAB) = 50°

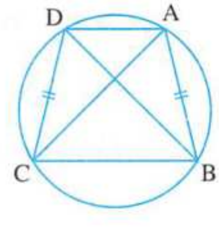


1 **Find with proof :** m ($\angle$ DME)

2 **Prove that :** XD = YE

[b] In the opposite figure :

ABCD is a quadrilateral inscribed in a circle in which AB = DC



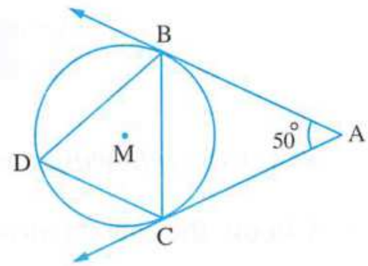
Prove that : AC = BD

3 [a] **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m (∠ A) = 50°

Find with proof : m (∠ BDC)



[b] **In the opposite figure :**

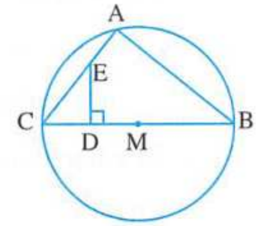
$\overline{BC}$ is a diameter in the circle M

, $\overline{ED} \perp \overline{BC}$

Prove that :

1 ABDE is a cyclic quadrilateral.

2 m (∠ CED) = $\frac{1}{2}$ m ($\widehat{AC}$)



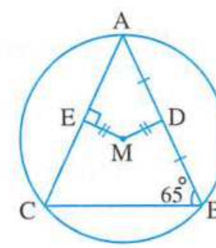
4 [a] **In the opposite figure :**

M is a circle , MD = ME

, D is the midpoint of $\overline{AB}$

, $\overline{ME} \perp \overline{AC}$, m (∠ ABC) = 65°

Find with proof : m (∠ BAC)



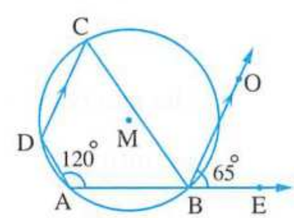
[b] **In the opposite figure :**

ABCD is a quadrilateral inscribed

in the circle M , $\overrightarrow{BO}$ // $\overline{DC}$

, m (∠ EBO) = 65° , m (∠ BAD) = 120°

Find with proof : m (∠ ADC)



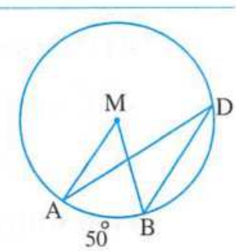
5 [a] **In the opposite figure :**

m ($\widehat{AB}$) = 50°

Find with proof :

1 m (∠ ADB)

2 m ($\widehat{ADB}$)



[b] **In the opposite figure :**

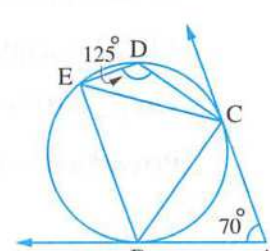
$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m (∠ A) = 70° , m (∠ CDE) = 125°

Prove that :

1 CB = CE

2 $\overrightarrow{BC}$ bisects ∠ ABE



19 Souhag Governorate

Answer the following questions : (Calculator is permitted)

1 Choose the correct answer :

1 In the cyclic quadrilateral , each two opposite angles are ....

(a) equal in measure. (b) supplementary.
(c) alternate. (d) complementary.

2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 2

3 The inscribed angle drawn in a semicircle is .... angle.

(a) an acute (b) a straight (c) a right (d) an obtuse

4 A rhombus whose two diagonal lengths are 6 cm. , 8 cm. , then its area is .... cm^2.

(a) 48 (b) 24 (c) 14 (d) 12

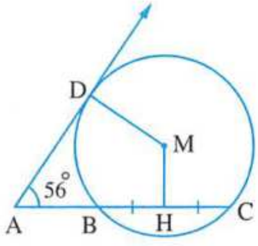
5 The measure of the exterior angle of the equilateral triangle equals ....°

(a) 60 (b) 108 (c) 120 (d) 135

6 The number of circles passing through three collinear points is ....

(a) infinite. (b) two. (c) one. (d) zero.

2 [a] In the opposite figure :



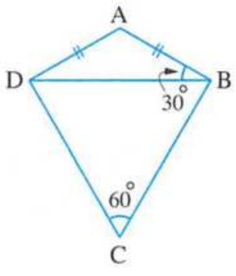
$\overrightarrow{AD}$ is a tangent to the circle M

, $\overrightarrow{AC}$ intersects the circle M at B , C

, m (∠ A) = 56° and H is the midpoint of $\overline{BC}$

Find with proof : m (∠ DMH)

[b] In the opposite figure :



ABCD is a quadrilateral , AB = AD

, m (∠ ABD) = 30° , m (∠ C) = 60°

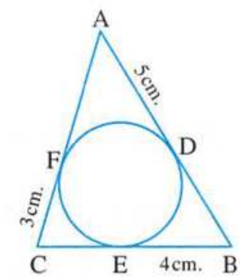
Prove that : ABCD is a cyclic quadrilateral.

3 [a] **In the opposite figure :**

A circle is drawn touching the sides of the triangle ABC , $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at D , E , F , AD = 5 cm.

, BE = 4 cm. , CF = 3 cm.

Find : the perimeter of Δ ABC

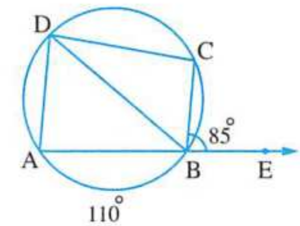


[b] **In the opposite figure :**

$E \in \overrightarrow{AB}$, $E \notin \overline{AB}$, m $(\widehat{AB}) = 110°$

, m ($\angle$ CBE) = 85°

Find : m ($\angle$ BDC)



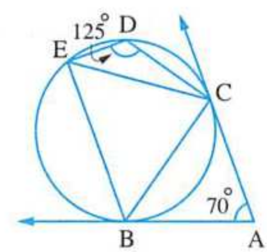
4 [a] **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m ($\angle$ A) = 70°

, m ($\angle$ CDE) = 125°

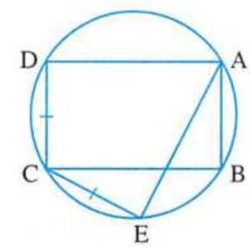
Prove that : CB = CE



[b] **In the opposite figure :**

ABCD is a rectangle inscribed in a circle , the chord $\overline{CE}$ is drawn where CE = CD

Prove that : AE = BC

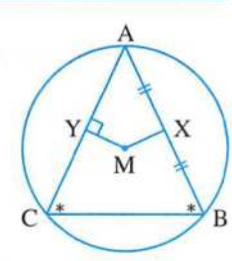


5 [a] **In the opposite figure :**

ABC is a triangle inscribed in the circle M in which m ($\angle$ B) = m ($\angle$ C) ,

X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

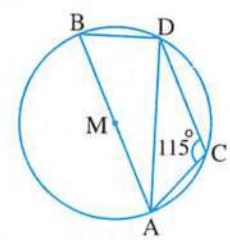


[b] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m ($\angle$ ACD) = 115°

Find : m ($\angle$ DAB)



20 Qena Governorate

Answer the following questions : ***(Calculators are permitted)***

1 Choose the correct answer from those given :

1 The length of a semicircle equals ....

(a) π r (b) 180° (c) $\frac{1}{2}\pi$ r (d) 2 π r

2 The sum of measures of the interior angles of a triangle equals ....

(a) 180° (b) 360° (c) 540° (d) 720°

3 The .... is a rhombus , one of its angles is a right angle.

(a) rectangle (b) square (c) parallelogram (d) trapezium

4 The measure of the inscribed angle equals .... the measure of the central angle, subtended by the same arc.

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

5 The measure of the exterior angle of the equilateral triangle equals ....

(a) 90° (b) 180° (c) 120° (d) 60°

6 The number of the common tangents of two circles touching externally equals ....

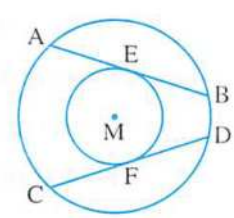
(a) 1 (b) 2 (c) 3 (d) 4

2 [a] Draw $\overline{AB}$ where AB = 5 cm. , then draw a circle passing through the two points A and B , the length of its radius is 3 cm. , using your geometric instruments (Don't remove the arcs) How many circles can be drawn ?

[b] **In the opposite figure :**

Two concentric circles of centre M , $\overline{AB}$ and $\overline{CD}$ are two chords in the greater circle and tangent-segments to the smaller circle at E and F

Prove that : AB = CD

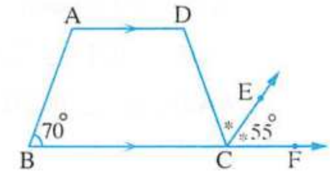


3 [a] **In the opposite figure :**

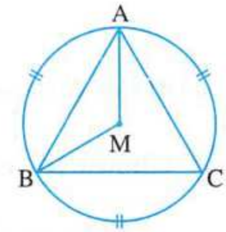
$\overrightarrow{AD}$ // $\overrightarrow{BC}$, F $\in \overrightarrow{BC}$, $\overrightarrow{CE}$ bisects $\angle$ DCF ,

m ($\angle$ B) = 70° , m ($\angle$ ECF) = 55°

Prove that : ABCD is a cyclic quadrilateral.



[b] In the opposite figure :



A , B and C are three points lie on the circle M

where : $m(\overset{\frown}{AB}) = m(\overset{\frown}{BC}) = m(\overset{\frown}{CA})$

1 **Find by proof :** $m(\angle ABM)$

2 **Prove that :** ΔABC is an equilateral triangle.

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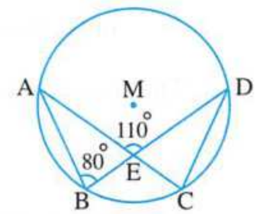
4 **[a] In the opposite figure :**

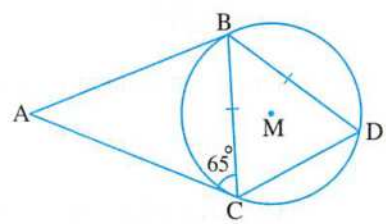


$\overline{AC}$ and $\overline{BD}$ are two chords in the circle M

, $\overline{AC} \cap \overline{BD} = \{E\}$, $m(\angle AED) = 110°$, $m(\angle B) = 80°$

Find by proof : $m(\angle D)$, $m(\overset{\frown}{AD})$

[b] In the opposite figure :



$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

at B and C , $m(\angle ACB) = 65°$

Find by proof : $m(\angle A)$, $m(\angle D)$

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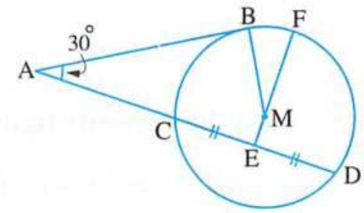
5 **[a] In the opposite figure :**



$\overline{AB}$ is a tangent-segment to the circle M at B

, $\overline{DC}$ is a chord in the circle M , $\overrightarrow{DC} \cap \overrightarrow{BA} = \{A\}$

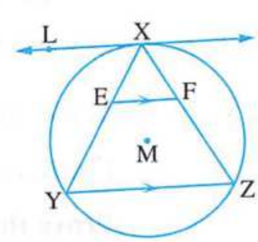
, E is the midpoint of $\overline{CD}$

, $\overrightarrow{EM} \cap$ the circle $M = \{F\}$, $m(\angle A) = 30°$

1 **Prove that :** ABME is a cyclic quadrilateral.

2 **Find :** $m(\overset{\frown}{BF})$

[b] In the opposite figure :



$\overleftrightarrow{LX}$ is a tangent to the circle at X , $\overline{EF} \parallel \overline{YZ}$

, where $\overline{YZ}$ is a chord in the circle M

Prove that : $\overrightarrow{XL}$ is a tangent to

the circle passing through the points X , E and F

21 Luxor Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1 If the diameter length of a circle is 8 cm. and the straight line L is at a distance of 4 cm. from its centre , then L is .... the circle.

(a) a tangent to (b) a secant to (c) outside (d) an axis of symmetry of

2 The measure of the inscribed angle which is drawn in $\frac{1}{4}$ a circle equals ....

(a) 45° (b) 90° (c) 120° (d) 135°

3 The two tangents to a circle at the two endpoints of a diameter of it are ....

(a) parallel. (b) perpendicular. (c) intersecting. (d) coincident.

4 The sum of measures of the accumulative angles at a point is ....

(a) 630° (b) 360° (c) 603° (d) 306°

5 The area of a square is 25 cm^2 , then its perimeter is .... cm.

(a) 5 (b) 10 (c) 15 (d) 20

6 The measure of the supplementary angle of the angle whose measure is 60° equals ....

(a) 30° (b) 90° (c) 120° (d) 180°

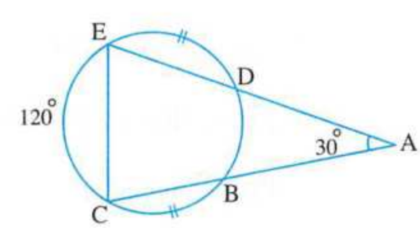
2 [a] In the opposite figure :

m (∠ A) = 30° , m ($\overset{\frown}{CE}$) = 120°

, m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{DE}$)

1 **Find :** m ($\overset{\frown}{BD}$)

2 **Prove that :** AB = AD



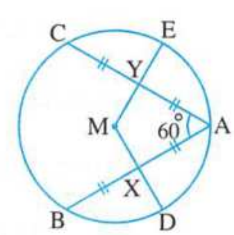
[b] In the opposite figure :

AB = AC , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$, m (∠ A) = 60°

1 **Find :** m (∠ DME)

2 **Prove that :** XD = YE

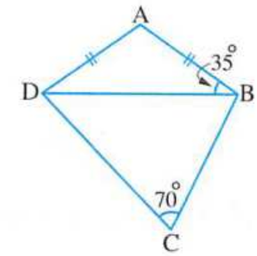


3 **[a] In the opposite figure :**

ABCD is a quadrilateral , AB = AD

, m (∠ ABD) = 35° , m (∠ C) = 70°

Prove that : ABCD is a cyclic quadrilateral.



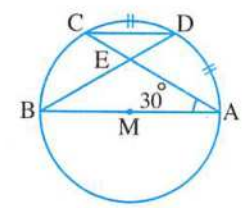
[b] In the opposite figure :

$\overline{AB}$ is a diameter of the circle M

, m (∠ CAB) = 30° , D is the midpoint of $\overset{\frown}{AC}$

Find : 1 m (∠ BDC)

2 m ($\overset{\frown}{AD}$)



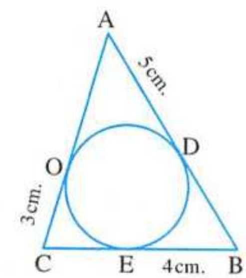
4 **[a] In the opposite figure :**

The sides of Δ ABC touches the

circle externally at D , E , O

If AD = 5 cm. , BE = 4 cm. , CO = 3 cm.

Find : the perimeter of Δ ABC



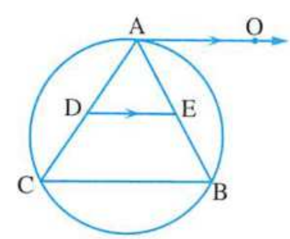
[b] In the opposite figure :

$\overrightarrow{AO}$ is a tangent to the circle

at A , $\overrightarrow{AO}$ // $\overline{DE}$

Prove that :

DEBC is a cyclic quadrilateral.



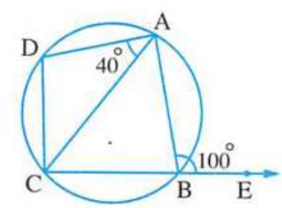
5 **[a] In the opposite figure :**

ABCD is a quadrilateral inscribed

in a circle , where m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{AD}$)

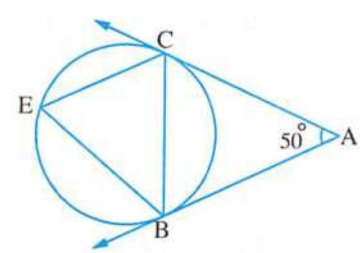


[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle

at B and C , m (∠ A) = 50°

Find : m (∠ BEC)



22 Aswan Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1 The area of the square whose side length is 6 cm. is .... cm^2.

(a) 12 (b) 24 (c) 36 (d) 60

2 M and N are two circles touching externally , their radii lengths are 3 cm. and 5 cm. , then MN = ... cm.

(a) 5 (b) 8 (c) 2 (d) 3

3 The angle whose measure is 50° complements an angle whose measure is ....°

(a) 40 (b) 60 (c) 90 (d) 180

4 If ABCD is a cyclic quadrilateral , $m(\angle A) = \frac{1}{2} m(\angle C)$, then $m(\angle A) =$ ....°

(a) 90 (b) 80 (c) 60 (d) 50

5 In Δ ABC , if $(AC)^2 = (AB)^2 + (BC)^2$, then ∠ B is .... angle.

(a) an acute (b) a right (c) an obtuse (d) a straight

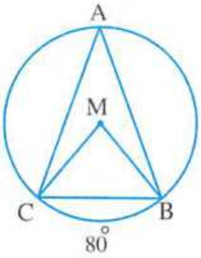
6 **In the opposite figure :**

M is a circle , if $m(\widehat{BC}) = 80°$

, then $m(\angle A) =$ ....°

(a) 10 (b) 20

(c) 30 (d) 40



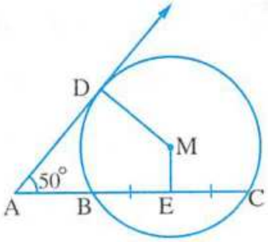
2 [a] In the opposite figure :

$\overrightarrow{AD}$ is a tangent to the circle M at D

, $\overrightarrow{AB}$ intersects the circle at B and C

, $m(\angle A) = 50°$, E is the midpoint of $\overline{BC}$

Find : $m(\angle DME)$

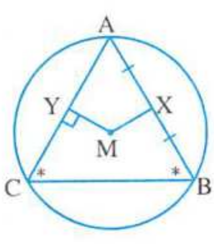


[b] In the opposite figure :

Δ ABC is a triangle inscribed in the circle M , $m(\angle B) = m(\angle C)$

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

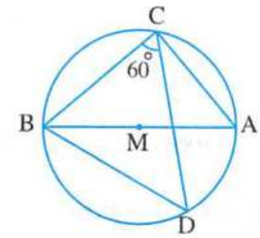


3 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M

, m (∠ DCB) = 60°

Find : m (∠ ABD)



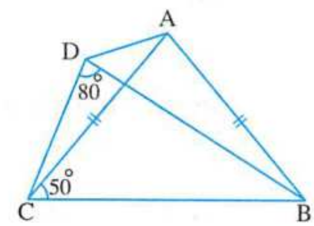
[b] **In the opposite figure :**

AB = AC , m (∠ BDC) = 80°

, m (∠ ACB) = 50°

Prove that :

The figure ABCD is a cyclic quadrilateral.



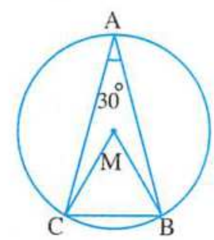
4 [a] **In the opposite figure :**

ABC is a triangle inscribed in the circle M

, m (∠ A) = 30°

1 **Find :** m (∠ BMC)

2 **Prove that :** Δ MBC is an equilateral triangle.

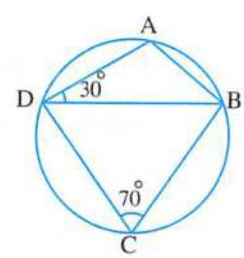


[b] **In the opposite figure :**

m (∠ ADB) = 30°

, m (∠ C) = 70°

Find : m (∠ ABD)

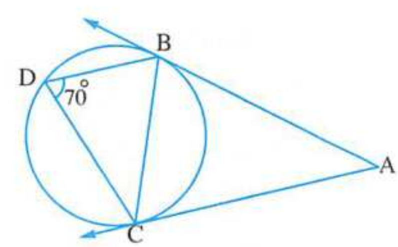


5 [a] **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

and m (∠ BDC) = 70°

Find : m (∠ A)



[b] **In the opposite figure :**

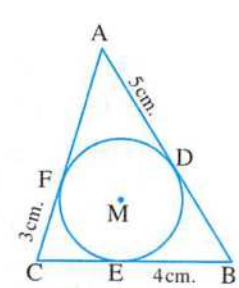
The inscribed circle M of Δ ABC touches

its sides $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ at D

, E and F respectively.

If AD = 5 cm. , BE = 4 cm. and CF = 3 cm.

, **find :** the perimeter of Δ ABC



23 New Valley Governorate

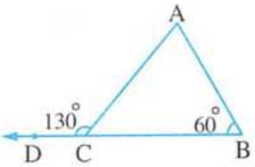
Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1 The circumference of the circle of radius length 7 cm. is ………… cm.

(a) 11 (b) 22 (c) 44 (d) 154

2 **In the opposite figure :**



If m (∠ B) = 60° , m (∠ ACD) = 130°

, C ∈ $\overline{BD}$, then m (∠ A) = …………°

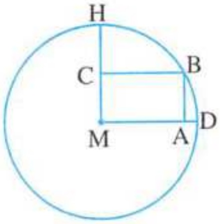
(a) 40 (b) 50

(c) 60 (d) 70

3 The measure of the inscribed angle drawn in a semicircle is …………°

(a) 45 (b) 90 (c) 120 (d) 180

4 **In the opposite figure :**



A circle of centre M

If MABC is a rectangle

, then the radius length of the circle equals …………

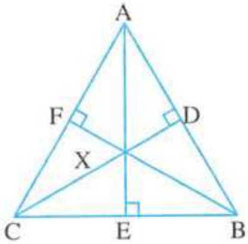
(a) BC (b) AC

(c) AM (d) AB

5 The straight line perpendicular to any chord from its midpoint is ………… of the circle.

(a) a chord (b) a radius (c) a diameter (d) an axis of symmetry

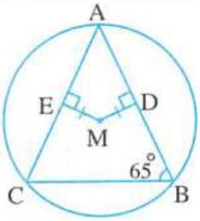
6 The number of cyclic quadrilaterals in the opposite figure is …………



(a) 1 (b) 3

(c) 6 (d) 9

2 [a] **In the opposite figure :**



If MD = ME , m (∠ B) = 65°

, $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

, **then find :** m (∠ A)

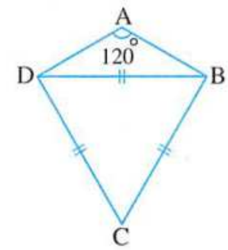
[b] In the opposite figure :

ABCD is a quadrilateral

in which m (∠ A) = 120°

, BC = CD = DB

Prove that : ABCD is a cyclic quadrilateral.

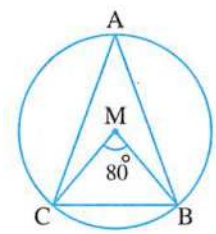


3 **[a] In the opposite figure :**

M is a circle , m (∠ BMC) = 80°

Find : 1 m (∠ A)

2 m (∠ MBC)



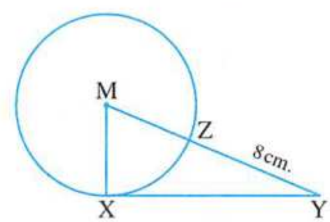
[b] In the opposite figure :

M is a circle with radius length 5 cm.

, YZ = 8 cm.

, $\overleftrightarrow{XY}$ is a tangent to the circle M at X

Find : the length of $\overline{XY}$



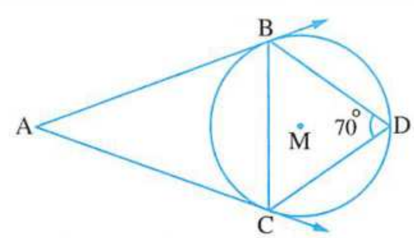
4 **[a] In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents

to the circle at B , C

, m (∠ D) = 70°

Find : m (∠ A)



[b] In the opposite figure :

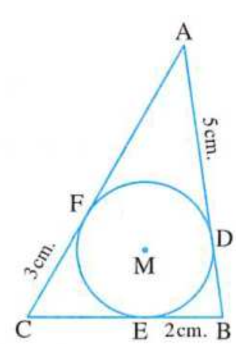
A circle is drawn touching

the sides of the triangle ABC

, $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ at D , E , F

, AD = 5 cm. , BE = 2 cm. , CF = 3 cm.

Find : the perimeter of Δ ABC



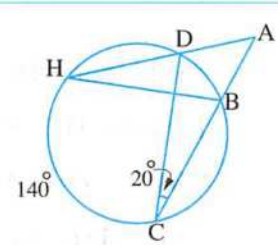
5 **[a] In the opposite figure :**

If m (∠ C) = 20°

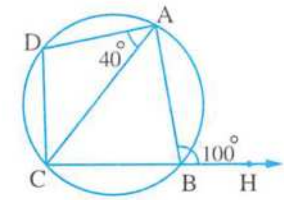
, m ($\overset{\frown}{CH}$) = 140°

, find :

1 m (∠ H) 2 m ($\overset{\frown}{BD}$) 3 m (∠ A)



[b] In the opposite figure :



m (∠ ABH) = 100°

, m (∠ CAD) = 40°

Prove that : m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{AD}$)

24 South Sinai Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 45° (b) 90° (c) 120° (d) 80°

2 The angle of tangency is included between ....

(a) two chords. (b) two tangents.

(c) a chord and a tangent. (d) a chord and a diameter.

3 ABCD is a cyclic quadrilateral , m (∠ A) = 120° , then m (∠ C) = ...°

(a) 60 (b) 120 (c) 90 (d) 180

4 M and N are two circles touching internally , their radii lengths are 5 cm. and 9 cm. , then MN = ... cm.

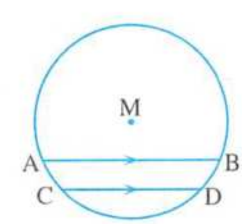
(a) 14 (b) 4 (c) 5 (d) 9

5 The number of symmetry axes of any circle is ....

(a) zero. (b) 1

(c) an infinite number. (d) 3

6 **In the opposite figure :**



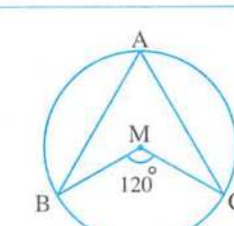
A circle of centre M in which $\overline{AB}$ // $\overline{CD}$

, then ....

(a) m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{BD}$) (b) AB = CD

(c) $\overline{AC}$ // $\overline{BD}$ (d) m ($\overset{\frown}{AC}$) > m ($\overset{\frown}{BD}$)

2 [a] In the opposite figure :



m (∠ CMB) = 120°

Find : m (∠ BAC)

[b] In the opposite figure :

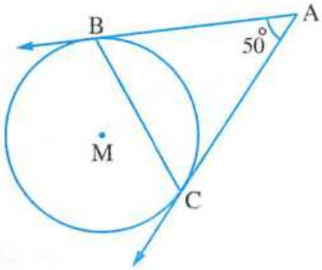
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are tangents to the circle M

, m (∠ BAC) = 50°

Find :

1 m (∠ ABC)

2 m (∠ ACB)

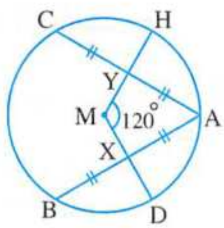


3 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$, m (∠ HMD) = 120°

1 **Find :** m (∠ BAC)

2 **Prove that :** DX = HY

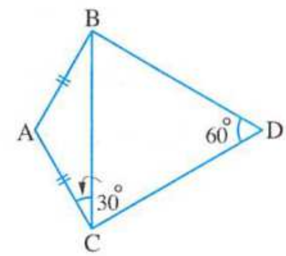


[b] In the opposite figure :

AB = AC , m (∠ BDC) = 60°

and m (∠ ACB) = 30°

Prove that : ABDC is a cyclic quadrilateral.

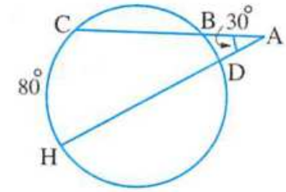


4 **[a] In the opposite figure :**

m ($\overset{\frown}{CH}$) = 80°

, m (∠ CAH) = 30°

Find : m ($\overset{\frown}{BD}$)

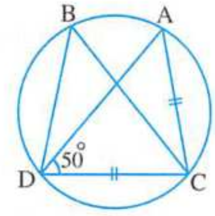


[b] In the opposite figure :

AC = CD

, m (∠ ADC) = 50°

Find : m (∠ CBD)



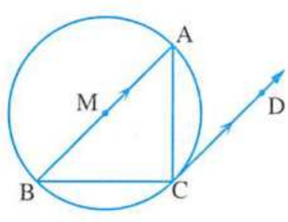
5 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{CD}$ is a tangent to the circle M at C

, $\overrightarrow{CD}$ // $\overline{AB}$

Find : m (∠ ABC) in degrees.

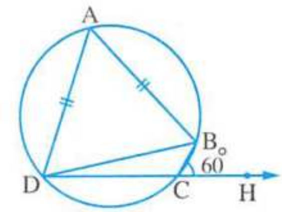


[b] In the opposite figure :

AB = AD , m (∠ BCH) = 60°

Prove that :

the triangle ABD is equilateral.



25 North Sinai Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1 M and N are two circles of radii lengths 9 cm. and 4 cm. respectively , MN = 5 cm. , then the two circles are ....

(a) touching externally. (b) touching internally.

(c) intersecting. (d) distant.

2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

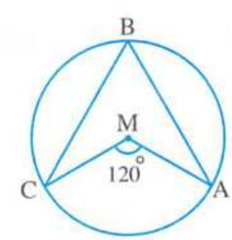
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

3 **In the opposite figure :**

m (∠ AMC) = 120° , then m (∠ ABC) = ...°

(a) 360 (b) 240

(c) 90 (d) 60



4 A rhombus whose diagonal lengths are 6 cm. , 8 cm. , then its area is .... cm^2

(a) 2 (b) 12 (c) 24 (d) 48

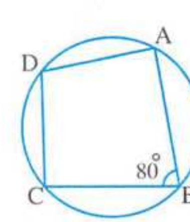
5 **In the opposite figure :**

ABCD is a cyclic quadrilateral , m (∠ B) = 80°

, then m (∠ D) = ...°

(a) 10 (b) 100

(c) 80 (d) 180



6 The number of symmetry axes of any circle is ....

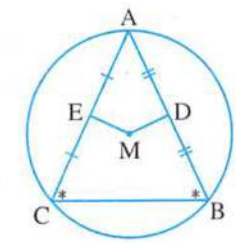
(a) 1 (b) 2 (c) 3 (d) an infinite number.

2 [a] **In the opposite figure :**

M is a circle in which : D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$, m (∠ B) = m (∠ C)

Prove that : MD = ME



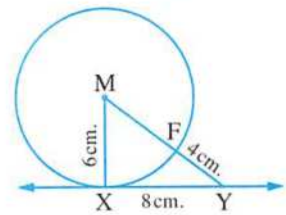
[b] **In the opposite figure :**

M is a circle of radius length 6 cm.

, XY = 8 cm. , $\overline{MY} \cap$ the circle M = {F}

, FY = 4 cm.

Prove that : $\overleftrightarrow{XY}$ is a tangent to the circle M at X



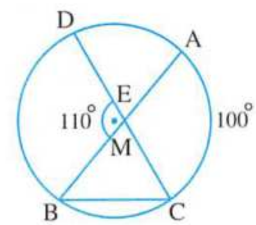
3 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $\overline{AB} \cap \overline{CD}$ = {E} , m (∠ DEB) = 110°

, m ($\overset{\frown}{AC}$) = 100°

Find : m (∠ DCB)

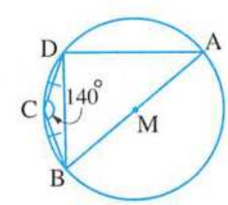


[b] **In the opposite figure :**

ABCD is a quadrilateral inscribed in a circle M

, M ∈ $\overline{AB}$, CB = CD , m (∠ BCD) = 140°

Find : m (∠ A) , m (∠ ADC)



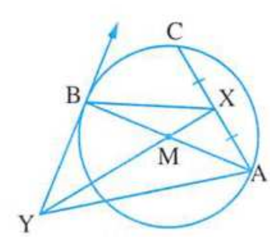
4 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, X is the midpoint of $\overline{AC}$, $\overrightarrow{YB}$ is a tangent to the circle M

, $\overrightarrow{XM} \cap \overrightarrow{BY}$ = {Y}

Prove that : AXBY is a cyclic quadrilateral.

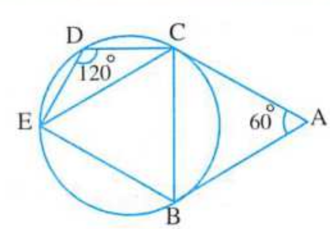


[b] Find the length and the measure of the arc , which is opposite to an inscribed angle of measure 45° in a circle whose radius length is 7 cm. (Consider $\pi = \frac{22}{7}$)

5 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are tangent-segments to the circle at B and C , m (∠ BAC) = 60° , m (∠ CDE) = 120°

Prove that : BCE is an equilateral triangle.

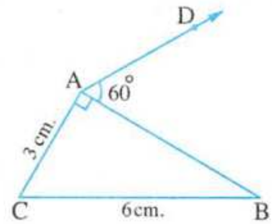


[b] **In the opposite figure :**

ABC is a right-angled triangle at A ,

AC = 3 cm. , BC = 6 cm. , m (∠ BAD) = 60°

Prove that : $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC



26 Red Sea Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The area of the circle whose radius length is 3 cm. equals .... cm^2.

(a) 9 π (b) 6 π (c) 12 π (d) 15 π

2 The number of symmetry axes of the circle is ....

(a) zero. (b) 1 (c) 2 (d) an infinite number.

3 The number of circles which pass through three non-collinear points is ....

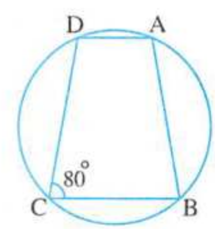
(a) 1 (b) 2 (c) 3 (d) zero

4 M and N are two circles touching externally , the lengths of their radii are 5 cm. and 3 cm. , then MN = ... cm.

(a) 8 (b) 2 (c) 9 (d) 6

5 **In the opposite figure :**

m (∠ A) = ...°

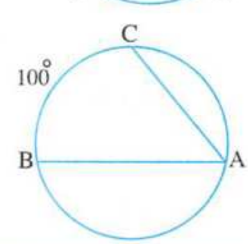


(a) 80 (b) 100

(c) 110 (d) 90

6 **In the opposite figure :**

m ($\overset{\frown}{BC}$) = 100° , then m (∠ A) = ...°



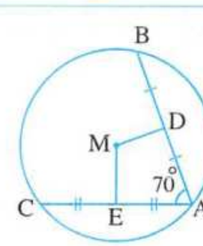
(a) 100 (b) 90

(c) 50 (d) 40

2 [a] **In the opposite figure :**

M is the centre of the circle , D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , m (∠ A) = 70°

Find : m (∠ DME)

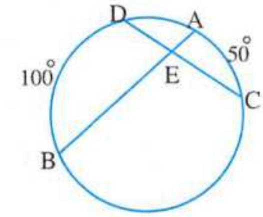


[b] In the opposite figure :

$\overline{AB} \cap \overline{CD} = \{E\}$

, $m(\overset{\frown}{AC}) = 50°$, $m(\overset{\frown}{BD}) = 100°$

Find : $m(\angle AEC)$

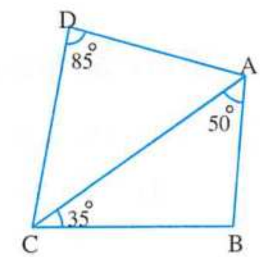


3 **[a] In the opposite figure :**

$m(\angle BAC) = 50°$, $m(\angle BCA) = 35°$

, $m(\angle D) = 85°$

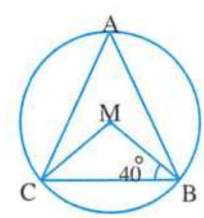
Prove that : ABCD is a cyclic quadrilateral.



[b] In the opposite figure :

A circle M , $m(\angle MBC) = 40°$

Find : $m(\angle A)$



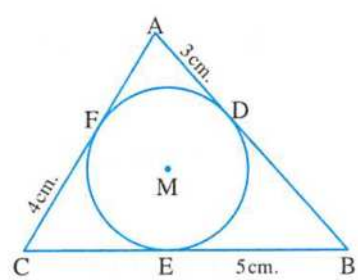
4 **[a] In the opposite figure :**

A circle M is drawn touching the sides of ΔABC

at D , E and F , BE = 5 cm.

, AD = 3 cm. , CF = 4 cm.

Find : the perimeter of ΔABC



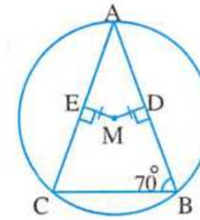
[b] In the opposite figure :

M is the centre of the circle

, $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

, MD = ME , $m(\angle B) = 70°$

Find : $m(\angle A)$

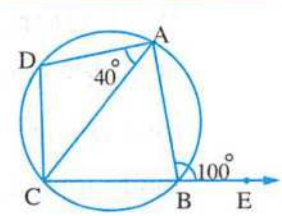


5 **[a] In the opposite figure :**

$E \in \overrightarrow{CB}$, $m(\angle ABE) = 100°$

, $m(\angle CAD) = 40°$

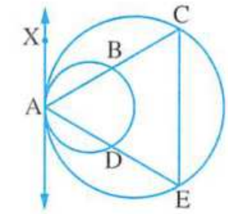
Prove that : $m(\overset{\frown}{CD}) = m(\overset{\frown}{AD})$



[b] In the opposite figure :

$\overleftrightarrow{AX}$ is a common tangent to the two circles touching internally at A

Prove that : $\overline{BD} \parallel \overline{CE}$



27 Matrouh Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1 The two opposite angles in the cyclic quadrilateral are ....

(a) equal in measure. (b) complementary.

(c) supplementary. (d) alternate.

2 The circumference of a circle equals ....

(a) π r (b) 2 π r (c) πr^2 (d) 2 π

3 The measure of the inscribed angle is .... the measure of the subtended arc.

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{5}$ (d) $\frac{1}{2}$

4 The square whose side length is 4 cm. , its area is .... cm^2.

(a) 4 (b) 8 (c) 16 (d) 24

5 The tangent to a circle of diameter length 6 cm. is at a distance of .... cm. from its centre.

(a) 6 (b) 12 (c) 3 (d) 2

6 ABC is a right-angled triangle at B , then $(AB)^2 + (BC)^2 =$ ....

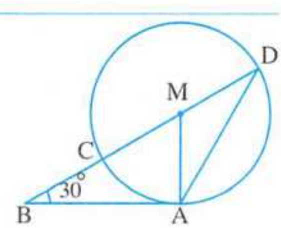
(a) $(AC)^2$ (b) $(AB)^2$ (c) $(BC)^2$ (d) $2\,(AC)^2$

2 [a] In the opposite figure :

M is a circle , $\overline{AB}$ is a tangent-segment to the circle M at A

, m (∠ B) = 30°

Find : m (∠ ADB)

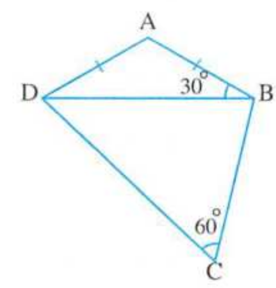


[b] In the opposite figure :

ABCD is a quadrilateral in which AB = AD

, m (∠ ABD) = 30° and m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.

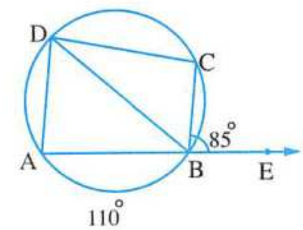


3 [a] **In the opposite figure :**

$E \in \overrightarrow{AB}$, $E \notin \overline{AB}$, $m(\widehat{AB}) = 110^\circ$

, m (∠ CBE) = 85°

Find : m (∠ BDC)



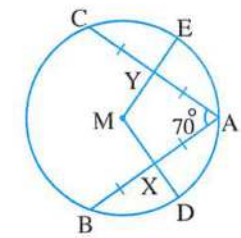
[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$, m (∠ CAB) = 70°

1 **Calculate :** m (∠ DME)

2 **Prove that :** XD = YE

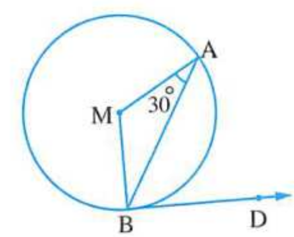


4 [a] **In the opposite figure :**

$\overrightarrow{BD}$ is a tangent to the circle M

, m (∠ BAM) = 30°

Find : m (∠ ABD) angle of tangency.

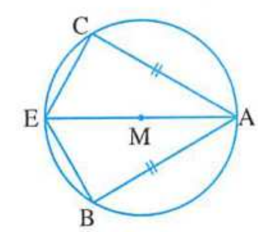


[b] **In the opposite figure :**

AB = AC , $E \in \widehat{BC}$

Prove that :

m (∠ AEB) = m (∠ AEC)



5 [a] **Complete the following :**

1 The line of centres of two intersecting circles is .... to the common chord and .... it.

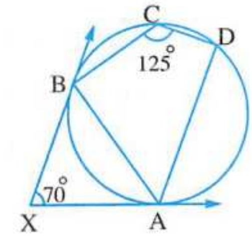
2 In the same circle , the measures of all inscribed angles subtended by the same arc are ....

[b] **In the opposite figure :**

$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B , m (∠ AXB) = 70°

, m (∠ DCB) = 125°

Prove that : $\overrightarrow{AB}$ bisects ∠ DAX



Answers of governorates' examinations of geometry

1 Cairo

1

1 d	2 b	3 a	4 d	5 c	6 b

2

[a] The measure of the arc $= \frac{1}{4} \times 360^\circ = 90^\circ$

, the length of the arc $= \frac{1}{4} \times 2 \times \frac{22}{7} \times 14 = 22$ cm.

[b] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC) = \frac{180^\circ - 80^\circ}{2} = 50^\circ$

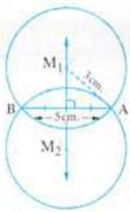
, $\because \overline{MC}$ is a radius $\therefore \overline{MC} \perp \overline{AC}$

$\therefore m(\angle ACM) = 90^\circ$

$\therefore m(\angle BCM) = 90^\circ - 50^\circ = 40^\circ$ (The req.)

3

[a]



$\therefore$ We can draw two circles.

[b] $\because m(\widehat{XY}) = m(\angle XMY) = 130^\circ$ (First req.)

, $\because m(\angle XZY) = \frac{1}{2} m(\widehat{XY}) = \frac{1}{2} \times 130^\circ = 65^\circ$ (Second req.)

$\therefore m(\angle LZX) = 180^\circ - 65^\circ = 115^\circ$

In ΔZXL : $\because ZL = ZX$

$\therefore m(\angle L) = m(\angle LXZ) = \frac{180^\circ - 115^\circ}{2} = 32^\circ\ 30'$ (Third req.)

4

[a] $\because \overrightarrow{HX}$ is a tangent to the circle M

$\therefore \overline{MX} \perp \overrightarrow{HX}$ $\therefore m(\angle HXM) = 90^\circ$

, $\because \overline{AB}$ is a common chord

, $\overleftrightarrow{MN}$ is the line of centres

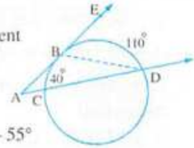
$\therefore \overline{AB} \perp \overleftrightarrow{MN}$ $\therefore m(\angle HYM) = 90^\circ$

$\therefore m(\angle HXM) + m(\angle HYM) = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ HXMY is a cyclic quadrilateral. (Q.E.D.)

[b] Const : Draw $\overline{BD}$

Proof : $\because \overrightarrow{AB}$ is a tangent



$\therefore m(\angle EBD)$

$= \frac{1}{2} m(\widehat{BD}) = 55^\circ$

$\therefore m(\angle ABD) = 180^\circ - 55^\circ$

$= 125^\circ$

, $\because m(\angle D) = \frac{1}{2} m(\widehat{BC}) = 20^\circ$

$\therefore$ In ΔABD :

$m(\angle A) = 180^\circ - (125^\circ + 20^\circ) = 35^\circ$ (The req.)

5

[a] $\because$ D is the midpoint of $\overline{XY}$

$\therefore \overline{MD} \perp \overline{XY}$ $\therefore m(\angle MDX) = 90^\circ$

, $\because$ H is the midpoint of $\overline{XZ}$

$\therefore \overline{MH} \perp \overline{XZ}$ $\therefore m(\angle MHX) = 90^\circ$

, $\because MD = MH$ $\therefore XY = XZ$ (1)

From the quadrilateral XDMH :

$\therefore m(\angle X) = 360^\circ - (90^\circ + 90^\circ + 120^\circ) = 60^\circ$ (2)

From (1) , (2) :

$\therefore \Delta XYZ$ is an equilateral triangle. (Q.E.D.)

[b] $\because \overleftrightarrow{XY} \parallel \overline{DH}$, $\overleftrightarrow{XZ}$ is a transversal

$\therefore m(\angle XHD) = m(\angle YXH)$ (alternate angles) (1)

, $\because m(\angle L)$ (inscribed) $= m(\angle YXZ)$ (tangency) (2)

From (1) and (2) : $\therefore m(\angle L) = m(\angle XHD)$

$\therefore$ DHZL is a cyclic quadrilateral. (Q.E.D.)

2 Giza

1

1 d	2 b	3 c	4 a	5 c	6 b

2

[a] $\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$ $\therefore m(\angle MDA) = 90^\circ$

, $\because$ H is the midpoint of $\overline{AC}$

$\therefore \overline{MH} \perp \overline{AC}$ $\therefore m(MHA) = 90^\circ$

From the quadrilateral ADMH :

$\therefore m(\angle DMH) = 360^\circ - (90^\circ + 90^\circ + 60^\circ)$

$= 120^\circ$ (The req)

[b] $\because m(\angle D) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

$\therefore m(\angle D) = \frac{1}{2} \times 140^\circ = 70^\circ$

, $\because \overline{AC} \parallel \overline{DB}$, $\overleftrightarrow{AD}$ is a transversal

$\therefore m(\angle DAC) + m(\angle D) = 180^\circ$

(two interior angles in the same side of the transversal)

$\therefore m(\angle DAC) = 180^\circ - 70^\circ = 110^\circ$ (The req.)

3

[a] $\because m(\angle DHB) = \frac{1}{2}\left[m(\overset{\frown}{DB}) + m(\overset{\frown}{AC})\right]$

$\therefore 110^\circ = \frac{1}{2}\left[m(\overset{\frown}{DB}) + 100^\circ\right]$

$\therefore 220^\circ = m(\overset{\frown}{DB}) + 100^\circ$ $\therefore m(\overset{\frown}{DB}) = 120^\circ$

$\therefore m(\angle DCB) = \frac{1}{2} m(\overset{\frown}{DB}) = \frac{1}{2} \times 120^\circ = 60^\circ$

(The req.)

[b] $\because m(\angle BCD) = \frac{1}{2} m(\angle BMD)$

(inscribed and central angles subtended by $\overset{\frown}{BD}$)

$\therefore m(\angle BCD) = \frac{1}{2} \times 130^\circ = 65^\circ$

$\because \overline{AB} \parallel \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal.

$\therefore m(\angle ABC) = m(\angle BCD) = 65^\circ$

(alternate angles) (1)

, $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC) = 65^\circ$ (2)

From (1) and (2) :

$\therefore m(\angle ACB) = m(\angle BCD) = 65^\circ$

$\therefore \overrightarrow{CB}$ bisects $\angle ACD$ (First req.)

In ΔABC :

$m(\angle A) = 180^\circ - (65^\circ + 65^\circ) = 50^\circ$ (Second req.)

4

[a] $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

$\therefore m(\angle ACB) = \frac{1}{2} \times 120^\circ = 60^\circ$ (1)

, $\because \overleftrightarrow{CD} \parallel \overline{AB}$ $\therefore m(\overset{\frown}{AC}) = m(\overset{\frown}{BC})$

$\therefore AC = BC$ (2)

From (1) and (2) :

$\therefore \Delta CAB$ is an equilateral triangle. (Q.E.D.)

[b] $\because$ X is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$ $\therefore \overline{MY} \perp \overline{AC}$

, $\because AB = AC$ $\therefore MX = MY$

, $\because MD = MH = r$

By subtracting :

$\therefore XD = YH$ (Q.E.D.)

5

[a] $\because \overline{DH} \parallel \overline{BC}$ $\therefore m(\overset{\frown}{BD}) = m(\overset{\frown}{CH})$

$\therefore m(\angle BAD) = m(\angle CAH)$

Adding $m(\angle BAC)$ to both sides

$\therefore m(\angle DAC) = m(\angle BAH)$ (Q.E.D.)

[b] $\because \overline{XY} \parallel \overleftrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle YXB) = m(\angle XBD)$ (alternate angles)

, $\because m(\angle ACB)$ (inscribed) $= m(\angle ABD)$

(tangency)

$\therefore m(\angle YXB) = m(\angle ACB)$

$\therefore$ AXYC is a cyclic quadrilateral. (Q.E.D.)

3 Alexandria

1

1 b 2 c 3 d 4 a 5 b 6 c

2

[a] $\because \overline{DH} \parallel \overline{BC}$ $\therefore m(\overset{\frown}{DB}) = m(\overset{\frown}{CH})$

$\therefore m(\angle DAB) = m(\angle CAH)$ (Q.E.D.)

[b] $\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$ $\therefore m(\angle ADM) = 90^\circ$

, $\because$ H is the midpoint of $\overline{AC}$

$\therefore \overline{MH} \perp \overline{AC}$ $\therefore m(\angle AHM) = 90^\circ$

From the quadrilateral ADMH :

$\therefore m(\angle DMH) = 360^\circ - (90^\circ + 90^\circ + 60^\circ)$

$= 120^\circ$ (The req.)

3

[a] $\because CB = CD$ $\therefore m(\overset{\frown}{CB}) = m(\overset{\frown}{CD})$ (1)

, $\because \overline{AC}$ is a diameter

$\therefore m(\overset{\frown}{ABC}) = m(\overset{\frown}{ADC})$ (2)

Subtracting (1) from (2) :

$\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{AD})$ (Q.E.D.)

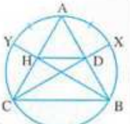
[b] $\because$ m $(\widehat{AX})$ = m $(\widehat{AY})$

$\therefore$ m (∠ ACX) = m (∠ ABY)

and they are drawn on $\overline{HD}$

and on one side of it

$\therefore$ BCHD is a cyclic quadrilateral.



(Q.E.D.)

4

[a] The length of $\widehat{AB} = \frac{108°}{360°} \times 2 \times 7 \times \frac{22}{7}$

= 13.2 cm. (The req.)

[b] $\because$ ABCD is a cyclic quadrilateral.

$\therefore$ m (∠ ADC) = m (∠ ABH) = 100°

In Δ ACD :

$\therefore$ m (∠ ACD) = 180° – (100° + 40°) = 40°

$\therefore$ m (∠ CAD) = m (∠ ACD)

$\therefore$ CD = AD (Q.E.D.)

5

[a] $\because$ $\overline{XC}$, $\overline{XA}$ are two tangent-segments

$\therefore$ XC = XA = 5 cm.

, $\because$ $\overline{YC}$, $\overline{YB}$ are two tangent-segments

$\therefore$ YC = YB = 8 cm.

$\therefore$ The perimeter of AXYB = 5 + 5 + 8 + 8 + 10

= 36 cm. (The req.)

[b] In Δ ABC : $\because$ CB = CA

$\therefore$ m (∠ B) = m (∠ BAC)

, $\because$ $\overline{AB}$ // $\overline{CD}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m (∠ ACD) = m (∠ BAC) (alternate angles)

$\therefore$ m (∠ B) = m (∠ ACD)

$\therefore$ $\overleftrightarrow{CD}$ is a tangent to the circle circumscribed about Δ ABC (Q.E.D.)

4 El-Kalyoubia

1

1 d 2 a 3 c 4 d 5 b 6 d

2

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MX} \perp \overline{AB}$ $\therefore$ m (∠ AXM) = 90°

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore$ $\overline{MY} \perp \overline{AC}$ $\therefore$ m (∠ AYM) = 90°

From the quadrilateral AXMY :

m (∠ DME) = 360° – (90° + 90° + 70°) = 110°

(First req.)

, $\because$ AB = AC $\therefore$ MX = MY

, $\because$ MD = ME = r

By subtracting : $\therefore$ XD = YE (Second req.)

[b] $\because$ m $(\widehat{BC})$ = m $(\widehat{DE})$

Adding m $(\widehat{BD})$ to both sides

$\therefore$ m $(\widehat{CD})$ = m $(\widehat{EB})$ $\therefore$ m (∠ C) = m (∠ E)

$\therefore$ In Δ ACE : AC = AE

, $\because$ m $(\widehat{CB})$ = m $(\widehat{ED})$ $\therefore$ CB = ED

$\therefore$ AB = AD (Q.E.D.)

3

[a] In Δ ABD : $\because$ AB = AD

$\therefore$ m (∠ ABD) = m (∠ ADB) = 30°

$\therefore$ m (∠ A) = 180° – (30° + 30°) = 120°

, $\because$ m (∠ A) + m (∠ C) = 120° + 60° = 180°

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b] $\because$ $\overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle

$\therefore$ AD = AF = 3 cm.

, $\because$ $\overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle

$\therefore$ BD = BE = 4 cm.

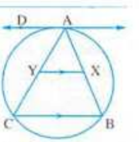
, $\because$ $\overline{CE}$, $\overline{CF}$ are two tangent-segments to the circle

$\therefore$ CE = CF = 2 cm.

$\therefore$ The perimeter of Δ ABC = 3 + 3 + 4 + 4 + 2 + 2

= 18 cm. (The req.)

2



[a] $\because$ $\overleftrightarrow{AD}$ is a tangent to the circle

$\therefore$ m (∠ DAC) (tangency)

= m (∠ B) (inscribed) (1)

, $\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m (∠ AXY) = m (∠ B) (2)

(corresponding angles)

From (1) and (2) : $\therefore$ m (∠ AXY) = m (∠ DAC)

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] $\because$ m (∠ AEC) = $\frac{1}{2}$ [m $(\widehat{AC})$ + m $(\widehat{BD})$]

$\therefore$ m (∠ AEC) = $\frac{1}{2}$ (50° + 80°) = 65° (The req.)

5

[a] ∵ $\overline{DE} \parallel \overline{BC}$

∴ m $(\widehat{DB})$ = m $(\widehat{EC})$ adding m $(\widehat{BC})$ to both sides

∴ m $(\widehat{DC})$ = m $(\widehat{EB})$

∴ m (∠ DAC) = m (∠ BAE) (Q.E.D.)

[b] ∵ m (∠ B) = $\frac{1}{2}$ m (∠ AMC) = 60° (1)

(inscribed and central angle subtended by $\widehat{AC}$)

, ∵ $\overleftrightarrow{CD} \parallel \overline{AB}$ ∴ m $(\widehat{AC})$ = m $(\widehat{CB})$

∴ AC = CB (2)

From (1) and (2) :

∴ Δ CAB is an equilateral triangle. (Q.E.D.)

5 El-Sharkia

1

1 a 2 d 3 b 4 c 5 d 6 b

2

[a] ∵ m (∠ A) = $\frac{1}{2}$ m (∠ BMC)

(inscribed and central angles subtended by $\widehat{BC}$)

∴ m (∠ A) = 65° (First req.)

, ∵ ABDC is a cyclic quadrilateral

∴ m (∠ D) = 180° − 65° = 115° (Second req.)

[b] ∵ $\overline{CD}$ is a diameter

∴ m (∠ CBD) = 90°

∴ m (∠ ABC) = 135° − 90° = 45°

, ∵ m (∠ D) (inscribed) = m (∠ ABC) (tangency) = 45°

∴ m (∠ D) + m (∠ ABD) = 45° + 135° = 180°

and they are interior angles in the same side of the transversal

∴ $\overline{DC} \parallel \overrightarrow{BA}$ (Q.E.D.)

3

[a] ∵ $\overrightarrow{AB}$ is a tangent ∴ $\overline{MB} \perp \overrightarrow{AB}$

∴ m (∠ ABM) = 90°

, ∵ $\overrightarrow{AC}$ is a tangent ∴ $\overline{MC} \perp \overrightarrow{AC}$

∴ m (∠ ACM) = 90°

, ∵ m (∠ ABM) + m (∠ ACM) = 90° + 90° = 180°

∴ ABMC is a cyclic quadrilateral. (Q.E.D. 1)

∴ m (∠ CMD) = m (∠ A) = 45°

, ∵ m (∠ MCD) = 90°

In Δ CMD : ∴ m (∠ D) = 180° − (90° + 45°) = 45°

∴ m (∠ CMD) = m (∠ D)

∴ CD = CM (Q.E.D. 2)

[b] ∵ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the smaller circle

∴ $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

, ∵ MD = ME = r

(radii lengths of the smaller circle)

∴ AB = AC (Q.E.D.)

4

[a] ∵ X is the midpoint of $\overline{AC}$

∴ $\overline{NX} \perp \overline{AC}$ ∴ m (∠ NXA) = 90°

, ∵ $\overline{AB}$ is the common chord

, $\overleftrightarrow{NM}$ is the line of centres

∴ $\overleftrightarrow{NM} \perp \overline{AB}$ ∴ m (∠ NYA) = 90°

From the quadrilateral AXNY :

∴ m (∠ BAC) = 360° − (90° + 90° + 80°) = 100° (The req.)

[b] ∵ ABCD is a cyclic quadrilateral

∴ m (∠ C) = 180° − 120° = 60°

, ∵ $\overrightarrow{BE} \parallel \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal

∴ m (∠ EBC) = m (∠ C) = 60° (alternate angles)

∴ m (∠ CBF) = 60° + 45° = 105°

∴ m (∠ CDA) = m (∠ CBF) = 105° (The req.)

5

[a] ∵ m $(\widehat{BD})$ = 2 m (∠ BED) = 20°

∴ m (∠ A) = $\frac{1}{2}$ [m $(\widehat{CE})$ − m $(\widehat{BD})$]

= $\frac{1}{2}$ (80° − 20°) = 30° (The req)

[b] ∵ In Δ ABC : m (∠ C) = 180° − (90° + 30°) = 60°

∴ m (∠ C) = m (∠ DAB) = 60°

∴ $\overrightarrow{AD}$ is a tangent to the circle passing through the points A , B and C (Q.E.D.)

6 El-Monofia

1

1 b 2 a 3 c 4 c 5 a 6 c

2

[a] ∵ $\overline{AB}$ is a diameter ∴ m $(\widehat{AB})$ = 180°

, ∵ $\overline{AB}$ // $\overline{CD}$

∴ m $(\widehat{AC})$ = m $(\widehat{BD})$ = $\frac{180° - 100}{2}$ = 40° (First req.)

∴ m (∠ AEC) = $\frac{1}{2}$ m $(\widehat{AC})$ = 20°

∴ 2 X − 10° = 20° ∴ 2 X = 30°

∴ X = 15° (Second req.)

[b] In Δ ABC :

∵ m (∠ B) = m (∠ C) ∴ AB = AC

, ∵ X is the midpoint of $\overline{AB}$ ∴ $\overline{MX}$ ⊥ $\overline{AB}$

, ∵ $\overline{MY}$ ⊥ $\overline{AC}$, AB = AC

∴ MX = MY (Q.E.D.)

3

[a] ∵ m (∠ ADC) = $\frac{1}{2}$ m (∠ AMC)

(inscribed and central angles subtended by $\widehat{AC}$)

∴ m (∠ ADC) = $\frac{1}{2}$ × 70 = 35° (First req.)

, ∵ $\overline{CD}$ // $\overline{AB}$, $\overleftrightarrow{AD}$ is a transversal

∴ m (∠ BAD) = m (∠ ADC) = 35° (alternate angles)

, ∵ $\overline{AB}$ is a diameter

∴ m (∠ ADB) = 90°

In Δ ABD : ∴ m (∠ ABD) = 180° − (90° + 35°)

= 55° (Second req.)

[b] ∵ $\overline{DE}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

∴ m (∠ AED) = m (∠ C) (corresponding angles)

, ∵ m (∠ C) (inscribed) = m (∠ BAX) (tangency)

∴ m (∠ AED) = m (∠ DAX)

∴ $\overleftrightarrow{AX}$ is a tangent to the circle passing through the points A , D and E (Q.E.D.)

4

[a] ∵ X is the midpoint of $\overline{CF}$

∴ $\overline{MX}$ ⊥ $\overline{CF}$ ∴ m (∠ MXE) = 90°

, ∵ $\overline{AB}$ is the common chord

, $\overleftrightarrow{MN}$ is the line of centres

∴ $\overleftrightarrow{MN}$ ⊥ $\overline{AB}$ ∴ m (∠ MDE) = 90°

From the quadrilateral XMDE :

∴ m (∠ XMD) = 360° − (90° + 90° + 52°) = 128° (The req.)

[b] In Δ BCD : ∵ BC = DC

∴ m (∠ BDC) = m (∠ CBD) = 35°

∴ m (∠ C) = 180° − (35° + 35°) = 110°

, ∵ m (∠ A) + m (∠ C) = 70° + 110° = 180°

∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] ∵ $\overline{BD}$, $\overline{BE}$ are two tangent-segments

∴ BD = BE = 6 cm.

∴ CE = 10 − 6 = 4 cm. The req.)

[b] In Δ ABE :

∵ AB = AE ∴ m (∠ AEB) = m (∠ B)

, ∵ m (∠ D) = m (∠ B) (properties of parallelogram)

∴ m (∠ AEB) = m (∠ D)

∴ AECD is a cyclic quadrilateral. (Q.E.D.)

7 El-Gharbia

1

1 b 2 d 3 c 4 b 5 c 6 c

2

[a] ∵ D is the midpoint of $\overline{AB}$

∴ $\overline{MD}$ ⊥ $\overline{AB}$ ∴ m (∠ ADM) = 90°

, ∵ E is the midpoint of $\overline{AC}$

∴ $\overline{ME}$ ⊥ $\overline{AC}$ ∴ m (∠ AEM) = 90°

From the quadrilateral MDAE :

∴ m (∠ DME) = 360° − (90° + 90° + 120°) = 60°

, ∵ m (∠ YMX) = m (∠ DME) = 60° (V.O.A)

, MY = MX = r

∴ Δ XMY is an equilateral triangle. (Q.E.D.)

[b] In Δ AMD :

∵ MA = MD = r

∴ m (∠ MAD) = m (∠ MDA) (1)

, ∵ $\overrightarrow{DA}$ bisects ∠ BDM

∴ m (∠ MDA) = m (∠ ADB) (2)

From (1) , (2) : ∴ m (∠ MAD) = m (∠ ADB)

and they are alternate angles

∴ $\overline{AM}$ // $\overline{BD}$

, ∵ $\overline{BD}$ ⊥ $\overleftrightarrow{AB}$ ∴ $\overline{MA}$ ⊥ $\overleftrightarrow{AB}$

∴ $\overleftrightarrow{AB}$ is a tangent to the circle M at A (Q.E.D.)

3

[a] $\because$ m ($\angle$ BCD) = $\frac{1}{2}$ m ($\angle$ BMD)

(inscribed and central angles subtended by $\widehat{BD}$)

$\therefore$ m ($\angle$BCD) = $\frac{1}{2}$ × 50° = 25°

, $\because$ $\overline{AB}$ is a diameter $\therefore$ m ($\angle$ ACB) = 90°

$\therefore$ m ($\angle$ ACD) = 25° + 90° = 115° (The req.)

[b] $\because$ $\overline{DE}$ // $\overline{BC}$ $\therefore$ m ($\widehat{BD}$) = m ($\widehat{CE}$)

$\therefore$ m ($\angle$ BAD) = m ($\angle$ CAE)

Adding m ($\angle$ BAC) to both sides

$\therefore$ m ($\angle$ DAC) = m ($\angle$ BAE) (Q.E.D.)

4

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ BAD) = 180° − 70° = 110°

, $\because$ ABFE is a cyclic quadrilateral and $\angle$ BAD is exterior of it.

$\therefore$ m ($\angle$ EFB) = m ($\angle$ BAD) = 110° (First req.)

$\therefore$ m ($\angle$ EFB) + m ($\angle$ BCD) = 110° + 70° = 180°

and they are interior angles in the same side of $\overleftrightarrow{FC}$

$\therefore$ $\overline{CD}$ // $\overline{EF}$ (Second req.)

[b] $\because$ $\overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

$\therefore$ XA = XB

$\therefore$ In Δ ABX :

m ($\angle$ XAB) = m ($\angle$ XBA) = $\frac{180° - 70°}{2}$ = 55°

, $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ BAD) = 180° − 125° = 55°

$\therefore$ m ($\angle$ XAB) = m ($\angle$ BAD)

$\therefore$ $\overrightarrow{AB}$ bisects $\angle$ DAX (Q.E.D.)

5

[a] $\because$ $\overline{AB}$ is the common chord

, $\overleftrightarrow{MN}$ is the line of centres $\therefore$ $\overline{MN} \perp \overline{AB}$

$\therefore$ $\overline{MD} \perp \overline{AB}$

, $\because$ $\overleftrightarrow{MX} \perp \overline{AC}$, AC = AB $\therefore$ MX = MD (1)

, $\because$ MY = ME (lengths of two radii) (2)

Subtracting (1) from (2) : $\therefore$ XY = DE (Q.E.D.)

[b] $\because$ $\overline{XY}$ // $\overleftrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m ($\angle$ DBX) = m ($\angle$ YXB) (alternate angles) (1)

, $\because$ m ($\angle$ C) (inscribed)

= m ($\angle$ ABD) (tangency) (2)

From (1) and (2) :

$\therefore$ m ($\angle$ C) = m ($\angle$ YXB)

$\therefore$ AXYC is a cyclic quadrilateral. (Q.E.D.)

8 El-Dakahlia

1

[a] 1 a 2 c 3 b

[b] $\because$ m ($\angle$ A) = $\frac{1}{2}$ [m ($\widehat{CE}$) − m ($\widehat{BD}$)]

$\therefore$ 20° = $\frac{1}{2}$ [80° − m ($\widehat{BD}$)]

$\therefore$ 40° = 80° − m ($\widehat{BD}$)

$\therefore$ m ($\widehat{BD}$) = 80° − 40° = 40°

, $\because$ $\overline{BC}$ is a diameter

$\therefore$ m ($\widehat{BC}$) = 180°

$\therefore$ m ($\widehat{DE}$) = 180° − (80° + 40°) = 60° (The req.)

2

[a] 1 b 2 c 3 d

[b] $\because$ m ($\angle$ BCD) = $\frac{1}{2}$ m ($\angle$ BMD)

(inscribed and central angles subtended by $\widehat{BD}$)

$\therefore$ m ($\angle$ BCD) = $\frac{1}{2}$ × 30° = 15° (First req.)

, $\because$ $\overline{AB}$ is a diameter

$\therefore$ m ($\angle$ ACB) = 90°

$\therefore$ m ($\angle$ ACD) = 90° + 15° = 105° (Second req.)

3

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ D) = m ($\angle$ ABE) = 100°

, $\because$ D is the midpoint of $\widehat{AC}$

$\therefore$ m ($\widehat{AD}$) = m ($\widehat{CD}$) $\therefore$ AD = CD

In Δ ACD :

$\therefore$ m ($\angle$ DAC) = m ($\angle$ DCA) = $\frac{180° - 100°}{2}$ = 40°

(The req.)

[b] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore$ AB = AC $\therefore$ $2x - 1 = x + 2$

$\therefore$ $2x - x = 2 + 1$ $\therefore$ $x = 3$ (First req.)

$\therefore$ AB = AC = 2 × 3 − 1 = 5 cm.

, BC = 7 − 3 = 4 cm.

$\therefore$ The perimeter of Δ ABC = 5 + 5 + 4 = 14 cm.

(Second req.)

4

[a] In Δ EBC : $\because$ BE = BC

$\therefore$ m ($\angle$ BEC) = m ($\angle$ C)

, $\because$ m ($\angle$ BAD) = m ($\angle$ C)

(properties of parallelogram)

$\therefore$ m (∠ BED) = m (∠ BAD)

and they are drawn on $\overline{BD}$ and on one side of it

$\therefore$ ABDE is a cyclic quadrilateral. (Q.E.D. 1)

$\therefore$ m (∠ AEB) = m (∠ ADB)

(drawn on $\overline{AB}$ and on one side of it)

, $\because$ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{BD}$ is a transversal

$\therefore$ m (∠ DBC) = m (∠ ADB) (alternate angles)

$\therefore$ m (∠ AEB) = m (∠ DBC) (Q.E.D. 2)

[b] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the smaller circle

$\therefore$ $\overline{MD} \perp \overline{AB}$ $\therefore$ m (∠ MDA) = 90°

, $\overline{ME} \perp \overline{AC}$ $\therefore$ m (∠ MEA) = 90°

From the quadrilateral ADME :

$\therefore$ m (∠ EMD) = 360° – (90° + 90° + 50°) = 130° (First req.)

, $\because$ MD = ME (two radii in the smaller circle)

$\therefore$ AB = AC (Second req.)

5

[a] In Δ AMC : $\because$ AM = MC = r

$\therefore$ m (∠ MAC) = m (∠ ACM)

$\because$ m (∠ BAC) = m (∠ MAC)

$\therefore$ m (∠ BAC) = m (∠ ACM) and they are alternate angles.

$\therefore$ $\overline{AB}$ // $\overline{CM}$

$\because$ D is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MD} \perp \overline{AB}$, $\because$ $\overline{AB}$ // $\overline{CM}$

$\therefore$ $\overline{DM} \perp \overline{CM}$ (Q.E.D.)

[b] $\because$ The figure ABCD is a cyclic quadrilateral

$\therefore$ m (∠ ABC) = 180° – 125° = 55° (1)

$\because$ $\overrightarrow{EA}$, $\overrightarrow{EB}$ are two tangents to the circle at A and B

$\therefore$ EA = EB $\because$ m (∠ E) = 70°

$\therefore$ m (∠ EAB) = $\frac{180° - 70°}{2}$ = 55°

$\because$ $\overrightarrow{EA}$ is a tangent to the circle at A

$\therefore$ m (∠ EAB) (tangency) = m (∠ ACB) (inscribed)

$\therefore$ m (∠ ACB) = 55° (2)

From (1) and (2) :

$\therefore$ m (∠ ACB) = m (∠ ABC) = 55°

$\therefore$ AB = AC (Q.E.D. 1)

$\therefore$ m (∠ BAC) = 180° – 2 × 55° = 70°

$\therefore$ m (∠ BAC) = m (∠ E) = 70°

$\therefore$ $\overrightarrow{AC}$ is a tangent to the circle passing through the vertices of Δ ABE (Q.E.D. 2)

9 Ismailia

1

1 c	2 b	3 d	4 a	5 d	6 c

2

[a] $\because$ m (∠ A) = $\frac{1}{2}$ m (∠ BMD) = $\frac{1}{2}$ × 150° = 75°

(inscribed and central angles subtended by $\widehat{BD}$)

, $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m (∠ C) = 180° – 75° = 105° (The req.)

[b] In Δ ABD : $\because$ AB = AD

$\therefore$ m (∠ ABD) = m (∠ ADB) = 30°

$\therefore$ m (∠ A) = 180° – (30° + 30°) = 120°

, $\because$ m (∠ A) + m (∠ C) = 120° + 60° = 180°

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\because$ m (∠ BDC) = m (∠ BAC)

(two inscribed angles subtended by $\widehat{BC}$)

$\therefore$ m (∠ BDC) = 30° (First req.)

$\because$ m ($\widehat{BC}$) = 2 m (∠ BAC) = 60°

, $\because$ $\overline{AB}$ is diameter in the circle M

$\therefore$ m ($\widehat{AB}$) = 180°

$\therefore$ m ($\widehat{AC}$) = 180° – 60° = 120°

, $\because$ D is the midpoint of $\widehat{AC}$

$\therefore$ m ($\widehat{AD}$) = $\frac{120°}{2}$ = 60°

$\therefore$ m (∠ ACD) = $\frac{1}{2}$ m ($\widehat{AD}$) = $\frac{1}{2}$ × 60° = 30°

$\therefore$ m (∠ BAC) = m (∠ ACD) but they are alternate angles

$\therefore$ $\overline{DC}$ // $\overline{AB}$ (Second req.)

[b] $\because$ $\overrightarrow{AD}$ is a tangent to the circle

$\therefore$ $\overline{MD} \perp \overrightarrow{AD}$ $\therefore$ m (∠ ADM) = 90°

, $\because$ E is the midpoint of $\overline{BC}$

$\therefore$ $\overline{ME} \perp \overline{BC}$ $\therefore$ m (∠ MEA) = 90°

$\therefore$ From the quadrilateral ADME :

m (∠ DME) = 360° – (90° + 90° + 65°) = 115° (The req.)

4

[a] $\because \overline{AX}$, $\overline{AZ}$ are two tangent-segments

$\therefore AX = AZ = 6$ cm.

$\therefore CZ = 10 - 6 = 4$ cm.

, $\because \overline{CZ}$, $\overline{CY}$ are two tangent-segments

$\therefore CZ = CY = 4$ cm.

, $\because \overline{BX}$, $\overline{BY}$ are two tangent-segments

$\therefore BX = BY$

$\because$ The perimeter of Δ ABC = 24 cm.

$BX + BY + 6 + 10 + 4 = 24$

$\therefore BX + BY = 4$ $\qquad \therefore BX = 2$ cm.

$\therefore AB = 6 + 2 = 8$ cm. (The req.)

[b] $\because m(\angle C) = \frac{1}{2} m(\angle BMD) = \frac{1}{2} \times 80^\circ = 40^\circ$

(inscribed and central angles subtended by $\widehat{BD}$)

, $\because \angle ABC$ is an exterior angle of Δ BCD

$\therefore m(\angle CDB) = 110^\circ - 40^\circ = 70^\circ$ (First req.)

, $\because m(\angle CBD) = 180^\circ - 110^\circ = 70^\circ$

$\because m(\angle CDB) = m(\angle CBD) = 70^\circ$

$\therefore$ In Δ CBD : CB = CD (Second req.)

5

[a] $\because m(\angle A) = \frac{1}{2} [m(\widehat{EC}) - m(\widehat{BD})]$

$\therefore 40^\circ = \frac{1}{2} [m(\widehat{EC}) - 60^\circ]$

$\therefore 80^\circ = m(\widehat{EC}) - 60^\circ$

$\therefore m(\widehat{EC}) = 80^\circ + 60^\circ = 140^\circ$ (First req.)

, $\because m(\widehat{BC}) = m(\widehat{ED})$

$\therefore m(\widehat{BC}) = m(\widehat{ED}) = \frac{360^\circ - (140^\circ + 60^\circ)}{2} = 80^\circ$

(Second req.)

[b] $\because \overline{XY} \parallel \overleftrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle DBX) = m(\angle YXB)$ (alternate angles) (1)

, $\because m(\angle C)$ (inscribed)

$= m(\angle ABD)$ (tangency) (2)

From (1) and (2) :

$\therefore m(\angle C) = m(\angle YXB)$

$\therefore$ AXYC is a cyclic quadrilateral. (Q.E.D.)

10 Suez

1

1 d 2 b 3 c 4 c 5 a 6 b

2

[a] $m(\angle ABD)$ (tangency) $= \frac{1}{2} m(\angle AMB)$ (central)

$= \frac{1}{2} \times 80^\circ = 40^\circ$ (First req.)

$m(\widehat{AB}) = m(\angle AMB) = 80^\circ$ (Second req.)

[b] $\because$ E is the midpoint of $\overline{AD}$

$\therefore \overline{ME} \perp \overline{AD}$ $\qquad \therefore m(\angle MEC) = 90^\circ$

, $\because \overline{BC}$ is a tangent-segment

$\therefore \overline{MB} \perp \overline{BC}$ $\qquad \therefore m(\angle MBC) = 90^\circ$

From the quadrilateral MBCE :

$\therefore m(\angle EMB) = 360^\circ - (90^\circ + 90^\circ + 50^\circ) = 130^\circ$

(The req.)

3

[a] $\because m(\widehat{AB}) = m(\widehat{BC}) = m(\widehat{AC})$

$\therefore m(\widehat{AB}) = \frac{360^\circ}{3} = 120^\circ$

$\therefore m(\angle C) = \frac{1}{2} m(\widehat{AB}) = \frac{1}{2} \times 120^\circ$

$= 60^\circ$ (The req.)

[b] $\because$ E is the midpoint of $\overline{AC}$ $\qquad \therefore \overline{ME} \perp \overline{AC}$

, $\because \overline{MD} \perp \overline{AB}$, MD = ME

$\therefore AB = AC$ (Q.E.D.)

4

[a] $m(\angle BDC)$ (inscribed) $= m(\angle ABC)$ (tangency)

$= 70^\circ$ (First req.)

$\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = 70^\circ$

$\therefore$ In Δ ABC :

$m(\angle BAC) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$

(Second req.)

[b] $\because \overline{AB} \parallel \overline{CD}$

$\therefore m(\widehat{BD}) = m(\widehat{AC}) = 30^\circ$

$\therefore m(\angle BED) = \frac{1}{2} m(\widehat{BD}) = \frac{1}{2} \times 30^\circ = 15^\circ$

(The req.)

5

[a] State by yourself.

[b] In Δ ABC :

$\because m(\angle B) = 180^\circ - (30^\circ + 50^\circ) = 100^\circ$

, $\because m(\angle B) + m(\angle D) = 100^\circ + 80^\circ = 180^\circ$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

11 Port Said

1

1 c   2 a   3 d   4 b   5 c   6 a

2

[a] ∵ $\overline{XY}$ is a tangent-segment

∴ $\overline{MX} \perp \overline{XY}$ ∴ m (∠ MXY) = 90°

∴ $(MY)^2 = (XY)^2 + (MX)^2 = 12^2 + 5^2 = 169$

∴ $MY = \sqrt{169} = 13$ cm.

∵ MX = MZ = r   ∴ MZ = 5 cm.

∴ YZ = 13 − 5 = 8 cm.   (The req.)

[b] ∵ $\overline{MO} \perp \overline{AB}$, $\overline{MH} \perp \overline{AC}$, AB = AC

∴ MO = MH

, ∵ MX = MY = r   ∴ OX = HY   (Q.E.D.)

3

[a] Mention by yourself.

[b] ∵ $\overline{BD}$ is a tangent

∴ $\overline{MB} \perp \overline{BD}$ ∴ m (∠ MBD) = 90°

, ∵ H is the midpoint of $\overline{AC}$

∴ $\overline{MH} \perp \overline{AC}$ ∴ m (∠ MHD) = 90°

, ∵ m (∠ MBD) + m (∠ MHD) = 90° + 90° = 180°

∴ DBMH is a cyclic quadrilateral.   (Q.E.D.)

4

[a] ∵ $\overrightarrow{AB}$ is a tangent

∴ $\overline{MB} \perp \overrightarrow{AB}$ ∴ m (∠ ABM) = 90°

In Δ AMB : ∴ m (∠ BMA) = 180° − (90° + 40°)

= 50°

∴ m (∠ BDC) = $\frac{1}{2}$ m (∠ BMC) = $\frac{1}{2}$ × 50° = 25°

(inscribed and central angles subtended by $\widehat{BC}$)

(The req.)

[b] ∵ $\overline{XY}$ // $\overline{BC}$ ∴ m ($\widehat{XB}$) = m ($\widehat{YC}$)

∴ m (∠ XAB) = m (∠ YAC)

Adding m (∠ BAC) to bath sides

∴ m (∠ XAC) = m (∠ YAB)   (Q.E.D.)

5

[a] ∵ $\overline{AD}$, $\overline{AQ}$ are two tangent-segments to the circle

∴ AD = AQ = 5 cm.

, ∵ $\overline{BD}$, $\overline{BH}$ are two tangent-segments to the circle

∴ BD = BH = 4 cm.

, ∵ $\overline{CH}$, $\overline{CQ}$ are two tangent-segments to the circle

∴ CH = CQ = 3 cm.

∴ The perimeter of Δ ABC = 5 + 5 + 4 + 4 + 3 + 3

= 24 cm.   (The req.)

[b] ∵ $\overleftrightarrow{AD}$ is a tangent to the circle

∴ m (∠ DAB) (tangency)

= m (∠ ACB) (inscribed)   (1)

, ∵ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{YC}$ is a transversal

∴ m (∠ AYX) = m (∠ ACB)

(corresponding angles)   (2)

From (1) and (2) :

∴ m (∠ DAB) = m (∠ AYX)

∴ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y   (Q.E.D.)

12 Damietta

1

1 c   2 d   3 a   4 c   5 b   6 c

2

[a] ∵ $\overrightarrow{AB}$ is a tangent

∴ $\overline{MA} \perp \overrightarrow{AB}$ ∴ m (∠ MAB) = 90°

, ∵ ∠ MBE is an exterior angle of Δ AMB

∴ m (∠ AMB) = 120° − 90° = 30°   (The req.)

[b] ∵ AB = CD (properties of rectangle)

, ∵ CE = CD   ∴ AB = CE

∴ m ($\widehat{AB}$) = m ($\widehat{CE}$)   (Q.E.D. 1)

Adding m ($\widehat{BE}$) to both sides

∴ m ($\widehat{AE}$) = m ($\widehat{BC}$)

∴ AE = BC   (Q.E.D. 2)

3

[a] ∵ X is the midpoint of $\overline{AB}$

∴ $\overline{MX} \perp \overline{AB}$ ∴ m (∠ MXA) = 90°

, ∵ Y is the midpoint of $\overline{AC}$

∴ $\overline{MY} \perp \overline{AC}$ ∴ m (∠ MYA) = 90°

From the quadrilateral AXMY :

∴ m (∠ EMD) = 360° − (90° + 90° + 80°)

= 100°   (First req.)

, ∵ AB = AC   ∴ MX = MY

, ∵ MD = ME = r

∴ XD = YE   (Second req.)

[b] In Δ ABD : $\because$ AB = AD

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ADB) = 30°

$\therefore$ m ($\angle$ A) = 180° − 2 × 30° = 120°

, $\because$ m ($\angle$ A) + m ($\angle$ C) = 120°+ 60° = 180°

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

4

[a] $\because$ m ($\angle$ D) = $\frac{1}{2}$ m ($\angle$ AMB)

(inscribed and central angles substanded by $\widehat{AB}$)

$\therefore$ m ($\angle$ D) = $\frac{1}{2}$ × 140° = 70°

, $\because$ $\overline{AC}$ // $\overline{DB}$, $\overleftrightarrow{AD}$ is a transversal

$\therefore$ m ($\angle$ DAC) + m ($\angle$ D) = 180°

(two interior angles in the same side of the transversal)

$\therefore$ m ($\angle$ DAC) = 180° − 70° = 110° (The req.)

[b] $\because$ $\overrightarrow{CB}$ bisects $\angle$ ACD

$\therefore$ m ($\angle$ ACB) = m ($\angle$ BCD) = 65°

, $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore$ AB = AC

$\therefore$ In Δ ABC :

m ($\angle$ ABC) = m ($\angle$ ACB) = 65°

$\therefore$ m ($\angle$ A) = 180° − (65° + 65°) = 50° (First req.)

, $\because$ m ($\angle$ D) (inscribed) = m ($\angle$ ACB) (tangency)

$\therefore$ m ($\angle$ D) = 65° (Second req.)

5

[a] m ($\angle$ ADB) = $\frac{1}{2}$ m ($\widehat{AB}$) = $\frac{1}{2}$ × 110° = 55° (First req.)

$\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ ADC) = m ($\angle$ CBE) = 85°

$\therefore$ m ($\angle$ BDC) = 85° − 55° = 30° (Second req.)

[b] In Δ ABC : $\because$ m ($\angle$ BAC) = 90° , AC = $\frac{1}{2}$ BC

$\therefore$ m ($\angle$ B) = 30°

$\therefore$ m ($\angle$ C) = 180° − (90° + 30°) = 60°

$\therefore$ m ($\angle$ C) = m ($\angle$ DAB) = 60°

$\therefore$ $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC (Q.E.D.)

13 Kafr El-Sheikh

1

[a] 1 b 2 a 3 d

[b] $\because$ E is the midpoint of $\overline{BC}$

$\therefore$ $\overline{ME} \perp \overline{BC}$ $\therefore$ m ($\angle$ AEM) = 90°

, $\because$ $\overrightarrow{AD}$ is a tangent to the circle

$\therefore$ $\overline{MD} \perp \overline{AD}$ $\therefore$ m ($\angle$ ADM) = 90°

From the quadrilateral ADME :

$\therefore$ m ($\angle$ DME) = 360° − (90° + 90° + 56°) = 124° (The req.)

2

[a] 1 c 2 b 3 a

[b] $\because$ m ($\widehat{BC}$) = 2 m ($\angle$ BAC) = 60°

, $\because$ $\overline{AB}$ is diameter in the circle M

$\therefore$ m ($\widehat{AB}$) = 180°

$\therefore$ m ($\widehat{AC}$) = 180° − 60° = 120°

, $\because$ D is the midpoint of $\widehat{AC}$

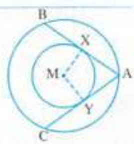
$\therefore$ m ($\widehat{AD}$) = m ($\widehat{DC}$) = $\frac{120°}{2}$ = 60° (First req.)

$\therefore$ m ($\angle$ ACD) = $\frac{1}{2}$ m ($\widehat{AD}$) = $\frac{1}{2}$ × 60° = 30°

$\therefore$ m ($\angle$ CAB) = m ($\angle$ ACD) but they are alternate angles

$\therefore$ $\overline{AB}$ // $\overline{DC}$ (Second req.)

3



[a] Construction :

Draw $\overline{MX}$, $\overline{MY}$

Proof :

$\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the smaller circle.

, $\overline{MX}$, $\overline{MY}$ are two radii

$\therefore$ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

, $\because$ MX = MY = r (radii of the smaller circle)

$\therefore$ AB = AC (Q.E.D.)

[b] In Δ ABC :

$\because$ AC = $\frac{1}{2}$ BC , m ($\angle$ BAC) = 90°

$\therefore$ m ($\angle$ B) = 30°

$\therefore$ m ($\angle$ C) = 180° − (90° + 30°) = 60°

$\therefore$ m ($\angle$ C) = m ($\angle$ BAD) = 60°

$\therefore$ $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC (Q.E.D.)

4

[a] $\because$ $\overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle

$\therefore AD = AF = 3$ cm. $\quad \therefore CF = 8 - 3 = 5$ cm.

, $\because \overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle

$\therefore BD = BE = 2$ cm.

, $\because \overline{CE}$, $\overline{CF}$ are two tangent-segments to the circle

$\therefore CE = CF = 5$ cm.

$\therefore BC = 2 + 5 = 7$ cm. (The req.)

[b] $\because m(\angle BCD) = \frac{1}{2} m(\angle M)$

(inscribed and central angles subtended by $\widehat{BD}$)

$\therefore m(\angle BCD) = \frac{1}{2} \times 130° = 65°$

$\because \overline{AB} // \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal

$\therefore m(\angle ABC) = m(\angle BCD) = 65°$ (alternate angles)

$\because \overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

$\therefore AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC) = 65°$

$\therefore$ In ΔABC :

$m(\angle A) = 180° - 2 \times 65° = 50°$ (The req.)

5

[a] State by yourself.

[b] $\because \overline{AB}$ is a diameter of the circle

$\therefore m(\angle ACB) = 90°$ (First req.)

, $\because m(\angle ACE) = m(\angle ADE) = 90°$

and they are drawn on $\overline{AE}$ and on one side of it

$\therefore$ ACDE is a cyclic quadrilateral. (Second req.)

14 El-Beheira

1

1 d	2 b	3 b	4 a	5 d	6 c

2

[a] In ΔABC : $\because m(\angle B) = m(\angle C)$

$\therefore AB = AC$

, $\because$ X is the midpoint of $\overline{AB}$ $\quad \therefore \overline{MX} \perp \overline{AB}$

, $\because \overline{MY} \perp \overline{AC}$

$\therefore MX = MY$ (Q.E.D.)

[b] $\because$ ABCD is a cyclic quadrilateral.

$\therefore m(\angle ADC) = m(\angle CBE) = 85°$

, $\because m(\angle ADB) = \frac{1}{2} m(\widehat{AB}) = \frac{1}{2} \times 110° = 55°$

$\therefore m(\angle BDC) = 85° - 55° = 30°$ (The req.)

3

[a] $\because \overline{CM} // \overline{AB}$, $\overleftrightarrow{AM}$ is a transversal

$\therefore m(\angle CMA) = m(\angle A) = 60°$ (alternate angles)

, $\because m(\angle B) = \frac{1}{2} m(\angle CMA)$

(inscribed and central angles subtended by $\widehat{AC}$)

$\therefore m(\angle B) = \frac{1}{2} \times 60° = 30°$ (The req.)

[b] $\because m(\angle E) = \frac{1}{2} [m(\widehat{CA}) - m(\widehat{BD})]$

$\therefore 30° = \frac{1}{2} [80° - m(\widehat{BD})]$

$\therefore 60° = 80° - m(\widehat{BD})$

$\therefore m(\widehat{BD}) = 80° - 60° = 20°$

, $\because \overline{BA}$ is a diameter in the circle

$\therefore m(\widehat{BA}) = 180°$

$\therefore m(\widehat{CD}) = 180° - (80° + 20°) = 80°$ (The req.)

4

[a] $\because$ X is the midpoint of $\overline{AC}$

$\therefore \overline{MX} \perp \overline{AC}$ $\quad \therefore m(\angle AXY) = 90°$

, $\because \overrightarrow{YB}$ is a tangent to the circle

$\therefore \overline{MB} \perp \overline{BY}$ $\quad \therefore m(\angle MBY) = 90°$

, $\because m(\angle AXY) = m(\angle ABY)$ and they are drawn on $\overline{AY}$ and on one side of it

$\therefore$ AXBY is a cyclic quadrilateral. (Q.E.D.)

[b] In ΔAMC : $\because AM = MC = r$

$\therefore m(\angle MAC) = m(\angle ACM)$

$\because m(\angle BAC) = m(\angle MAC)$

$\therefore m(\angle BAC) = m(\angle ACM)$ and they are alternate angles.

$\therefore \overline{AB} // \overline{CM}$

$\because$ D is the midpoint of $\overline{AB}$ $\quad \therefore \overline{MD} \perp \overline{AB}$

, $\because \overline{AB} // \overline{CM}$

$\therefore \overline{DM} \perp \overline{CM}$ (Q.E.D.)

5

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangents

$\therefore AB = AC$

$\therefore$ In ΔABC :

$m(\angle ABC) = m(\angle ACB) = \frac{180° - 40°}{2} = 70°$

, $\because m(\angle D)$ (inscribed) $= m(\angle ABC)$ (tangency)

$\therefore m(\angle D) = 70°$ (The req.)

[b] ∵ $\overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

∴ XA = XB

∴ In Δ ABX :

m (∠ XAB) = m (∠ XBA) = $\frac{180° - 50}{2}$ = 65°

, ∵ ABCD is a cyclic quadrilateral

∴ m (∠ BAD) + m (∠ DCB) = 180°

∴ m (∠ BAD) = 180° – 115° = 65°

∴ m (∠ XAB) = m (∠ BAD)

∴ $\overrightarrow{AB}$ bisects ∠ DAX (Q.E.D.1)

, ∵ m (∠ ADB) (inscribed)
= m (∠ XAB) (tangency) = 65°

∴ m (∠ BAD) = m (∠ ADB)

∴ BD = BA (Q.E.D.2)

15 El-Fayoum

1

1 d 2 c 3 c 4 d 5 d 6 c

2

[a] ∵ $\overrightarrow{CE}$ bisects ∠ DCF

∴ m (∠ DCF) = 2 × 53° = 106°

, ∵ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{DC}$ is a transversal

∴ m (∠ D) = m (∠ DCF) = 106° (alternate angles)

∴ m (∠ B) + m (∠ D) = 74° + 106° = 180°

∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b] ∵ D is the midpoint of $\overline{AB}$

∴ $\overline{MD} \perp \overline{AB}$ ∴ m (∠ ADM) = 90°

, ∵ E is the midpoint of $\overline{AC}$

∴ $\overline{ME} \perp \overline{AC}$ ∴ m (∠ AEM) = 90°

From the quadrilateral ADME :

∴ m (∠ DME) = 360° – (90° + 90° + 65°) = 115° (The req.)

3

[a] ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents ∴ AB = AC

∴ In Δ ABC :

m (∠ ABC) = m (∠ ACB) = $\frac{180° - 70°}{2}$ = 55°

∴ m (∠ D) (inscribed) = m (∠ ABC) (tangency)
= 55°

In Δ BCD : ∵ BD = BC

∴ m (∠ D) = m (∠ BCD) = 55°

∴ m (∠ DBC) = 180° – 2 × 55° = 70°

∴ m (∠ ABD) = 55° + 70° = 125° (The req.)

[b] ∵ MC = MA = r ∴ MC = MA = 10 – 4 = 6 cm.

, ∵ $\overrightarrow{BA}$ is a tangent to the circle M

∴ $\overline{MA} \perp \overrightarrow{BA}$ ∴ m (∠ MAB) = 90°

∴ $(AB)^2 = (MB)^2 - (MA)^2 = 10^2 - 6^2 = 64$

∴ AB = $\sqrt{64}$ = 8 cm. (The req.)

4

[a] ∵ $\overline{MX} \perp \overline{AB}$ ∴ X is the midpoint of $\overline{AB}$

∴ AB = 2 × BX = 2 × 5 = 10 cm.

, ∵ $\overline{MY} \perp \overline{CD}$, MX = MY

∴ CD = AB = 10 cm. (The req.)

[b] In Δ ABD : ∵ AB = AD

∴ m (∠ ABD) = m (∠ ADB) = 30°

∴ m (∠ A) = 180° – 2 × 30° = 120°

, ∵ m (∠ A) + m (∠ C) = 120° + 60° = 180°

∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] ∵ AB = AC

∴ m ($\widehat{AB}$) = m ($\widehat{AC}$)

∴ m (∠ AEB) = m (∠ AEC) (Q.E.D.)

[b] ∵ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

∴ m (∠ AXY) = m (∠ ABC) (corresponding angles)

, ∵ m (∠ ABC) (inscribed) = m (∠ CAD) (tangency)

∴ m (∠ AXY) = m (∠ YAD)

∴ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

16 Beni Suef

1

1 b 2 d 3 d 4 c 5 a 6 c

2

[a] In Δ ABD : ∵ AB = AD

∴ m (∠ ABD) = m (∠ ADB) = 40°

∴ m (∠ A) = 180° – 2 × 40° = 100°

, ∵ m (∠ A) + m (∠ C) = 100° + 80° = 180°

∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b] $\because m(\angle E) = \frac{1}{2}[m(\widehat{AC}) - m(\widehat{BD})]$

$\therefore 20^\circ = \frac{1}{2}[80^\circ - m(\widehat{BD})]$

$\therefore 40^\circ = 80^\circ - m(\widehat{BD})$

$\therefore m(\widehat{BD}) = 80^\circ - 40^\circ = 40^\circ$

$\because \overline{AB}$ is a diameter $\therefore m(\widehat{AB}) = 180^\circ$

$\therefore m(\widehat{CD}) = 180^\circ - (80^\circ + 40^\circ) = 60^\circ$ (The req.)

3

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle MXA) = 90^\circ$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\therefore m(\angle MYA) = 90^\circ$

From the quadrilateral AXMY

$\therefore m(\angle DME) = 360^\circ - (90^\circ + 90^\circ + 60^\circ) = 120^\circ$ (First req.)

, $\because AB = AC$ $\therefore MX = MY$

, $\because MD = ME = r$ $\therefore XD = YE$ (Second req.)

[b] $\because m(\angle BDC)$ (inscribed)

$= m(\angle ABC)$ (tangency) $= 65^\circ$

, $\because \overrightarrow{AB}, \overrightarrow{AC}$ are two tangents $\therefore AB = AC$

$\therefore$ In ΔABC : $m(\angle ABC) = m(\angle ACB) = 65^\circ$

$\therefore m(\angle BAC) = 180^\circ - 2 \times 65^\circ = 50^\circ$ (The req.)

4

[a] $\because$ E is the midpoint of $\overline{AD}$ $\therefore \overline{ME} \perp \overline{AD}$

$\therefore m(\angle MEC) = 90^\circ$

, $\because \overline{BC}$ is a tangent-segment $\therefore \overline{BC} \perp \overline{AB}$

$\therefore m(\angle MBC) = 90^\circ$

$\therefore m(\angle MEC) + m(\angle MBC) = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ EMBC is a cyclic quadrilateral. (Q.E.D.)

[b] $\because \overline{MC} \parallel \overline{AB}$, $\overleftrightarrow{AM}$ is a transversal

$\therefore m(\angle AMC) = m(\angle MAB) = 60^\circ$ (alternate angles)

, $\because m(\angle B) = \frac{1}{2} m(\angle AMC)$

(inscribed and central angles subtended by $\widehat{AC}$

$\therefore m(\angle B) = \frac{1}{2} \times 60^\circ = 30^\circ$ (The req.)

5

[a] $\because \overline{AB} \parallel \overline{XY}$ $\therefore m(\widehat{AX}) = m(\widehat{BY})$ (1)

, $\because m(\widehat{XC}) = m(\widehat{YC})$ (2)

adding (1) , (2) : $\therefore m(\widehat{AC}) = m(\widehat{BC})$

$\therefore AC = BC$ (Q.E.D.)

[b] $\because \overrightarrow{XA}, \overrightarrow{XB}$ are two tangents to the circle

$\therefore XA = XB$

$\therefore$ In ΔABX :

$m(\angle XAB) = m(\angle XBA) = \frac{180^\circ - 70^\circ}{2} = 55^\circ$ (1)

, $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle DAB) = 180^\circ - 125^\circ = 55^\circ$ (2)

From (1) and (2) :

$\therefore m(\angle DAB) = m(\angle XAB)$

$\therefore \overrightarrow{AB}$ bisects $\angle DAX$ (Q.E.D.)

17 El-Menia

1

1 c 2 a 3 d 4 c 5 c 6 b

2

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

, $\because \overline{MY} \perp \overline{AC}$, $AB = AC$

$\therefore MX = MY$ (Q.E.D.)

[b] $\because MB = MC = r$

$\therefore$ In ΔMBC :

$m(\angle MBC) = m(\angle MCB) = 25^\circ$

$\therefore m(\angle BMC) = 180^\circ - 2 \times 25^\circ = 130^\circ$

, $\because m(\angle BAC) = \frac{1}{2} m(\angle BMC)$

(inscribed and central angles subtended by $\widehat{BC}$)

$\therefore m(\angle BAC) = \frac{1}{2} \times 130^\circ = 65^\circ$ (The req.)

3

[a] In ΔABC :

$\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = 50^\circ$

$\therefore m(\angle A) = 180^\circ - 2 \times 50^\circ = 80^\circ$

, $\because m(\angle A) + m(\angle D) = 80^\circ + 100^\circ = 180^\circ$

$\therefore$ ABDC is a cyclic quadrilateral. (Q.E.D.)

[b] $\because m(\angle BDC)$ (inscribed)

$= m(\angle BAC)$ (tangency) $= 70^\circ$,

$\because \overrightarrow{AB}, \overrightarrow{AC}$ are two tangents to the circle

$\therefore AB = AC$

$\therefore$ In ΔABC : $m(\angle ABC) = m(\angle ACB) = 70^\circ$

$\therefore m(\angle A) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$ (The req.)

4

[a] ∵ $\overleftrightarrow{CD} \parallel \overline{AB}$ ∴ $m(\widehat{AC}) = m(\widehat{BC})$

∴ AC = BC (Q.E.D.)

[b] ∵ ABCD is a cyclic quadrilateral

∴ m (∠ D) = m (∠ ABE) = 110°

In Δ ADC :

∴ m (∠ ACD) = 180° – (110° + 35°) = 35°

∴ m (∠ ACD) = m (∠ CAD) = 35°

∴ $m(\widehat{DA}) = m(\widehat{DC})$ (Q.E.D.)

5

[a] In Δ ADE :

∵ AE = DE ∴ m (∠ DAC) = m (∠ ADB)

∴ $m(\widehat{DC}) = m(\widehat{AB})$

∴ m (∠ EBC) = m (∠ ECB)

In Δ EBC : ∴ EC = EB (Q.E.D.)

[b] ∵ m (∠ CMB) = 2 m (∠ CAB) = 2 × 50° = 100°

(central and inscribed angles subtended by $\widehat{CB}$)

∴ m (reflex ∠ CMB) = 360° – 100° = 260°

(The req.)

18 Assiut

1

1 d	2 c	3 b	4 a	5 d	6 c

2

[a] ∵ X is the midpoint of $\overline{AB}$

∴ $\overline{MX} \perp \overline{AB}$ ∴ m (∠ MXA) = 90°

, ∵ Y is the midpoint of $\overline{AC}$

∴ $\overline{MY} \perp \overline{AC}$ ∴ m (∠ MYA) = 90°

From the quadrilateral AXMY :

∴ m (∠ DME) = 360° – (90° + 90° + 50°) = 130°

(First req.)

, ∵ AB = AC ∴ MX = MY

, ∵ MD = ME = r ∴ XD = YE (Second req.)

[b] ∵ AB = DC ∴ $m(\widehat{AB}) = m(\widehat{DC})$

adding $m(\widehat{BC})$ to both sides

∴ $m(\widehat{AC}) = m(\widehat{BD})$ ∴ AC = BD (Q.E.D.)

3

[a] ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents ∴ AB = AC

In Δ ABC :

∴ m (∠ ABC) = m (∠ ACB) = $\frac{180° - 50°}{2}$ = 65°

∴ m (∠ BDC) (inscribed)

= m (∠ ABC) (tangency) = 65° (The req.)

[b] ∵ $\overline{BC}$ is a diameter ∴ m (∠ A) = 90°

, ∵ $\overline{ED} \perp \overline{BC}$ ∴ m (∠ EDB) = 90°

, ∵ m (∠ A) + m (∠ EDB) = 90° + 90° = 180°

∴ ABDE is a cyclic quadrilateral. (Q.E.D. 1)

∴ m (∠ CED) = m (∠ B)

, ∵ m (∠ B) = $\frac{1}{2}$ $m(\widehat{AC})$

∴ m (∠ CED) = $\frac{1}{2}$ $m(\widehat{AC})$ (Q.E.D. 2)

4

[a] ∵ D is the midpoint of $\overline{AB}$ ∴ $\overline{MD} \perp \overline{AB}$

, ∵ $\overline{ME} \perp \overline{AC}$, MD = ME ∴ AB = AC

∴ In Δ ABC :

m (∠ ABC) = m (∠ ACB) = 65°

∴ m (∠ BAC) = 180° – 2 × 65° = 50° (The req.)

[b] ∵ ABCD is a cyclic quadrilateral.

∴ m (∠ BCD) + m (∠ BAD) = 180°

∴ m (∠ BCD) = 180° – 120° = 60°

, ∵ $\overrightarrow{BO} \parallel \overline{DC}$, $\overleftrightarrow{BC}$ is a transversal.

∴ m (∠ CBO) = m (∠ BCD) = 60°

(alternate angles)

∴ m (∠ CBE) = 60° + 55° = 115°

, ∵ ∠ CBE is an exterior angle of a cyclic quadrilateral

∴ m (∠ ADC) = m (∠ CBE) = 115° (The req.)

5

[a] m (∠ ADB) = $\frac{1}{2}$ $m(\widehat{AB})$ = $\frac{1}{2}$ × 50° = 25°

(First req.)

, $m(\widehat{ADB})$ = 360° – 50° = 310° (Second req.)

[b] ∵ BCDE is a cyclic quadrilateral

∴ m (∠ CBE) + m (∠ D) = 180°

∴ m (∠ CBE) = 180° – 125° = 55°

, ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

∴ AB = AC

∴ In Δ ABC :

m (∠ ACB) = m (∠ ABC) = $\frac{180° - 70°}{2}$ = 55°

, ∵ m (∠ BEC) = (inscribed)

= m (∠ ACB) (tangency) = 55°

∴ In Δ CBE : m (∠ CBE) = m (∠ BEC) = 55°

∴ CB = CE (Q.E.D. 1)

, ∵ m (∠ CBE) = (∠ ABC) = 55°

∴ $\overrightarrow{BC}$ bisects ∠ ABE (Q.E.D. 2)

19 Souhag

1

1 b 2 a 3 c 4 b 5 c 6 d

2

[a] $\because$ H is the midpoint of $\overline{BC}$

$\therefore \overline{MH} \perp \overline{BC}$ $\therefore$ m ($\angle$ MHA) = 90°

, $\because \overrightarrow{AD}$ is a tangent $\therefore \overline{MD} \perp \overline{AD}$

$\therefore$ m ($\angle$ MDA) = 90°

$\therefore$ From the quadrilateral ADMH :

$\therefore$ m ($\angle$ DMH) = 360° – (90° + 90° + 56°) = 124° (The req.)

[b] In Δ ABD : $\because$ AB = AD

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ADB) = 30°

$\therefore$ m ($\angle$ A) = 180° – 2 × 30° = 120°

, $\because$ m ($\angle$ A) + m ($\angle$ C) = 120° + 60° = 180°

$\therefore$ ABCD is a cyclic quadrilateral (Q.E.D.)

3

[a] $\because \overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle

$\therefore$ AD = AF = 5 cm.

, $\because \overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle

$\therefore$ BD = BE = 4 cm.

, $\because \overline{CE}$, $\overline{CF}$ are two tangent-segments to the circle

$\therefore$ CE = CF = 3 cm.

$\therefore$ The perimeter of Δ ABC = 5 + 5 + 4 + 4 + 3 + 3 = 24 cm. (The req.)

[b] $\because \angle$ CBE is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m ($\angle$ ADC) = m ($\angle$ CBE) = 85°

, $\because$ m ($\angle$ ADB) = $\frac{1}{2}$ m ($\widehat{AB}$)

$= \frac{1}{2} \times 110° = 55°$

$\therefore$ m ($\angle$ BDC) = 85° – 55° = 30° (The req.)

4

[a] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangent to the circle

$\therefore$ AB = AC

In Δ ABC :

m ($\angle$ ABC) = m ($\angle$ ACB) = $\frac{180° - 70°}{2}$ = 55°

$\therefore$ m ($\angle$ BEC) (inscribed) = m ($\angle$ ABC) (tangency) = 55°

, $\because$ BCDE is a cyclic quadrilateral.

$\therefore$ m ($\angle$ CBE) + m ($\angle$ CDE) = 180°

$\therefore$ m ($\angle$ CBE) = 180° – 125° = 55°

In Δ BCE : $\therefore$ m ($\angle$ BEC) = m ($\angle$ CBE)

$\therefore$ CB = CE (Q.E.D.)

[b] $\because$ AB = CD (properties of the rectangle)

, $\because$ CE = CD $\therefore$ AB = CE

$\therefore$ m ($\widehat{AB}$) = m ($\widehat{CE}$) and adding m ($\widehat{BE}$) to both sides.

$\therefore$ m ($\widehat{AE}$) = m ($\widehat{BC}$)

$\therefore$ AE = BC (Q.E.D.)

5

[a] In Δ ABC :

$\because$ m ($\angle$ B) = m ($\angle$ C) $\therefore$ AB = AC

, $\because$ X is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$

, $\because \overline{MY} \perp \overline{AC}$, AB = AC

$\therefore$ MX = MY (Q.E.D.)

[b] $\because$ ABDC is a cyclic quadrilateral

$\therefore$ m ($\angle$ B) + m ($\angle$ C) = 180°

$\therefore$ m ($\angle$ B) = 180° – 115° = 65°

, $\because \overline{AB}$ is a diameter $\therefore$ m ($\angle$ ADB) = 90°

In Δ ABD :

$\therefore$ m ($\angle$ DAB) = 180° – (90° + 65°) = 25° (The req.)

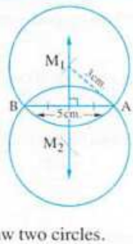
20 Qena

1

1 a 2 a 3 b 4 a 5 c 6 c

2

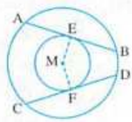
[a]



$\therefore$ We can draw two circles.

[b] Construction :

Draw $\overline{ME}$, $\overline{MF}$



Proof :

$\because \overline{AB}$, $\overline{CD}$ are two tangent-segments to the smaller circle

$\therefore \overline{ME} \perp \overline{AB}$, $\overline{MF} \perp \overline{CD}$

, $\because$ ME = MF = r (radii lengths of the smaller circle)

$\therefore$ AB = CD (Q.E.D.)

3

[a] $\because \overrightarrow{CE}$ bisects $\angle$ DCF

$\therefore m(\angle DCF) = 2 \times 55^\circ = 110^\circ$

, $\because \overline{AD} \parallel \overrightarrow{BC}$, $\overleftrightarrow{CD}$ is a transversal

$\therefore m(\angle D) = m(\angle DCF) = 110^\circ$ (alternate angles)

$\therefore m(\angle B) + m(\angle D) = 70^\circ + 110^\circ = 180^\circ$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b] $\because m(\widehat{AB}) = m(\widehat{BC}) = m(\widehat{AC}) = \frac{360^\circ}{3} = 120^\circ$

$\therefore m(\angle AMB) = m(\widehat{AB}) = 120^\circ$

, $\because$ MA = MB = r

$\therefore m(\angle ABM) = m(\angle BAM) = \frac{180^\circ - 120^\circ}{2}$

$= 30^\circ$ (First req.)

$\because m(\widehat{AB}) = m(\widehat{BC}) = m(\widehat{AC})$

$\therefore$ AB = BC = AC

$\therefore \Delta$ ABC is an equilateral triangle. (Second req.)

4

[a] $\because m(\angle ABD) = m(\angle ACD) = 80^\circ$

(two inscribed angles subtended by $\widehat{AD}$)

, $\angle$ AED is an exterior angle of Δ ECD

$\therefore m(\angle D) = 110^\circ - 80^\circ = 30^\circ$ (First req.)

$m(\widehat{AD}) = 2\, m(\angle B) = 2 \times 80^\circ = 160^\circ$ (Second req.)

[b] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore$ AB = AC

$\therefore m(\angle ABC) = m(\angle ACB) = 65^\circ$

$\therefore m(\angle A) = 180^\circ - 2 \times 65^\circ = 50^\circ$ (First req.)

, $m(\angle D)$ (inscribed) = $m(\angle ACB)$ (tangency)

$= 65^\circ$ (Second req.)

5

[a] $\because \overline{AB}$ is a tangent-segment

$\therefore \overline{MB} \perp \overline{AB}$ $\therefore m(\angle MBA) = 90^\circ$

, $\because$ E is the midpoint of $\overline{CD}$

$\therefore \overline{ME} \perp \overline{CD}$ $\therefore m(\angle MEA) = 90^\circ$

$\therefore m(\angle MBA) + m(\angle MEA) = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ ABME is a cyclic quadrilateral (First req.)

$\therefore m(\angle BMF) = m(\angle A) = 30^\circ$

$\therefore m(\widehat{BF}) = m(\angle BMF) = 30^\circ$ (Second req.)

[b] $\because m(\angle XZY)$ (inscribed) = $m(\angle LXY)$ (tangency)

, $\because \overline{EF} \parallel \overline{YZ}$, $\overleftrightarrow{XZ}$ is a transversal

$\therefore m(\angle XFE) = m(\angle XZY)$ (corresponding angles)

$\therefore m(\angle XFE) = m(\angle LXE)$

$\therefore \overleftrightarrow{LX}$ is a tangent to the circle passing through the points X , E and F (Q.E.D.)

21 Luxor

1

1 a 2 d 3 a 4 b 5 d 6 c

2

[a] $\because m(\angle A) = \frac{1}{2}\left[m(\widehat{EC}) - m(\widehat{BD})\right]$

$\therefore 30^\circ = \frac{1}{2}\left[120^\circ - m(\widehat{BD})\right]$

$\therefore 60^\circ = 120^\circ - m(\widehat{BD})$

$\therefore m(\widehat{BD}) = 120^\circ - 60^\circ = 60^\circ$ (First req.)

, $\because m(\widehat{BC}) = m(\widehat{DE})$ $\therefore$ BC = DE

By adding $m(\widehat{BD})$ to both sides.

$\therefore m(\widehat{CD}) = m(\widehat{EB})$ $\therefore m(\angle C) = m(\angle E)$

In Δ ACE : $\therefore$ AC = AE

, $\because$ BC = DE

$\therefore$ AB = AD (Second req.)

[b] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle AXM) = 90^\circ$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\therefore m(\angle AYM) = 90^\circ$

From the quadrilateral AXMY :

$\therefore m(\angle DME) = 360^\circ - (90^\circ + 90^\circ + 60^\circ) = 120^\circ$

(First req.)

, $\because$ AB = AC $\therefore$ MX = MY

, $\because$ MD = ME = r

$\therefore$ XD = YE (Second req.)

3

[a] $\because$ AB = AD

$\therefore m(\angle ABD) = m(\angle ADB) = 35^\circ$

$\therefore m(\angle A) = 180^\circ - 2 \times 35^\circ = 110^\circ$

$\therefore m(\angle A) + m(\angle C) = 110^\circ + 70^\circ = 180^\circ$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b] $\because$ m ($\angle$ BDC) = m ($\angle$ BAC)

(two inscribed angles subtended by $\widehat{BC}$)

$\therefore$ m ($\angle$ BDC) = 30° (First req.)

, $\because$ m ($\widehat{BC}$) = 2 m ($\angle$ BAC) = 60°

, $\overline{AB}$ is a diameter in the circle M

$\therefore$ m ($\widehat{AB}$) = 180°

$\therefore$ m ($\widehat{AC}$) = 180° – 60° = 120°

, $\because$ D is the midpoint of $\widehat{AC}$

$\therefore$ m ($\widehat{AD}$) = $\frac{120°}{2}$ = 60° (Second req.)

4

[a] $\because$ $\overline{AD}$, $\overline{AO}$ are two tangent-segments to the circle

$\therefore$ AD = AO = 5 cm.

, $\because$ $\overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle

$\therefore$ BD = BE = 4 cm.

, $\because$ $\overline{CE}$, $\overline{CO}$ are two tangent-segments to the circle

$\therefore$ CE = CO = 3 cm.

$\therefore$ The perimeter of Δ ABC = 5 + 5 + 4 + 4 + 3 + 3

= 24 cm. (The req.)

[b] $\because$ $\overrightarrow{AO}$ // $\overline{DE}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m ($\angle$ AED) = m ($\angle$ EAO) (alternate angles)

, $\because$ m ($\angle$ C) (inscribed) = m ($\angle$ BAO)(tangency)

$\therefore$ m ($\angle$ C) = m ($\angle$ AED)

$\therefore$ DEBC is a cyclic quadrilateral. (Q.E.D.)

5

[a] $\because$ $\angle$ ABE is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m ($\angle$ D) = m ($\angle$ ABE) = 100°

In Δ ACD :

$\because$ m ($\angle$ ACD) = 180° – (100° + 40°) = 40°

$\therefore$ m ($\angle$ ACD) = m ($\angle$ CAD)

$\therefore$ CD = AD

$\therefore$ m ($\widehat{CD}$) = m ($\widehat{AD}$) (Q.E.D.)

[b] $\because$ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents

$\therefore$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = $\frac{180° - 50°}{2}$ = 65°

$\therefore$ m ($\angle$ BEC) (inscribed)

= m ($\angle$ ABC) (tangency) = 65° (The req.)

22 Aswan

1

1 c 2 b 3 a 4 c 5 b 6 d

2

[a] $\because$ $\overrightarrow{AD}$ is a tangent

$\therefore$ $\overline{MD} \perp \overline{AD}$ $\therefore$ m ($\angle$ MDA) = 90°

, $\because$ E is the midpoint of $\overline{BC}$

$\therefore$ $\overline{ME} \perp \overline{BC}$ $\therefore$ m ($\angle$ MEA) = 90°

From the quadrilateral ADME :

$\therefore$ m ($\angle$ DME) = 360° – (50° + 90° + 90°)

= 130° (The req.)

[b] In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

, $\because$ X is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MX} \perp \overline{AB}$, $\because$ $\overline{MY} \perp \overline{AC}$

$\therefore$ MX = MY (Q.E.D.)

3

[a] $\because$ $\overline{AB}$ is a diameter

$\therefore$ m ($\angle$ ACB) = 90°

$\therefore$ m ($\angle$ DCA) = 90° – 60° = 30°

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ACD) = 30°

(Two inscribed angles subtended by $\widehat{AD}$) (The req.)

[b] In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = 50°

$\therefore$ m ($\angle$ BAC) = 180° – 2 × 50° = 80°

$\therefore$ m ($\angle$ BAC) = m ($\angle$ BDC)

and they are drawn on $\overline{BC}$ and on one side of it

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

4

[a] m ($\angle$ BMC) = 2 m ($\angle$ BAC) = 2 × 30° = 60° (1)

(central and inscribed angles subtended by $\widehat{BC}$)

(First req.)

, $\because$ MB = MC = r (2)

From (1) and (2) :

$\therefore$ Δ MBC is an equilateral triangle (Second req.)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ A) = 180° – 70° = 110°

In Δ ABD :

∴ m (∠ ABD) = 180° – (110° + 30°) = 40° (The req.)

5

[a] ∵ m (∠ BDC) (inscribed)

= m (∠ ABC) (tangency) = 70°

, ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents

∴ AB = AC

∴ m (∠ ABC) = m (∠ ACB) = 70°

In Δ ABC :

∴ m (∠ A) = 180° – (70° + 70°) = 40° (The req.)

[b] ∵ $\overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle

∴ AD = AF = 5 cm.

, ∵ $\overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle

∴ BD = BF = 4 cm.

, ∵ $\overline{CE}$, $\overline{CF}$ are two tangent-segments to the circle

∴ CE = CF = 3 cm.

∴ The perimeter of Δ ABC = 5 + 5 + 4 + 4 + 3 + 3

= 24 cm. (The req.)

23 New Valley

1

1 c 2 d 3 b 4 b 5 d 6 b

2

[a] ∵ MD = ME

, $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

∴ AB = AC ∴ m (∠ B) = m (∠ C) = 65°

∴ m (∠ A) = 180° – 2 × 65° = 50° (The req.)

[b] ∵ BC = CD = DB ∴ Δ BCD is equilateral

∴ m (∠ C) = 60°

∴ m (∠ C) + m (∠ A) = 60° + 120° = 180°

∴ ABCD is a cyclic quadrilateral (Q.E.D.)

3

[a] m (∠ A) = $\frac{1}{2}$ m (∠ BMC) = $\frac{1}{2}$ × 80° = 40° (First req.)

(inscribed and central angles subtended by $\overset{\frown}{BC}$)

, ∵ MB = MC = r

∴ m (∠ MBC) = m (∠ MCB) = $\frac{180° - 80°}{2}$ = 50° (Second req.)

[b] ∵ $\overleftrightarrow{XY}$ is a tangent

∴ $\overline{MX} \perp \overleftrightarrow{XY}$ ∴ m (∠ MXY) = 90°

∴ In Δ MXY : $(XY)^2 = (MY)^2 - (MX)^2$

$= (13)^2 - 5^2 = 144$

∴ XY = $\sqrt{144}$ = 12 cm. (The req.)

4

[a] ∵ m (∠ BDC) (inscribed)

= m (∠ ABC) (tangency) = 70°

, ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents

∴ AB = AC

∴ m (∠ ABC) = m (∠ ACB) = 70°

In Δ ABC :

∴ m (∠ A) = 180° – (70° + 70°) = 40° (The req.)

[b] ∵ $\overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle

∴ AD = AF = 5 cm.

, ∵ $\overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle

∴ BD = BE = 2 cm.

, ∵ $\overline{CE}$, $\overline{CF}$ are two tangent-segments to the circle

∴ CE = CF = 3 cm.

∴ The perimeter of Δ ABC = 5 + 5 + 2 + 2 + 3 + 3

= 20 cm. (The req.)

5

[a] m (∠ H) = m (∠ C) = 20°

(Two inscribed angles subtended by $\overset{\frown}{BD}$) (First req.)

m ($\overset{\frown}{BD}$) = 2 m (∠ C) = 2 × 20° = 40° (Second req.)

∴ m (∠ A) = $\frac{1}{2}$ [m ($\overset{\frown}{CH}$) – m ($\overset{\frown}{BD}$)]

= $\frac{1}{2}$ (140° – 40°) = 50° (Third req.)

[b] ∵ ABCD is a cyclic quadrilateral

∴ m (∠ D) = m (∠ ABH) = 100°

∴ m (∠ ACD) = 180° – (100° + 40°) = 40°

∴ m (∠ CAD) = m (∠ ACD)

∴ m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{AD}$) (Q.E.D.)

24 South Sinai

1

1 b 2 c 3 a 4 b 5 c 6 a

2

[a] $\because$ m ($\angle$ BAC) = $\frac{1}{2}$ m ($\angle$ BMC)

(inscribed and central angles subtended by $\widehat{BC}$)

$\therefore$ m ($\angle$ BAC) = $\frac{1}{2} \times 120° = 60°$ (The req.)

[b] $\because$ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents

$\therefore$ AB = AC

$\therefore$ In Δ ABC :

m ($\angle$ ABC) = m ($\angle$ ACB) = $\frac{180° - 50°}{2}$

= 65° (The req.)

3

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MX} \perp \overline{AB}$ $\therefore$ m ($\angle$ MXA) = 90°

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore$ $\overline{MY} \perp \overline{AC}$ $\therefore$ m ($\angle$ MYA) = 90°

From the quadrilateral AXMY :

$\therefore$ m ($\angle$ BAC) = 360° − (90° + 90° + 120°) = 60°

(First req.)

, $\because$ AB = AC $\therefore$ MX = MY

, $\because$ MD = MH = r $\therefore$ DX = HY (Second req.)

[b] In Δ ABC :

$\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = 30°

$\therefore$ m ($\angle$ A) = 180° − 2 × 30° = 120°

$\therefore$ m ($\angle$ A) + m ($\angle$ D) = 120° + 60° = 180°

$\therefore$ ABDC is a cyclic quadrilateral. (Q.E.D.)

4

[a] $\because$ m ($\angle$ A) = $\frac{1}{2}\left[m(\widehat{CH}) - m(\widehat{BD})\right]$

$\therefore$ $30° = \frac{1}{2}\left[80° - m(\widehat{BD})\right]$

$\therefore$ $60° = 80° - m(\widehat{BD})$

$\therefore$ $m(\widehat{BD}) = 80° - 60° = 20°$ (The req.)

[b] In Δ ACD : $\because$ AC = CD

$\therefore$ m ($\angle$ CAD) = m ($\angle$ ADC) = 50°

$\therefore$ m ($\angle$ CBD) = m ($\angle$ CAD) = 50°

(two inscribed angles subtended by $\widehat{CD}$)

(The req.)

5

[a] $\because$ $\overline{AB}$ is a diameter $\therefore$ m ($\angle$ BCA) = 90°

, $\because$ $\overrightarrow{CD}$ // $\overline{AB}$ $\therefore$ $m(\widehat{AC}) = m(\widehat{BC})$

$\therefore$ AC = BC

$\therefore$ In Δ ABC :

m ($\angle$ ABC) = m ($\angle$ BAC) = $\frac{180° - 90°}{2} = 45°$

(The req.)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ A) = m ($\angle$ BCH) = 60°

, $\because$ AB = AD

$\therefore$ Δ ABD is equilateral. (Q.E.D.)

25 North Sinai

1

1 b 2 a 3 d 4 c 5 b 6 d

2

[a] In Δ ABC :

$\because$ m ($\angle$ B) = m ($\angle$ C) $\therefore$ AB = AC

, $\because$ D is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MD} \perp \overline{AB}$

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore$ $\overline{ME} \perp \overline{AC}$ $\therefore$ MD = ME (Q.E.D.)

[b] $\because$ MF = MX = r = 6 cm.

$\therefore$ MY = 6 + 4 = 10 cm.

In Δ MAXY

$\because$ $(MY)^2 = (10)^2 = 100$

, $(MX)^2 + (XY)^2 = 6^2 + 8^2 = 100$

$\therefore$ $(MY)^2 = (MX)^2 + (XY)^2$ $\therefore$ $\overline{MX} \perp \overleftrightarrow{XY}$

$\therefore$ $\overleftrightarrow{XY}$ is a tangent to the circle at X (Q.E.D.)

3

[a] $\because$ m ($\angle$ DEB) = $\frac{1}{2}\left[m(\widehat{AC}) + m(\widehat{BD})\right]$

$\therefore$ $110° = \frac{1}{2}\left[100° + m(\widehat{BD})\right]$

$\therefore$ $220° = 100° + m(\widehat{BD})$

$\therefore$ $m(\widehat{BD}) = 220° - 100° = 120°$

$\therefore$ m ($\angle$ DCB) = $\frac{1}{2}$ $m(\widehat{BD}) = \frac{1}{2} \times 120° = 60°$

(The req.)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ A) = 180° − 140° = 40° (First req.)

, $\because$ $\overline{AB}$ is a diameter

$\therefore$ m ($\angle$ ADB) = 90°

In Δ BCD : $\because$ CD = CB

$\therefore$ m ($\angle$ CDB) = m ($\angle$ CBD) = $\frac{180° - 140°}{2}$ = 20°

$\therefore$ m ($\angle$ ADC) = 90° + 20° = 110° (Second req.)

4

[a] $\because$ $\overrightarrow{YB}$ is a tangent , $\overline{AB}$ is a diameter

$\therefore$ $\overline{AB} \perp \overrightarrow{YB}$ $\therefore$ m ($\angle$ ABY) = 90°

, $\because$ X is the midpoint of $\overline{AC}$

$\therefore$ $\overline{MX} \perp \overline{AC}$ $\therefore$ m ($\angle$ MXA) = 90°

$\therefore$ m ($\angle$ ABY) = m ($\angle$ AXY) = 90°

and they are drawn on $\overline{AY}$ and on one side of it

$\therefore$ AXBY is a cyclic quadrilateral. (Q.E.D.)

[b] The measure of the arc = 2 × 45° = 90°

The length of the arc = $\frac{90°}{360°} \times 2 \times \frac{22}{7} \times 7$ = 11 cm. (The req.)

5

[a] $\because$ BCDE is a cyclic quadrilateral

$\therefore$ m ($\angle$ CBE) = 180° − 120° = 60°

, $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore$ AB = AC

$\therefore$ In Δ ABC :

m ($\angle$ ABC) = m ($\angle$ ACB) = $\frac{180° - 60°}{2}$ = 60°

$\therefore$ m ($\angle$ CEB) (inscribed)

= m ($\angle$ ABC) (tangency) = 60°

In Δ EBC : $\because$ m ($\angle$ CBE) = m ($\angle$ CEB) = 60°

$\therefore$ m ($\angle$ BCE) = 180° − 2 × 60° = 60°

$\therefore$ Δ BCE is an equilateral triangle. (Q.E.D.)

[b] In Δ ABC : $\because$ m ($\angle$ BAC) = 90°

, AC = $\frac{1}{2}$ BC $\therefore$ m ($\angle$ B) = 30°

$\therefore$ m ($\angle$ C) = 180° − (90° + 30°) = 60°

$\therefore$ m ($\angle$ C) = m ($\angle$ BAD) = 60°

$\therefore$ $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC (Q.E.D.)

26 Red Sea

1

1 a 2 d 3 a 4 a 5 b 6 c

2

[a] $\because$ D is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MD} \perp \overline{AB}$ $\therefore$ m ($\angle$ ADM) = 90°

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore$ $\overline{ME} \perp \overline{AC}$ $\therefore$ m ($\angle$ AEM) = 90°

$\therefore$ From the quadrilateral ADME :

$\therefore$ m ($\angle$ DME) = 360° − (90° + 90° + 70°) = 110° (The req.)

[b] m ($\angle$ AEC) = $\frac{1}{2}$ $\left[m(\widehat{AC}) + m(\widehat{BD})\right]$

= $\frac{1}{2}$ (50° + 100°) = 75° (The req.)

3

[a] In Δ ABC : $\because$ m ($\angle$ B) = 180° − (50° + 35°) = 95°

$\therefore$ m ($\angle$ B) + m ($\angle$ D) = 95° + 85° = 180°

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b] In Δ MBC : $\because$ MB = MC = r

$\therefore$ m ($\angle$ MBC) = m ($\angle$ MCB) = 40°

$\therefore$ m ($\angle$ BMC) = 180° − 2 × 40° = 100°

, $\because$ m ($\angle$ A) = $\frac{1}{2}$ m ($\angle$ BMC)

(inscribed and central angles subtended by $\widehat{BC}$)

$\therefore$ m ($\angle$ A) = $\frac{1}{2}$ × 100° = 50° (The req.)

4

[a] $\because$ $\overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle

$\therefore$ AD = AF = 3 cm.

, $\because$ $\overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle

$\therefore$ BD = BE = 5 cm.

, $\because$ $\overline{CE}$, $\overline{CF}$ are two tangent-segments to the circle

$\therefore$ CE = CF = 4 cm.

$\therefore$ The perimeter of Δ ACB = 5 + 5 + 4 + 4 + 3 + 3 = 24 cm. (The req.)

[b] $\because$ MD = ME

, $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

$\therefore$ AB = AC

$\therefore$ In Δ ABC :

m ($\angle$ B) = m ($\angle$ C) = 70°

$\therefore$ m ($\angle$ A) = 180° − 2 × 70° = 40° (The req.)

5

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ D) = m ($\angle$ ABE) = 100°

In Δ ACD : $\therefore$ m ($\angle$ ACD) = 180° – (100° + 40°)
= 40°

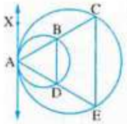
$\therefore$ m ($\angle$ CAD) = m ($\angle$ ACD) = 40°

$\therefore$ m ($\widehat{CD}$) = m ($\widehat{AD}$) (Q.E.D.)

[b] $\because$ $\overleftrightarrow{AX}$ is a common tangent for two circles

$\therefore$ m ($\angle$ BDA) (inscribed)
= m ($\angle$ BAX) (tangency)

, m ($\angle$ CEA) (inscribed)
= m ($\angle$ CAX) (tangency)



$\therefore$ m ($\angle$ BDA) = m ($\angle$ CEA)

and they are corresponding angles

$\therefore$ $\overline{BD}$ // $\overline{CE}$ (Q.E.D.)

27 Matrouh

1

1 c 2 b 3 d 4 c 5 c 6 a

2

[a] $\because$ $\overline{AB}$ is a tangent-segment

$\therefore$ $\overline{MA} \perp \overline{AB}$ $\therefore$ m ($\angle$ MAB) = 90°

In Δ MAB :

$\therefore$ m ($\angle$ AMB) = 180° – (90° + 30°) = 60°

$\therefore$ m ($\angle$ ADC) = $\frac{1}{2}$ m ($\angle$ AMC)

(inscribed and central angles subtended by $\widehat{AC}$)

$\therefore$ m ($\angle$ ADB) = $\frac{1}{2}$ × 60° = 30° (The req.)

[b] In Δ ABD : $\because$ AB = AD

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ADB) = 30°

$\therefore$ m ($\angle$ A) = 180° – 2 × 30° = 120°

$\therefore$ m ($\angle$ A) + m ($\angle$ C) = 120° + 60° = 180°

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\because$ $\angle$ CBE is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m ($\angle$ ADC) = m ($\angle$ CBE) = 85°

, $\because$ m ($\angle$ ADB) = $\frac{1}{2}$ m ($\widehat{AB}$) = $\frac{1}{2}$ × 110° = 55°

$\therefore$ m ($\angle$ BDC) = 85° – 55° = 30° (The req.)

[b] $\because$ X is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MX} \perp \overline{AB}$ $\therefore$ m ($\angle$ AXM) = 90°

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore$ $\overline{MY} \perp \overline{AC}$ $\therefore$ m ($\angle$ AYM) = 90°

From the quadrilateral AXMY :

$\therefore$ m ($\angle$ DME) = 360° – (90° + 90° + 70°) = 110° (First req.)

, $\because$ AB = AC $\therefore$ MX = MY

, $\because$ MD = ME = r $\therefore$ XD = YE (Second req.)

4

[a] $\because$ $\overrightarrow{BD}$ is a tangent

$\therefore$ $\overline{MB} \perp \overrightarrow{BD}$ $\therefore$ m ($\angle$ MBD) = 90°

, $\because$ MA = MB = r

$\therefore$ In Δ MAB :

m ($\angle$ MBA) = m ($\angle$ MAB) = 30°

$\therefore$ m ($\angle$ ABD) = 90° – 30° = 60° (The req.)

[b] $\because$ AB = AC

$\therefore$ m ($\widehat{AB}$) = m ($\widehat{AC}$)

$\therefore$ m ($\angle$ AEB) = m ($\angle$ AEC) (Q.E.D.)

5

[a] 1 perpendicular , bisects

2 equal

[b] $\because$ $\overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

$\therefore$ XA = XB

$\therefore$ In Δ ABX

m ($\angle$ XAB) = m ($\angle$ XBA) = $\frac{180° - 70°}{2}$ = 55° (1)

, $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ BAD) + m ($\angle$ DCB) = 180°

$\therefore$ m ($\angle$ BAD) = 180° – 125° = 55° (2)

From (1) and (2) :

$\therefore$ m ($\angle$ XAB) = m ($\angle$ BAD) = 55°

$\therefore$ $\overrightarrow{AB}$ bisects $\angle$ DAX (Q.E.D.)

Answer the following questions :

1 Choose the correct answer from those given :

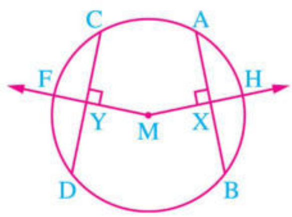
1. The slope of the straight line $3\,x + 2\,y = 1$ is ....
   (a) $\frac{2}{3}$ (b) $-\frac{3}{2}$ (c) $-\frac{2}{3}$ (d) $\frac{3}{2}$
2. M and N are two intersecting circles , their radii lengths are 3 cm. and 5 cm. , then MN ∈ ....
   (a)]8 , ∞[(b)]3 , 5[(c)]0 , 2[(d)]2 , 8[
3. The measurement of any angle of the regular hexagon is ....
   (a) 90° (b) 108° (c) 120° (d) 135°
4. ABCD is a cyclic quadrilateral , m (∠ A) = 70° , then m (∠ C) equals ....
   (a) 25° (b) 20° (c) 110° (d) 100°
5. In Δ ABC , if $(AB)^2 = (AC)^2 + (BC)^2$, then ∠ B is ....
   (a) acute. (b) obtuse. (c) right. (d) reflex.
6. The measure of the inscribed angle drawn in a semicircle equals ....
   (a) 130° (b) 90° (c) 50° (d) 180°

2 [a] In the opposite figure :

$\overline{AB}$ and $\overline{CD}$ are two chords equal in length in the circle M

, $\overrightarrow{MX} \perp \overline{AB}$, $\overrightarrow{MY} \perp \overline{CD}$

Prove that : HX = FY

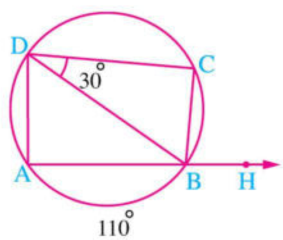


[b] In the opposite figure :

H ∈ $\overrightarrow{AB}$, m ($\widehat{AB}$) = 110°

, m (∠ CDB) = 30°

Find : m (∠ HBC)

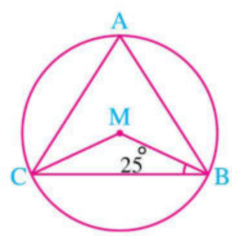


3 [a] In the opposite figure :

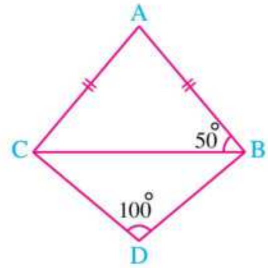
ABC is a triangle drawn in the circle M

, m (∠ MBC) = 25°

Find : m (∠ BAC)



[b] In the opposite figure :



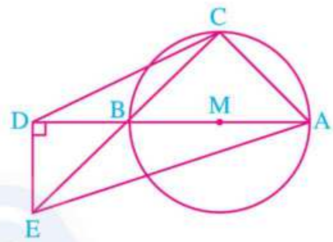
AB = AC , m (∠ D) = 100°

, m (∠ ABC) = 50°

Prove that : ABDC is a cyclic quadrilateral.

---

4 [a] In the opposite figure :



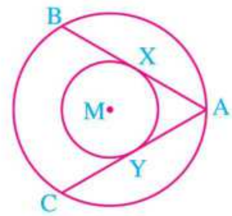
$\overline{AB}$ is a diameter in the circle M

, $D \in \overrightarrow{AB}$, $D \notin \overline{AB}$, $\overrightarrow{DE} \perp \overrightarrow{AB}$

, $C \in \widehat{AB}$, $\overrightarrow{CB} \cap \overrightarrow{DE} = \{E\}$

Prove that : ACDE is a cyclic quadrilateral

[b] In the opposite figure :



Two concentric circles of centre M

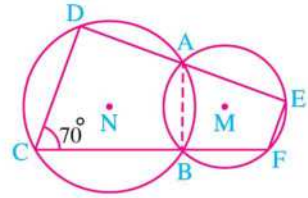
, $\overline{AB}$ and $\overline{AC}$ are two chords in the greater circle

and tangents to the smaller circle at X and Y respectively.

Prove that : AB = AC

---

5 [a] In the opposite figure :



M and N are two intersecting circles at A and B

, $\overleftrightarrow{AD}$ is drawn to intersect the circle M at E and

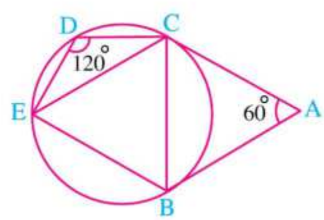
the circle N at D , $\overleftrightarrow{AB}$ is drawn to intersect the circle M at

F and the circle N at C , m (∠ BCD) = 70°

1 **Find :** m (∠ EFB)

2 **Prove that :** $\overline{CD} \parallel \overline{EF}$

[b] In the opposite figure :



$\overline{AB}$ and $\overline{AC}$ are tangent-segments to the circle at B and C

, m (∠ BAC) = 60° , m (∠ CDE) = 120°

Prove that : 1 Δ BCE is an equilateral triangle.

2 $\overline{AC} \parallel \overline{BE}$

Model Exam 2

MATHEMATICS (Geometry) — 30 Marks

Answer the following questions :

1 Choose the correct answer from those given :

1. ∠ A and ∠ B are two complementary angles , ∠ B and ∠ C are two supplementary angles , m (∠ A) = 30° , then m (∠ C) = ...°

   (a) 30 (b) 60 (c) 90 (d) 120

2. If the surface of the circle M ⋂ the surface of the circle N = {A} and the radius length of one of them equals 3 cm. and MN = 8 cm. , then the radius length of the other circle equals .... cm.

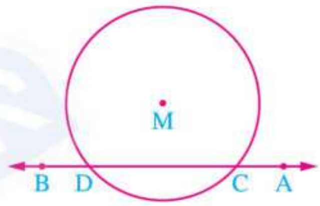
   (a) 5 (b) 6 (c) 11 (d) 16

3. **In the opposite figure :**

   $\overleftrightarrow{AB}$ ⋂ the surface of the circle M = ...

   (a) {C , D} (b) $\overline{CD}$

   (c) $\overleftrightarrow{CD}$ (d) ∅



4. A circle can be drawn passing through the vertices of a ....

   (a) rhombus. (b) parallelogram. (c) trapezium. (d) rectangle.

5. The rhombus whose two diagonal lengths are 12 cm. and 16 cm. , then its side length equals .... cm.

   (a) 6 (b) 8 (c) 10 (d) 20

6. **In the opposite figure :**

   If the side length of the square = 10 cm.

   , then the surface area of the circle = ... cm^2

   (a) 100 π (b) 25 π

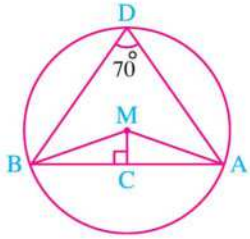
   (c) 50 π (d) 40 π

2 [a] In the opposite figure :

$\overline{AB}$ is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$, m (∠ ADB) = 70°

Find : m (∠ AMC)

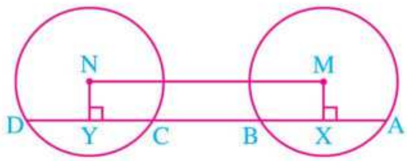


[b] In the opposite figure :

M and N are two congruent circles

, AB = CD , $\overline{MX} \perp \overline{AB}$ and $\overline{NY} \perp \overline{CD}$

Prove that : The figure MXYN is a rectangle.



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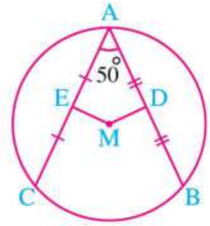
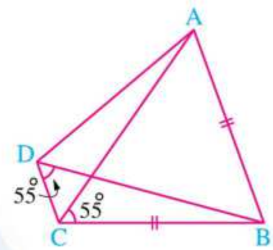
3 [a] **In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two chords in the circle M , D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$ and m (∠ BAC) = 50°

Find : m (∠ DME)

[b] **In the opposite figure :**



AB = BC

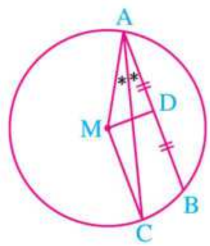
, m (∠ ACB) = 55°

and m (∠ BDC) = 55°

Prove that : The figure ABCD is a cyclic quadrilateral.

---

4 [a] **In the opposite figure :**



$\overline{AB}$ is a chord in the circle M

, $\overrightarrow{AC}$ bisects ∠ BAM and intersects the circle M at C

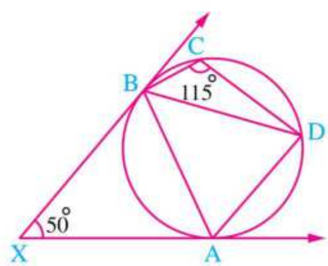
If D is the midpoint of $\overline{AB}$

, prove that : $\overline{DM} \perp \overline{CM}$

[b] $\overline{AB}$ is a diameter in the circle M , $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are two tangents to the circle M , $\overrightarrow{CM}$ intersects the circle M at X and Y respectively and intersects $\overleftrightarrow{BD}$ at E **Prove that :** CX = YE

---

5 [a] **In the opposite figure :**



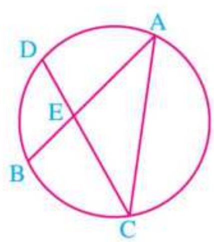
$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

, m (∠ AXB) = 50° , m (∠ DCB) = 115°

Prove that : 1 $\overrightarrow{AB}$ bisects ∠ DAX

2 BD = BA

[b] **In the opposite figure :**



$\overline{AB}$ and $\overline{CD}$ are two equal chords in length in the circle

, $\overline{AB} \cap \overline{CD} = \{E\}$

Prove that : The triangle ACE is an isosceles triangle.

Model Exam 3

MATHEMATICS (Geometry) — **30** Marks

Answer the following questions :

1 Choose the correct answer from those given :

1. The measure of the inscribed angle is .... the measure of the central angle subtended by the same arc.
   (a) half (b) twice (c) quarter (d) third

2. The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.
   (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

3. Two distant circles M and N with radii lengths 6 cm. and 8 cm. respectively , then MN .... 14 cm.
   (a) < (b) > (c) = (d) ≤

4. The angle of measure 40° is the complemented angle of the angle of measure ....°
   (a) 320 (b) 140 (c) 60 (d) 50

5. The area of the rhombus with diagonal lengths 6 cm. , 8 cm. is .... cm^2.
   (a) 2 (b) 14 (c) 24 (d) 48

6. In the cyclic quadrilateral ABCD , if $m(\angle A) = \frac{1}{2} m(\angle C)$, then $m(\angle A) =$ ....°
   (a) 20 (b) 30 (c) 60 (d) 120

2 [a] In the opposite figure :

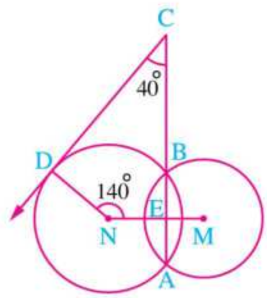
M and N are two intersecting circles at A and B

, $C \in \overrightarrow{AB}$, $\overline{AC} \cap \overline{MN} = \{E\}$

, D ∈ the circle N , $m(\angle DNM) = 140°$

and $m(\angle C) = 40°$

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle N at D



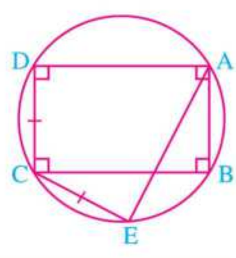
[b] In the opposite figure :

ABCD is a rectangle inscribed in a circle

, the chord $\overline{CE}$ is drawn

where CE = CD

Prove that : AE = BC



3 **[a]** State two cases of the cyclic quadrilateral.

[b] In the opposite figure :

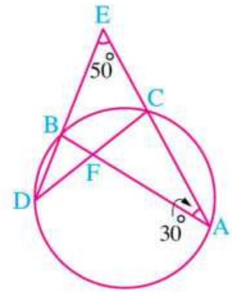
$\overline{AB} \cap \overline{CD} = \{F\}$, $\overrightarrow{AC} \cap \overrightarrow{DB} = \{E\}$

, m (∠ A) = 30°

, m (∠ E) = 50°

Find : 1 m $(\widehat{AD})$

2 m (∠ AFD)

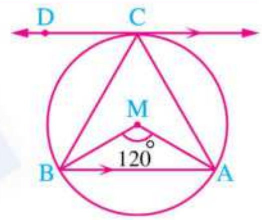


4 **[a] In the opposite figure :**

$\overleftrightarrow{CD}$ is a tangent to the circle at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$, m (∠ AMB) = 120°

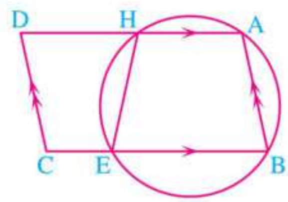
Prove that : The triangle CAB is an equilateral triangle.



[b] In the opposite figure :

ABCD is a parallelogram.

Prove that : HDCE is a cyclic quadrilateral.

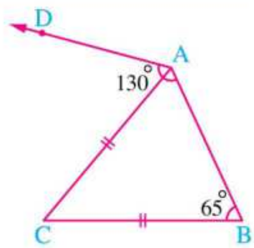


5 **[a] In the opposite figure :**

AC = BC , m (∠ ABC) = 65°

, m (∠ DAB) = 130°

Prove that : $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC



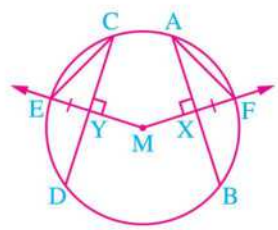
[b] In the opposite figure :

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle at F

, $\overrightarrow{MY} \perp \overline{CD}$ and intersects the circle at E , FX = EY

Prove that : 1 AB = CD   2 AF = CE

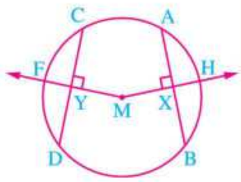


Answers of model 1

1

1 b	2 d	3 c
4 c	5 a	6 b

2

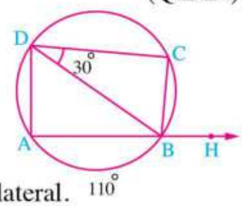


[a] ∵ AB = CD , $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

∴ MX = MY

, ∵ MH = MF = r

∴ HX = FY (Q.E.D.)



[b] ∵ m (∠ ADB) = $\frac{1}{2}$ m ($\widehat{AB}$)

= $\frac{1}{2}$ × 110°

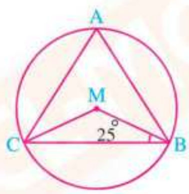
= 55°

, ∵ ABCD is a cyclic quadrilateral.

∴ m (∠ HBC) = m (∠ CDB) + m (∠ ADB)

= 30° + 55° = 85° (The req.)

3



[a] In Δ BMC :

∵ MB = MC = r

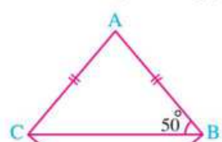
∴ m (∠ MCB) = m (∠ MBC) = 25°

∴ m (∠ BMC) = 180° – (25° + 25°) = 130°

, ∵ m (∠ BAC) = $\frac{1}{2}$ m (∠ BMC)

(inscribed and central angles subtended by $\widehat{BC}$)

∴ m (∠ BAC) = $\frac{1}{2}$ × 130° = 65° (The req.)



[b] In Δ ABC :

∵ AB = AC

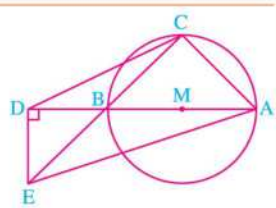
∴ m (∠ ACB) = m (∠ ABC) = 50°

∴ m (∠ A) = 180° – (50° + 50°) = 80°

, ∵ m (∠ A) + m (∠ D) = 80° + 100° = 180°

∴ ABDC is a cyclic quadrilateral. (Q.E.D.)

4



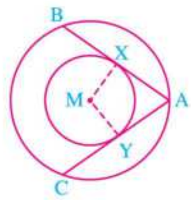
[a] ∵ $\overline{AB}$ is a diameter of the circle.

∴ m (∠ ACB) = 90°

∴ m (∠ ACE) = m (∠ ADE)

and they are drawn on $\overline{AE}$ and on one side of it

∴ ACDE is a cyclic quadrilateral. (Q.E.D.)



[b] Construction :

Draw $\overline{MX}$, $\overline{MY}$

Proof :

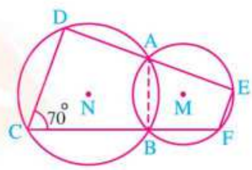
∵ $\overline{AB}$, $\overline{AC}$ are two tangents to the smaller circle at X , Y respectively

∴ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

, ∵ MX = MY = r (radius length of the smaller circle)

∴ AB = AC (Q.E.D.)

5



[a] ∵ ABCD is a cyclic quadrilateral

∴ m (∠ BAD) = 180° – 70° = 110°

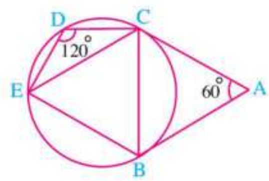
, ∵ ABFE is a cyclic quadrilateral and ∠ BAD is exterior of it.

∴ m (∠ EFB) = m (∠ BAD) = 110° (First req.)

∴ m (∠ EFB) + m (∠ BCD) = 110° + 70° = 180°

and they are interior angles in the same side of $\overleftrightarrow{FC}$

∴ $\overline{CD}$ // $\overline{EF}$ (Second req.)



[b] ∵ $\overline{AB}$, $\overline{AC}$ are tangent-segments to the circle

∴ AB = AC

∴ m (∠ ACB) = $\frac{180° - 60°}{2}$ = 60° (1)

∴ m (∠ BEC) (inscribed) = m (∠ ACB) (tangency) = 60° (2)

, ∵ EBCD is a cyclic quadrilateral

∴ m (∠ EBC) = 180° – 120° = 60° (3)

∴ From (2) , (3) in Δ EBC :

∴ m (∠ BCE) = 60°

∴ Δ BCE is equilateral. (Q.E.D. 1)

From (1) , (3) : ∴ m (∠ ACB) = m (∠ EBC) and they are alternate angles

∴ $\overline{AC}$ // $\overline{BE}$ (Q.E.D. 2)

Answers of model 2

1

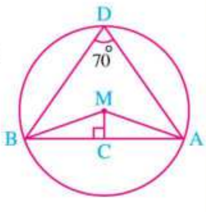
1 d	2 a	3 b
4 d	5 c	6 b

2

[a] ∵ m (∠ AMB) = 2 m (∠ ADB)

= 2 × 70° = 140°

(central and inscribed angles subtended by $\overset{\frown}{AB}$)



In Δ ABM : ∵ $\overline{MC} \perp \overline{AB}$

, MA = MB = r

∴ $\overrightarrow{MC}$ bisects ∠ AMB

∴ m (∠ AMC) = $\frac{1}{2}$ m (∠ AMB) = $\frac{1}{2}$ × 140° = 70°

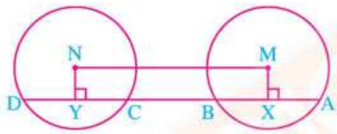
(The req.)

[b] ∵ M , N are two congruent circles

, AB = CD

, $\overline{MX} \perp \overline{AB}$

, $\overline{NY} \perp \overline{CD}$



∴ MX = NY , $\overline{MX}$ // $\overline{NY}$

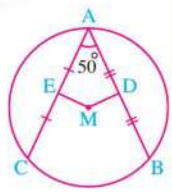
∴ MXYN is a rectangle. (Q.E.D.)

3

[a] ∵ D is the midpoint of $\overline{AB}$

∴ $\overline{MD} \perp \overline{AB}$

∴ m (∠ ADM) = 90°



, ∵ E is the midpoint of $\overline{AC}$

∴ $\overline{ME} \perp \overline{AC}$ ∴ m (∠ AEM) = 90°

From the quadrilateral ADME :

∴ m (∠ DME) = 360° – (90° + 90° + 50°) = 130°

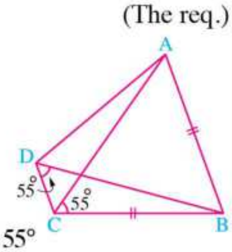
(The req.)

[b] In Δ ABC :

∵ AB = BC

∴ m (∠ BAC)

= m (∠ ACB) = 55°



, ∵ m (∠ BDC) = m (∠ BAC) = 55°

and they are drawn on $\overline{BC}$ and on one side of it

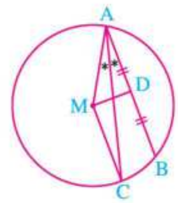
∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

4

[a] In Δ AMC :

∵ AM = MC = r

∴ m (∠ MAC) = m (∠ ACM)



, ∵ m (∠ BAC) = m (∠ MAC)

∴ m (∠ BAC) = m (∠ ACM)

and they are alternate angles.

∴ $\overline{AB}$ // $\overline{CM}$

, ∵ D is the midpoint of $\overline{AB}$

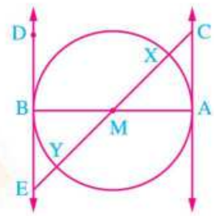
∴ $\overline{MD} \perp \overline{AB}$, ∵ $\overline{AB}$ // $\overline{CM}$

∴ $\overline{DM} \perp \overline{CM}$ (Q.E.D.)

[b] ∵ $\overleftrightarrow{AC}$ is a tangent to the circle M at A

∴ $\overline{MA} \perp \overleftrightarrow{AC}$

∴ m (∠ CAM) = 90°



, ∵ $\overleftrightarrow{BD}$ is a tangent to the circle M at B

∴ $\overline{MB} \perp \overleftrightarrow{BD}$ ∴ m (∠ EBM) = 90°

∴ In ΔΔ CAM , EBM :

$\left\{\begin{array}{l}\text{m (∠ CAM) = m (∠ EBM) = 90°} \\ \text{m (∠ AMC) = m (∠ BME) (V.O.A.)} \\ \text{MA = MB (lengths of two radii)}\end{array}\right.$

∴ The two triangles are congruent and we deduce that CM = EM

, ∵ XM = YM (lengths of two radii)

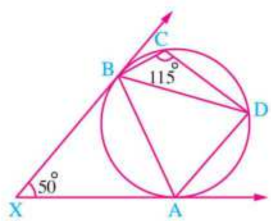
, by subtracting

∴ CX = YE (Q.E.D.)

5

[a] ∵ $\overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

∴ XA = XB



∴ In Δ ABX

m (∠ XAB) = m (∠ XBA) = $\frac{180° - 50°}{2}$ = 65°

, ∵ ABCD is a cyclic quadrilateral

∴ m (∠ BAD) + m (∠ DCB) = 180°

∴ m (∠ BAD) = 180° – 115° = 65°

∴ m (∠ XAB) = m (∠ BAD)

∴ $\overrightarrow{AB}$ bisects ∠ DAX (Q.E.D.1)

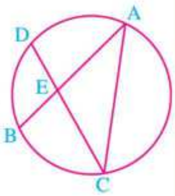
, ∵ m (∠ ADB) (inscribed)

= m (∠ XAB) (tangency) = 65°

∴ m (∠ BAD) = m (∠ ADB)

∴ In Δ ABD : BD = BA (Q.E.D.2)

[b] ∵ AB = CD



∴ m $(\widehat{AB})$ = m $(\widehat{CD})$

Subtracting m $(\widehat{BD})$ from both sides

∴ m $(\widehat{AD})$ = m $(\widehat{BC})$

∴ m (∠ ACD) = m (∠ BAC)

∴ In Δ ACE : AE = CE

∴ Δ ACE is an isosceles triangle. (Q.E.D.)

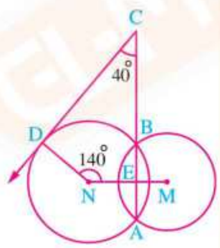
Answers of model 3

1

1 a	2 a	3 b
4 d	5 c	6 c

2

[a] ∵ $\overleftrightarrow{MN}$ is the line of centres



, $\overline{AB}$ is the common chord.

∴ $\overline{AB} \perp \overline{MN}$

∴ m (∠ BEN) = 90°

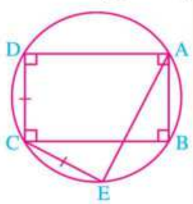
In the quadrilateral CDNE :

∴ m (∠ CDN) = 360° – (140° + 40° + 90°) = 90°

∴ $\overleftrightarrow{ND} \perp \overleftrightarrow{CD}$

∴ $\overrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

[b] ∵ AB = CD



(properties of the rectangle)

, ∵ CE = CD

∴ AB = CE

∴ m $(\widehat{AB})$ = m $(\widehat{CE})$ and adding m $(\widehat{BE})$ to both sides.

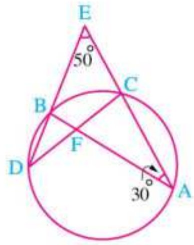
∴ m $(\widehat{AE})$ = m $(\widehat{BC})$

∴ AE = BC (Q.E.D.)

3

[a] State by yourself.

[b] ∵ m $(\widehat{BC})$ = 2 m (∠ A)



= 2 × 30° = 60°

, ∵ m (∠ E)

$= \frac{1}{2}\left[m(\widehat{AD}) - m(\widehat{BC})\right]$

∴ $50° = \frac{1}{2}\left[m(\widehat{AD}) - 60°\right]$

∴ 100° = m $(\widehat{AD})$ – 60°

∴ m $(\widehat{AD})$ = 160° (First req.)

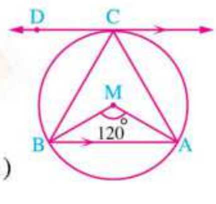
, ∵ m (∠ AFD) $= \frac{1}{2}\left[m(\widehat{AD}) + m(\widehat{BC})\right]$

∴ m (∠ AFD) $= \frac{1}{2}\left[160° + 60°\right]$ = 110°

(Second req.)

4

[a] ∵ m (∠ ACB)



$= \frac{1}{2}$ m (∠ AMB) = 60°

(inscribed and central angles subtended the same arc $\widehat{AB}$) (1)

∵ $\overleftrightarrow{CD}$ // $\overline{AB}$

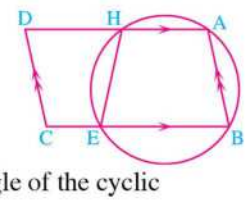
∴ m $(\widehat{AC})$ = m $(\widehat{BC})$

∴ AC = BC (2)

From (1) and (2) :

∴ Δ CAB is equilateral. (Q.E.D.)

[b] ∵ $\overline{AB}$ // $\overline{DC}$, $\overleftrightarrow{AD}$ is a transversal to them.



∴ m (∠ A) + m (∠ D) = 180° (1)

but ∠ CEH is an exterior angle of the cyclic quadrilateral ABEH

∴ m (∠ CEH) = m (∠ A) (2)

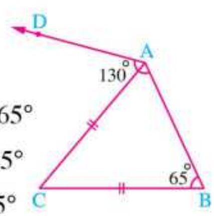
From (1) and (2) :

∴ m (∠ CEH) + m (∠ D) = 180°

∴ HDCE is a cyclic quadrilateral. (Q.E.D.)

5

[a] In Δ ABC :



∵ AC = BC

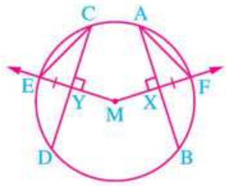
∴ m (∠ BAC) = m (∠ ABC) = 65°

∴ m (∠ CAD) = 130° – 65° = 65°

, ∵ m (∠ B) = m (∠ CAD) = 65°

∴ $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC (Q.E.D.)

[b] ∵ MF = ME
(lengths of two radii)



, XF = YE

∴ MX = MY

∵ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

∴ AB = CD (Q.E.D.1)

∵ $\overline{MX} \perp \overline{AB}$

∴ X is the midpoint of $\overline{AB}$

∴ $AX = \frac{1}{2} AB$, ∵ $\overline{MY} \perp \overline{CD}$

∴ Y is the midpoint of $\overline{CD}$

∴ $CY = \frac{1}{2} CD$, ∵ AB = CD

∴ AX = CY

∴ In ΔΔ AXF , CYE

$$\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90^\circ \end{cases}$$

∴ Δ AXF ≡ Δ CYE ∴ AF = CE (Q.E.D.2)

Model Examinations of the School Book on Geometry

Model 1

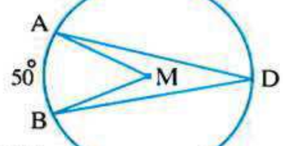
Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 The inscribed angle drawn in a semicircle is ....

(a) an acute. (b) an obtuse. (c) a straight. (d) a right.

2 **In the opposite figure :**



Circle of centre M

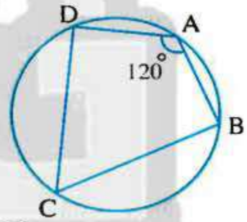
If m $(\widehat{AB}) = 50°$, then m (∠ ADB) = ...

(a) 25° (b) 50° (c) 100° (d) 150°

3 The number of symmetric axes of any circle is ....

(a) zero (b) 1 (c) 2 (d) an infinite number.

4 **In the opposite figure :**



If m (∠ A) = 120° , then m (∠ C) = ...

(a) 60° (b) 90°

(c) 120° (d) 180°

5 If the straight line L is a tangent to the circle M of diameter length 8 cm. , then the distance between L and the centre of the circle equals .... cm.

(a) 3 (b) 4 (c) 6 (d) 8

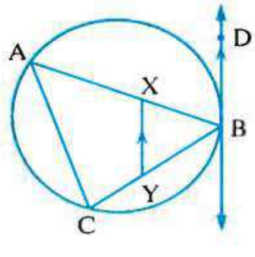
6 The surface of the circle M ∩ the surface of the circle N = {A} and the radius length of one of them is 3 cm. and MN = 8 cm. , then the radius length of the other circle equals .... cm.

(a) 5 (b) 6 (c) 11 (d) 16

2 [a] Complete and prove that :

In a cyclic quadrilateral , each two opposite angles are ....

[b] In the opposite figure :



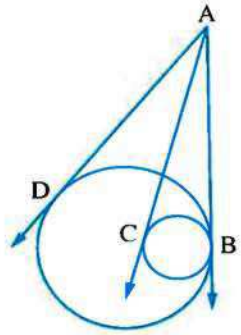
ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, $X \in \overline{AB}$, $Y \in \overline{BC}$ where $\overline{XY} \parallel \overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.

3 **[a] In the opposite figure :**



Two circles are touching internally at B

, $\overrightarrow{AB}$ is a common tangent

, $\overrightarrow{AC}$ is a tangent to the smaller circle at C

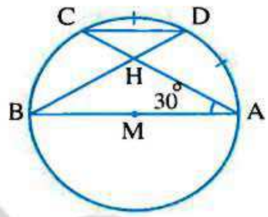
, $\overrightarrow{AD}$ is a tangent to the greater circle at D

, AC = 15 cm. , AB = $(2x - 3)$ cm.

and AD = $(y - 2)$ cm.

Find : The value of each of x and y

[b] In the opposite figure :



$\overline{AB}$ is a diameter in the circle M

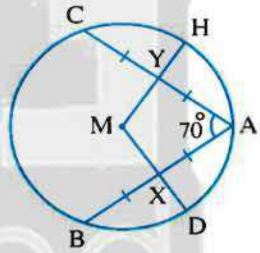
, C $\in$ the circle M , m ($\angle$ CAB) = 30°

, D is midpoint of $\overset{\frown}{AC}$, $\overline{DB} \cap \overline{AC} = \{H\}$

1 **Find :** m ($\angle$ BDC) and m $(\overset{\frown}{AD})$

2 **Prove that :** $\overline{AB} \parallel \overline{DC}$

4 **[a] In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in circle M

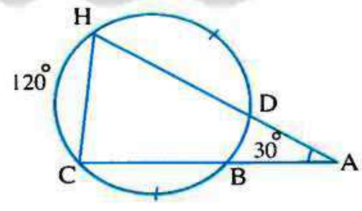
, X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$

, m ($\angle$ CAB) = 70°

1 **Calculate :** m ($\angle$ DMH)

2 **Prove that :** XD = YH

[b] In the opposite figure :



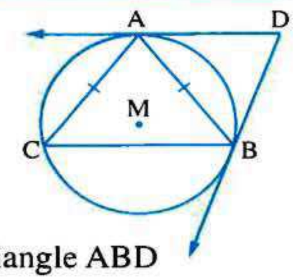
m ($\angle$ A) = 30° , m $(\overset{\frown}{HC})$ = 120°

, m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{DH})$

1 **Find :** m ($\overset{\frown}{BD}$ the minor)

2 **Prove that :** AB = AD

5 **[a] In the opposite figure :**



$\overrightarrow{DA}$ and $\overrightarrow{DB}$ are two tangents of the circle M

and AB = AC

Prove that :

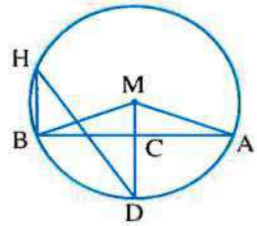
$\overline{AC}$ is a tangent to the circle passing through the vertices of the triangle ABD

[b] In the opposite figure :

C is the midpoint of $\overline{AB}$, $\overrightarrow{MC} \cap$ the circle M = {D}

, m (∠ MAB) = 20°

Find : m (∠ BHD) and m ($\widehat{ADB}$)



Model 2

1 Choose the correct answer from those given :

1 The measure of the arc which equals half the measure of the circle equals ....

(a) 360° (b) 180° (c) 120° (d) 90°

2 The number of common tangents of two touching circles externally equals ....

(a) 0 (b) 1 (c) 2 (d) 3

3 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 45° (b) 90° (c) 120° (d) 80°

4 The angle of tangency is included between ....

(a) two chords. (b) two tangents.

(c) a chord and a tangent. (d) a chord and a diameter.

5 ABCD is a cyclic quadrilateral , m (∠ A) = 60° , then m (∠ C) = ...

(a) 60° (b) 30° (c) 90° (d) 120°

6 If M , N are two touching circles internally , their radii lengths are 5 cm. , 9 cm.

, then MN = ... cm.

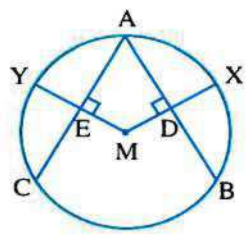
(a) 14 (b) 4 (c) 5 (d) 9

2 [a] In the opposite figure :

AB = AC , $\overline{MD} \perp \overline{AB}$,

$\overline{ME} \perp \overline{AC}$

Prove that : XD = YE



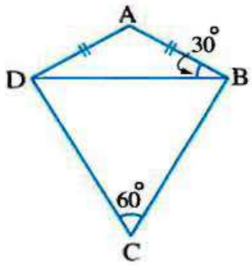
[b] In the opposite figure :

ABCD is a quadrilateral in which AB = AD ,

m (∠ ABD) = 30° ,

m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.



3 [a] State two cases of a cyclic quadrilateral.

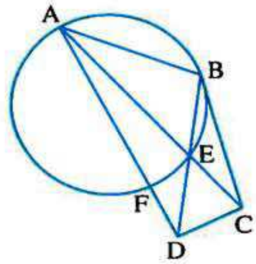
[b] In the opposite figure :

$\overline{BC}$ is a tangent at B ,

E is the midpoint of $\widehat{BF}$

Prove that :

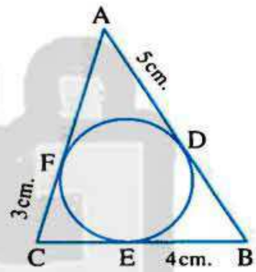
ABCD is a cyclic quadrilateral.



4 **[a] In the opposite figure :**

A circle is drawn touches the sides of a triangle ABC , $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at D , E , F , AD = 5 cm , BE = 4 cm. , CF = 3 cm.

Find the perimeter of Δ ABC

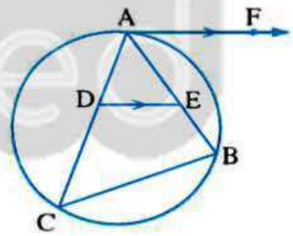


[b] In the opposite figure :

$\overrightarrow{AF}$ is a tangent to the circle at A , $\overrightarrow{AF}$ // $\overrightarrow{DE}$

Prove that :

DEBC is a cyclic quadrilateral.



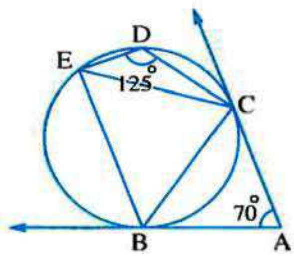
5 **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C , m (∠ A) = 70° , m (∠ CDE) = 125°

Prove that :

1 CB = CE

2 $\overrightarrow{AC}$ // $\overline{BE}$



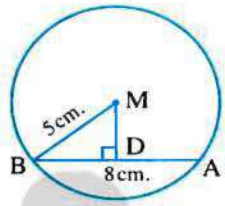
Model examination for the merge students

Answer the following questions in the same paper : (Calculator is allowed)

1 Complete each of the following :

1. The longest chord in the circle is called ……………
2. The straight line passing through the center of the circle and the midpoint of any chord is ……………
3. The two tangent-segments drawn to a circle from a point outside it are …………… in length.
4. **In the opposite figure :**

   The length of $\overline{MD}$ = …………… cm.



5. The number of symmetry axes of a circle is ……………
6. If $\overline{AC}$ is a diameter in a circle M , then m $(\widehat{AC})$ = …………… °

2 Choose the correct answer from those given :

1. If A ∈ the circle M of diameter length 6 cm.

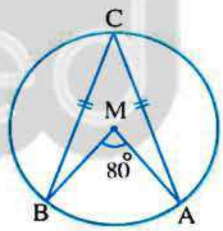
   , then MA = …………… cm.

   (a) 3 (b) 4
   (c) 5 (d) 6

2. **In the opposite figure :**

   m (∠ ACB) = ……………

   (a) 40° (b) 80°
   (c) 90° (d) 180°



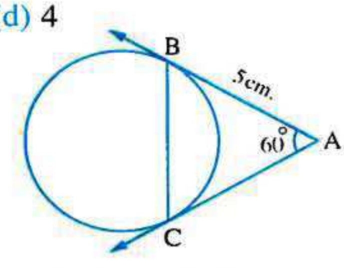
3. The number of the common tangents of two distant circles is ……………

   (a) 1 (b) 2 (c) 3 (d) 4

4. **In the opposite figure :**

   The length of $\overline{BC}$ = …………… cm.

   (a) 3 (b) 4
   (c) 5 (d) 6



5. The number of circles which can be drawn passing through the endpoints of a line segment $\overline{AB}$ equals ……………

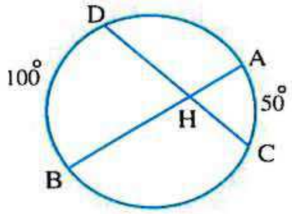
   (a) 1 (b) 2 (c) 3 (d) an infinite number.

6 **In the opposite figure :**

m (∠ AHC) = ...

(a) 25° (b) 50°

(c) 75° (d) 100°



3 Put (✓) for the correct statement , (✗) for the incorrect statement :

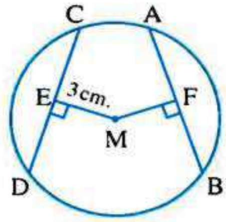
1 If M , N are two touching externally circles with radii lengths are $r_1 = 5$ cm. , $r_2 = 3$ cm. , then MN = 15 cm. ( )

2 **In the opposite figure :**

If AB = CD ,

ME = 3 cm. , then

MF = 3 cm. ( )

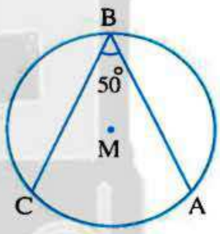


3 The quadrilateral ABCD is a cyclic quadrilateral if

m (∠ A) + m (∠ C) = 90° ( )

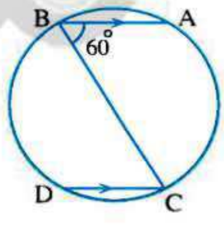
4 **In the opposite figure :**

m $(\overset{\frown}{AC})$ = 100° ( )



5 **In the opposite figure :**

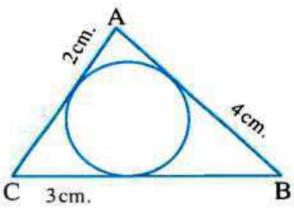
m $(\overset{\frown}{AB})$ + m $(\overset{\frown}{CD})$ = 300° ( )



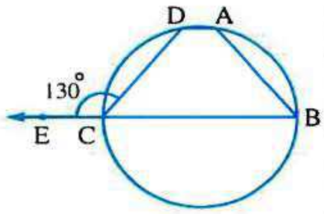
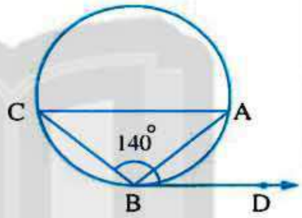
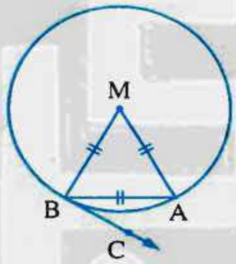
6 **In the opposite figure :**

The perimeter of

Δ ABC = 9 cm. ( )



4 **Join from the column (A) to the suitable one of the column (B) :**

(A)	(B)
1 The measure of the inscribed angle which is drawn in a semicircle equals	• 130°
2 **In the opposite figure :** m (∠ A) = ... 	• 40°
3 **In the opposite figure :** $\overrightarrow{BD}$ is a tangent at B , m (∠ DBC) = 140° , then m (∠ A) = ... 	• 90°
4 The radius of the circumcircle of the vertices of right-angled triangle of hypotenuse length 10 cm. equals .... cm.	• 30°
5 **In the opposite figure :** Δ MAB is an equilateral triangle , $\overrightarrow{BC}$ is a tangent at B , , then m (∠ ABC) = ... 	• 2 : 1
6 The ratio between the measures of the central angle and inscribed angle subtended by the same arc is	• 5

Governorates' Examinations on Geometry

1 Cairo Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 The area of the rhombus with diagonal lengths 6 cm. , 8 cm. is .... cm^2

(a) 2 (b) 14 (c) 24 (d) 48

2 Two distant circles M and N with radii lengths 6 cm and 8 cm respectively , then MN .... 14 cm.

(a) < (b) > (c) = (d) ≥

3 The measure of the inscribed angle is .... the measure of the central angle subtended by the same arc.

(a) half (b) twice (c) quarter (d) third

4 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

5 In the cyclic quad. ABCD , if $m(\angle A) = \frac{1}{2} m(\angle C)$, then $m(\angle A) =$ ....°

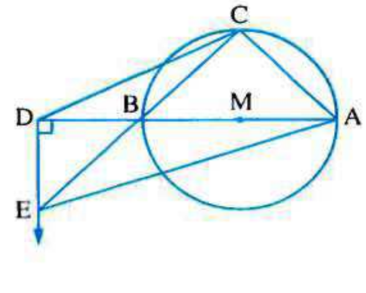
(a) 20 (b) 30 (c) 60 (d) 120

6 The angle of measure 40° is the complemented angle of the angle of measure .... °

(a) 320 (b) 140 (c) 60 (d) 50

2 [a] Mention two cases of the cyclic quadrilateral.

[b] **In the opposite figure :**



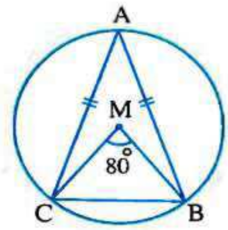
$\overline{AB}$ is a diameter of the circle M , $D \in \overrightarrow{AB}$, $D \notin \overline{AB}$, $\overrightarrow{DE} \perp \overrightarrow{AB}$, $C \in \widehat{AB}$, $\overrightarrow{CB} \cap \overrightarrow{DE} = \{E\}$

1 **Find :** $m(\angle ACB)$

2 **Prove that :** The figure ACDE is a cyclic quadrilateral.

3 [a] Find the measure of the arc which represents $\frac{1}{3}$ of the measure of the circle.

[b] **In the opposite figure :**



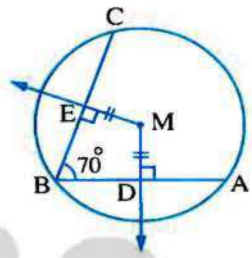
Δ ABC is drawn inside the circle M

, AB = AC , m ($\angle$ BMC) = 80°

Find : 1 m ($\angle$ ABC)

2 The measure of the major arc $\overset{\frown}{BC}$

4 [a] **In the opposite figure :**



$\overline{AB}$ and $\overline{BC}$ are two chords in the circle M , $\overrightarrow{MD} \perp \overline{AB}$

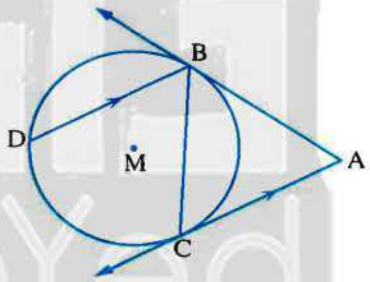
, $\overrightarrow{ME} \perp \overline{CB}$, MD = ME

, m ($\angle$ ABC) = 70°

1 **Find :** m ($\angle$ DME)

2 **Prove that :** AB = CB

[b] **In the opposite figure :**



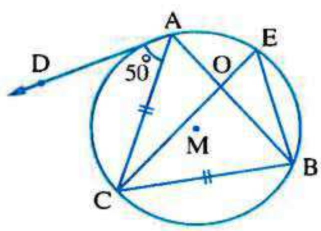
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M at B and C respectively

, $\overrightarrow{BD}$ // $\overrightarrow{AC}$

Prove that : $\overrightarrow{BC}$ bisects $\angle$ ABD

5 [a] Using the geometric tools , draw $\overline{AB}$ with length 6 cm , and then draw a circle passing through the two points A , B with radius length 4 cm. What is the length of the radius of the smallest circle passing through the two points A and B ?

[b] **In the opposite figure :**



A circle M , AC = BC

, $\overrightarrow{AD}$ is a tangent to the circle at A , m ($\angle$ CAD) = 50°

1 **Find :** m ($\angle$ ABC) , m ($\angle$ BEC)

2 **Prove that :**

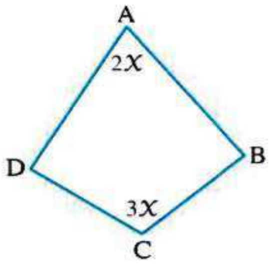
$\overrightarrow{BC}$ is a tangent to the circle passing through the vertices of the triangle BEO

2 Giza Governorate

Answer the following questions :

1 Choose the correct answer :

1 In the opposite figure :



ABCD is a cyclic quadrilateral

, m (∠ A) = 2 X , m (∠ C) = 3 X

, then the value of X = ...°

(a) 20 (b) 30

(c) 32 (d) 36

2 If the ratio between the perimeters of two squares is 1 : 2 , then the ratio between their areas equals ....

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

3 The measure of the inscribed angle in a semicircle equals ....°

(a) 45 (b) 90 (c) 120 (d) 180

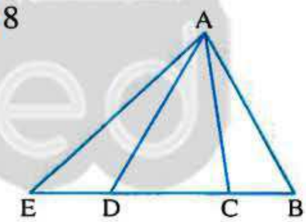
4 The median of the triangle divides its surface into two triangles ....

(a) congruent. (b) equal in area. (c) isosceles. (d) right-angled.

5 If the two circles M , N are touching internally , their radii lengths are 3 cm. , 5 cm. , then MN = ... cm.

(a) 3 (b) 5 (c) 2 (d) 8

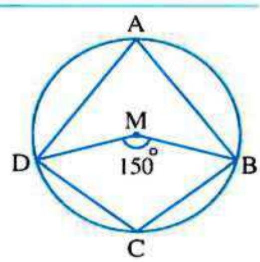
6 The number of triangles in the opposite figure equals ....



(a) 3 (b) 4

(c) 5 (d) 6

2 [a] In the opposite figure :

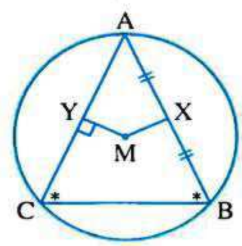


A circle of centre M

, m (∠ BMD) = 150°

Find with proof : m (∠ C)

[b] In the opposite figure :



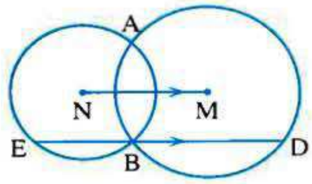
ABC is an inscribed triangle in a circle M

in which m (∠ B) = m (∠ C)

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

3 [a] **In the opposite figure :**

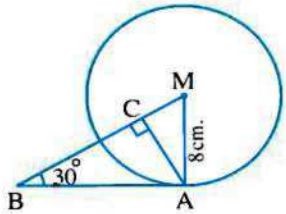


M , N are two intersecting circles at A , B

, $\overleftrightarrow{BD}$ // $\overleftrightarrow{MN}$ and intersects the two circles at D , E

Prove that : DE = 2 MN

[b] **In the opposite figure :**

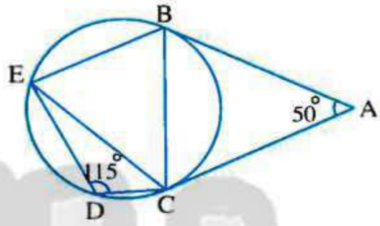


$\overrightarrow{AB}$ is a tangent to the circle M at A

, MA = 8 cm. , m (∠ ABM) = 30° , $\overline{AC} \perp \overline{MB}$

Find : The length of each of $\overline{AB}$, $\overline{AC}$

4 [a] **In the opposite figure :**



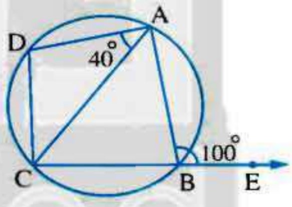
$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle at B , C

, m (∠ A) = 50° , m (∠ CDE) = 115°

Prove that : 1 $\overrightarrow{BC}$ bisects ∠ ABE

2 CB = CE

[b] **In the opposite figure :**

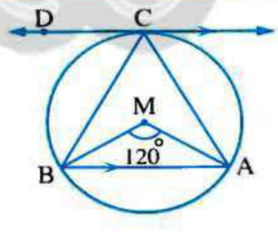


m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{AD}$)

5 [a] **In the opposite figure :**



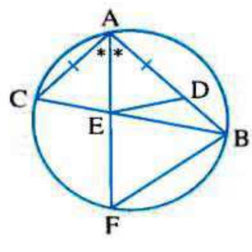
$\overleftrightarrow{CD}$ is a tangent to the circle M at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$

, m (∠ AMB) = 120°

Prove that : Δ CAB is an equilateral triangle.

[b] **In the opposite figure :**



AC = AD , $\overrightarrow{AE}$ bisects ∠ BAC

and cuts $\overline{BC}$ at E and the circle at F

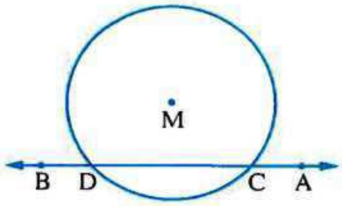
Prove that : BDEF is a cyclic quadrilateral.

3 Alexandria Governorate

Answer the following questions : (Calculators are permitted)

1 Choose the correct answer from those given :

1 In the opposite figure :



$\overleftrightarrow{AB} \cap$ the surface of the circle M = ...

(a) $\{C , D\}$ (b) $\overline{CD}$
(c) $\overleftrightarrow{CD}$ (d) $\varnothing$

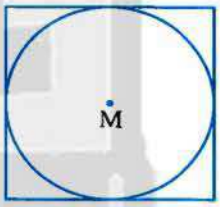
2 ∠ A and ∠ B are two complementary angles , ∠ B and ∠ C are two supplementary angles , m (∠ A) = 30° , then m (∠ C) = ...°

(a) 30 (b) 60 (c) 90 (d) 120

3 If the surface of the circle M ∩ the surface of the circle N = {A} and the radius length of one of them equals 3 cm and MN = 8 cm. , then the radius length of the other circle equals .... cm.

(a) 5 (b) 6 (c) 11 (d) 16

4 In the opposite figure :



If the side length of the square = 10 cm.
, then the surface area of the circle = ... cm^2

(a) 100 π (b) 25 π
(c) 50 π (d) 40 π

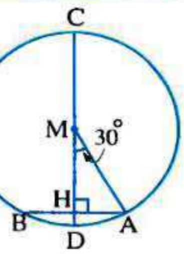
5 A circle can be drawn passing through the vertices of a ....

(a) rhombus (b) parallelogram (c) trapezium (d) rectangle

6 The rhombus whose two diagonal lengths are 12 cm. and 16 cm. , then its side length equals .... cm.

(a) 6 (b) 8 (c) 10 (d) 20

2 [a] In the opposite figure :



$\overline{CD}$ is a diameter in the circle M
, AB = 10 cm. , $\overline{MH} \perp \overline{AB}$
, m (∠ AMD) = 30°
Find : The length of $\overline{CD}$

[b] ABCD is a quadrilateral inscribed in a circle , E is a point outside the circle , $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are two tangents to the circle at A and B , if m (∠ AEB) = 70° and m (∠ ADC) = 125° **, prove that :** AB = AC

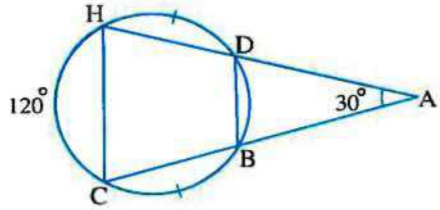
3 **[a] In the opposite figure :**

m (∠ A) = 30° , m ($\overset{\frown}{HC}$) = 120°

, m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{DH}$)

1 **Find :** m ($\overset{\frown}{BD}$) «the minor arc»

2 **Prove that :** AB = AD



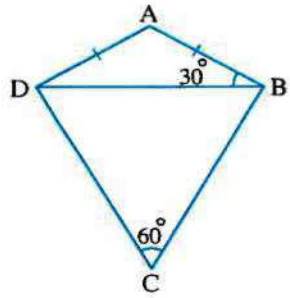
[b] In the opposite figure :

ABCD is a quadrilateral , AB = AD

, m (∠ ABD) = 30°

, m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.

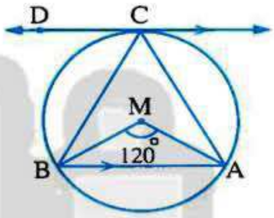


4 **[a] In the opposite figure :**

$\overleftrightarrow{CD}$ is a tangent to the circle at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$, m (∠ AMB) = 120°

Prove that : The triangle CAB is an equilateral triangle.



[b] In the opposite figure :

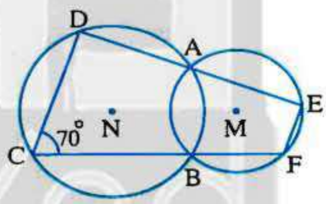
M and N are two intersecting circles at A and B

, $\overleftrightarrow{AD}$ is drawn to intersect the circle M at E and the circle N at D

, $\overleftrightarrow{BC}$ is drawn to intersect the circle M at F and the circle N at C

, m (∠ C) = 70°

Prove that : $\overline{CD}$ // $\overline{EF}$

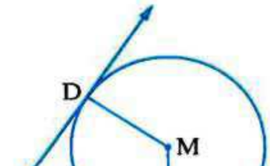


5 **[a] In the opposite figure :**

AC = BC , m (∠ ABC) = 65°

, m (∠ DAB) = 130°

Prove that : $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC



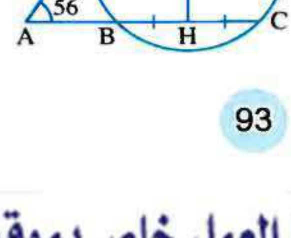
[b] In the opposite figure :

$\overrightarrow{AD}$ is a tangent to the circle M

, $\overrightarrow{AC}$ intersects the circle M at B , C

, m (∠ A) = 56° and H is the midpoint of $\overline{BC}$

Find with proof : m (∠ DMH)



4 El-Kalyoubia Governorate

Answer the following questions :

1 Choose the correct answer :

1 ABC is a triangle in which : $(AB)^2 > (BC)^2 + (AC)^2$, then $\angle$ C is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

2 If M and N are two intersecting circles whose radii length are 5 cm and 2 cm. , then MN $\in$ ....

(a) $]3 , 7[$ (b) $[3 , 7[$ (c) $]3 , 7]$ (d) $[3 , 7]$

3 If Δ ABC ~ Δ XYZ , m ($\angle$ A) = 50° , m ($\angle$ B) = 60° , then m ($\angle$ Z) = ...°

(a) 90 (b) 110 (c) 10 (d) 70

4 The measure of the central angle which is opposite to an arc of length $\frac{1}{3}\pi r$ equals ....°

(a) 30 (b) 60 (c) 120 (d) 240

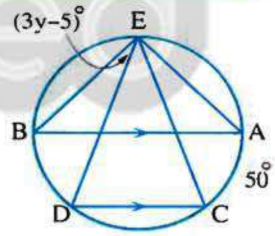
5 ABC is a right-angled triangle at B , $\overline{BD} \perp \overline{AC}$ where $\overline{BD} \cap \overline{AC} = \{D\}$, then the projection of $\overline{BD}$ on $\overleftrightarrow{AC}$ is ....

(a) A (b) B (c) C (d) D

6 If ABCD is a cyclic quadrilateral , then m ($\angle$ BAC) = m ($\angle$ ....)

(a) BCA (b) DBA (c) BDC (d) ACD

2 [a] In the opposite figure :



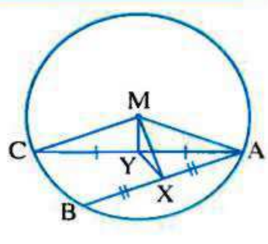
$\overline{AB} // \overline{CD}$, m ($\widehat{AC}$) = 50°

, m ($\angle$ BED) = (3 y − 5)°

Find : The value of y

[b] Using your geometric tools , draw $\overline{AB}$ with length 4 cm , then draw a circle passing through the two points A and B whose diameter length is 5 cm.
How many circles can be drawn ? (Don't erase the arcs).

3 [a] In the opposite figure :



A circle with centre M

, X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.

Prove that : 1 AXYM is a cyclic quadrilateral.

2 m ($\angle$ MXY) = m ($\angle$ MCY)

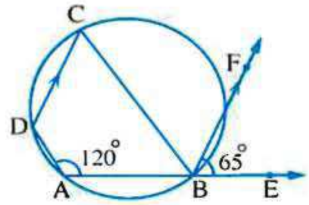
[b] In the opposite figure :

m (∠ A) = 120° , m (∠ EBF) = 65°

, $\overline{DC}$ // $\overrightarrow{BF}$

Find with proof :

1 m (∠ C) 2 m (∠ D)



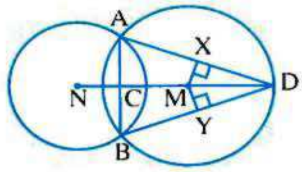
4 **[a] In the opposite figure :**

Circle M ⋂ circle N = {A , B}

, $\overleftrightarrow{AB}$ ⋂ $\overleftrightarrow{MN}$ = {C} , D ∈ $\overleftrightarrow{MN}$

, $\overline{MX}$ ⊥ $\overline{AD}$, $\overline{MY}$ ⊥ $\overline{BD}$

Prove that : MX = MY



[b] ABC is a triangle inscribed in a circle , $\overleftrightarrow{AD}$ is a tangent to the circle at A

, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$, where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

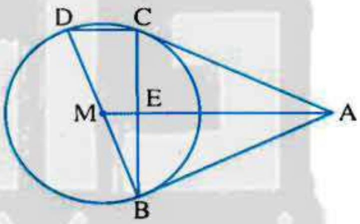
5 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AM}$ ⋂ $\overline{CB}$ = {E}

and $\overline{BD}$ is a diameter of the circle.

Prove that : $\overline{AM}$ // $\overline{CD}$



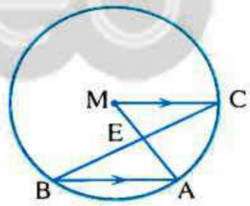
[b] In the opposite figure :

$\overline{AB}$ is a chord in the circle M

, $\overline{CM}$ // $\overline{AB}$

, $\overline{BC}$ ⋂ $\overline{AM}$ = {E}

Prove that : BE > AE



5 El-Sharkia Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from the given ones :**

1 A circle can be drawn passing through the vertices of a ....

(a) rhombus. (b) rectangle. (c) trapezium. (d) parallelogram.

2 A circle with diameter length 10 cm. , the straight line L is distant from its centre by 5 cm. , then the straight line L is ....

(a) a tangent. (b) a secant.

(c) outside the circle. (d) a diameter of the circle.

3 The number of common tangents of two touching circles externally equals ....

(a) zero (b) 1 (c) 2 (d) 3

4 If M , N are two touching circles externally , the lengths of their radii are 2 cm. , 4 cm. respectively , then the area of the circle with diameter $\overline{MN}$ equals .... cm^2

(a) 36 π (b) 9 π (c) 16 π (d) 4 π

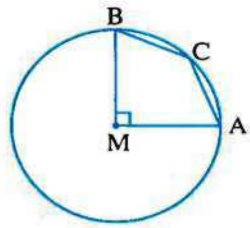
5 **In the opposite figure :**

A circle M , $\overline{MA} \perp \overline{MB}$

, then m ($\angle$ ACB) = ...

(a) 45° (b) 90°

(c) 145° (d) 135°



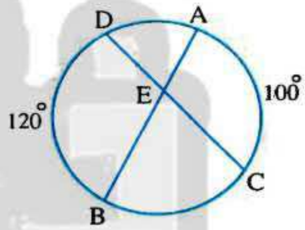
6 **In the opposite figure :**

m ($\overset{\frown}{AC}$) = 100° , m ($\overset{\frown}{DB}$) = 120°

, then m ($\angle$ AEC) = ...

(a) 110° (b) 55°

(c) 70° (d) 100°



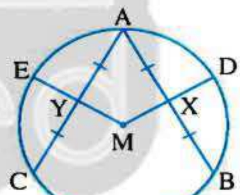
2 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two equal chords in circle M

, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$

Prove that : XD = YE

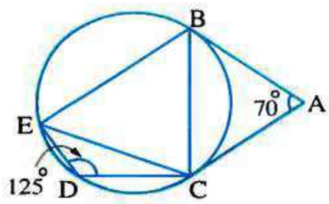


[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C

, m ($\angle$ A) = 70° , m ($\angle$ CDE) = 125°

Prove that : $\overrightarrow{BC}$ bisects $\angle$ ABE



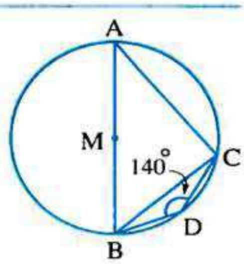
3 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m ($\overset{\frown}{BD}$) = m ($\overset{\frown}{DC}$) , m ($\angle$ BDC) = 140°

Find with proof : 1 m ($\angle$ ABC)

2 m ($\overset{\frown}{ABD}$)



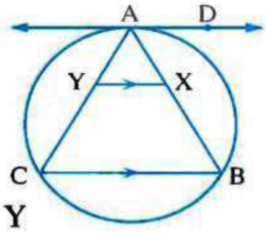
[b] In the opposite figure :

$\overleftrightarrow{AD}$ is a tangent to the circle at A , X $\in \overline{AB}$

, Y $\in \overline{AC}$ and $\overline{XY}$ // $\overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle which passes through the points A , X and Y

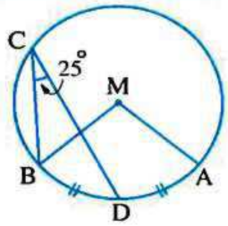


4 **[a] In the opposite figure :**

A circle M , D is the midpoint of $\widehat{AB}$

, m (∠ DCB) = 25°

Find : m (∠ AMB)



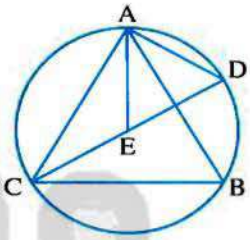
[b] In the opposite figure :

ABC is an equilateral triangle drawn in the circle

, D $\in \widehat{AB}$, E $\in \overline{DC}$, where AD = DE

Prove that : 1 Δ ADE is an equilateral triangle.

2 m (∠ DAB) = m (∠ EAC)



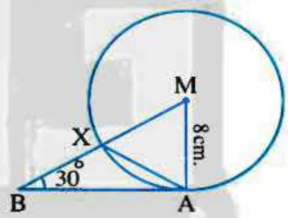
5 **[a] In the opposite figure :**

$\overline{AB}$ is a tangent-segment to the circle M at A

, AM = 8 cm. , m (∠ ABM) = 30°

1 **Find :** The length of $\overline{AB}$

2 **Prove that :** Δ XAB is an isosceles triangle.

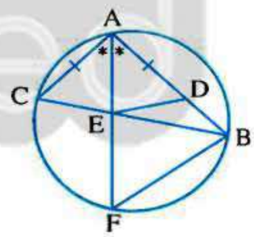


[b] In the opposite figure :

AD = AC

, $\overrightarrow{AF}$ bisects ∠ BAC

Prove that : DBFE is a cyclic quadrilateral.



6 El-Monofia Governorate

Answer the following questions : ***(Calculators are permitted)***

1 **Choose the correct answer from those given :**

1 The axis of symmetry of a circle is ....

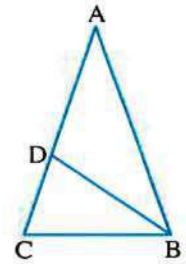
(a) the diameter. (b) the chord.

(c) the straight line passing through the center. (d) the tangent.

2 XYZ is a triangle. If $(XY)^2 - (YZ)^2 > (XZ)^2$, then $\angle$ Y is ....

(a) acute. (b) right. (c) obtuse. (d) reflex.

3 **In the opposite figure :**



If AB = AC , BC = BD = AD

, then m ($\angle$ A) = ...°

(a) 30 (b) 36

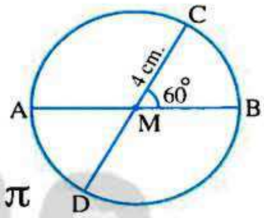
(c) 45 (d) 72

4 ABCD is a cyclic quadrilateral in which m ($\angle$ A) = 2 m ($\angle$ C)

, then m ($\angle$ C) = ...°

(a) 30 (b) 60 (c) 90 (d) 120

5 **In the opposite figure :**



A circle M , MC = 4 cm.

, m ($\angle$ CMB) = 60°

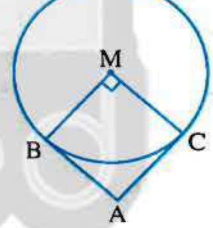
, then the length of $\widehat{BD}$ = ... cm.

(a) 4π (b) 8π (c) $\frac{8}{3}\pi$ (d) 16π

6 If Y $\in \overline{XZ}$ and XY = 2 YZ , then the area of the square drawn on $\overline{XY}$ = ...
The area of the square drawn on $\overline{XZ}$

(a) $\frac{9}{4}$ (b) $\frac{4}{9}$ (c) 2 (d) $\frac{1}{2}$

2 [a] **In the opposite figure :**

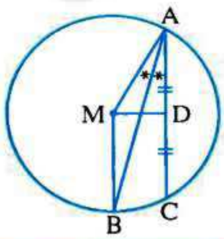


$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, m ($\angle$ BMC) = 90°

Prove that : ABMC is a square.

[b] **In the opposite figure :**



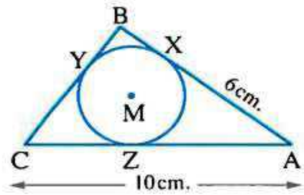
$\overline{AC}$ is a chord in the circle M

, $\overrightarrow{AB}$ bisects $\angle$ CAM

, D is the midpoint of $\overline{AC}$

Prove that : $\overline{DM} \perp \overline{MB}$

3 [a] **In the opposite figure :**



$\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ are tangents to the circle M at X

, Y and Z respectively , AC = 10 cm.

, AX = 6 cm. and the perimeter of Δ ABC = 24 cm.

1 **Find :** The length of $\overline{AB}$

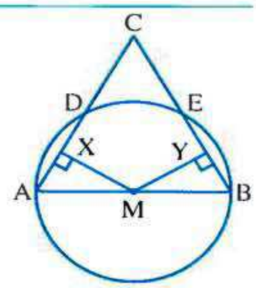
2 Determine the type of Δ ABC according to the measures of its angles.

[b] ABC is a triangle inscribed in a circle , $X \in \widehat{AB}$, $Y \in \widehat{AC}$ where $m(\widehat{AX}) = m(\widehat{AY})$, $\overline{CX} \cap \overline{AB} = \{D\}$ and $\overline{BY} \cap \overline{AC} = \{E\}$

Prove that : 1 The figure BCED is a cyclic quadrilateral.

2 $m(\angle DEB) = m(\angle XAB)$

4 **[a] In the opposite figure :**



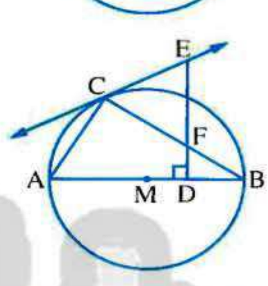
$\overline{AB}$ is a diameter in the circle M

, CA = CB , $\overline{MX} \perp \overline{DA}$

, $\overline{MY} \perp \overline{EB}$

Prove that : CD = CE

[b] In the opposite figure :



$\overline{AB}$ is a diameter in the circle M

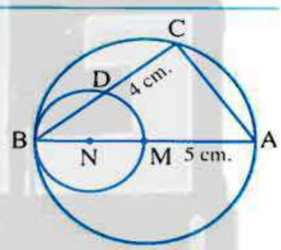
, $\overleftrightarrow{EC}$ is a tangent to the circle M at C , $\overline{ED} \perp \overline{AB}$

, where $\overline{ED} \cap \overline{CB} = \{F\}$

Prove that : 1 The figure ADFC is a cyclic quadrilateral.

2 Δ ECF is an isosceles triangle.

5 **[a] In the opposite figure :**

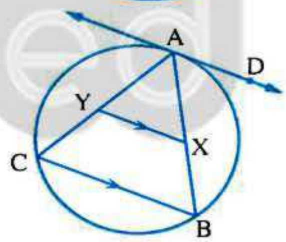


M , N are two circles touching internally at B

, AM = 5 cm. , CD = 4 cm.

Find with proof : The length of $\overline{AC}$

[b] In the opposite figure :



ABC is a triangle inscribed in a circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A , $\overline{XY}$ // $\overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

7 El-Gharbia Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 A square whose diagonal length is 10 cm. , then its surface area equals .... cm^2.

(a) 40 (b) 50 (c) 80 (d) 100

2 ABC is a triangle in which $(AC)^2 > (AB)^2 + (BC)^2$, then $\angle$ BAC is ....

(a) acute. (b) obtuse. (c) right. (d) straight.

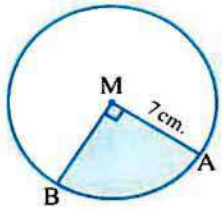
3 M and N are two intersecting circles at two points and the two radii lengths are 3 cm. and 5 cm. , then MN ∈ ....

(a)]8 , ∞[(b)]2 , ∞[(c)]0 , 2[(d)]2 , 8[

4 ABCD is a cyclic quadrilateral in which m (∠ A) = 3 m (∠ C) , then m (∠ A) = ...°

(a) 90 (b) 45 (c) 135 (d) 120

5 **In the opposite figure :**

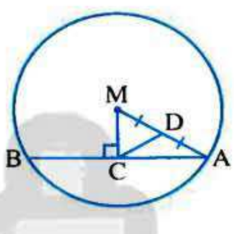


$\overline{MA}$, $\overline{MB}$ are two radii perpendicular in the circle M whose radius length is 7 cm.

, then the perimeter of the shaded part = ... cm. $\left(\pi = \frac{22}{7}\right)$

(a) 14 (b) 11 (c) $38\frac{1}{2}$ (d) 25

6 **In the opposite figure :**



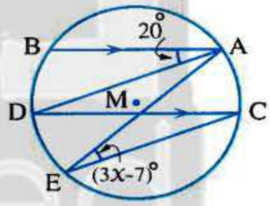
$\overline{AB}$ is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$, D is the midpoint of $\overline{MA}$, CD = 3 cm.

, then the surface area of the circle M = ... π cm²

(a) 3 (b) 6 (c) 9 (d) 36

2 [a] **In the opposite figure :**

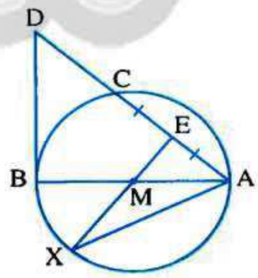


$\overline{AB}$ // $\overline{CD}$, m (∠ BAD) = 20°

, m (∠ AEC) = $(3X - 7)°$

What is the value of X ?

[b] **In the opposite figure :**

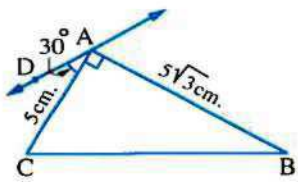


$\overline{AB}$ is a diameter in the circle M , $\overline{BD}$ is a tangent-segment to the circle M at B , E is the midpoint of $\overline{AC}$ and $\overrightarrow{EM}$ intersects the circle M at X

Prove that : 1 The figure MEDB is a cyclic quadrilateral.

2 m (∠ BAX) = $\frac{1}{2}$ m (∠ D)

3 [a] **In the opposite figure :**



ABC is a right-angled triangle at A

, AC = 5 cm. , AB = $5\sqrt{3}$ cm.

, m (∠ DAC) = 30°

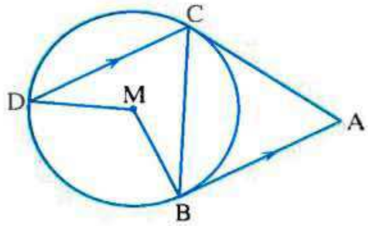
Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC

[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M at B and C

, $\overline{AB}$ // $\overline{CD}$

Prove that : $\overrightarrow{CB}$ bisects ∠ ACD



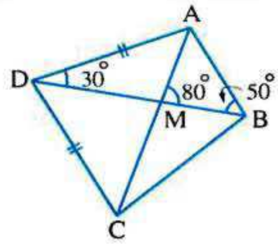
4 **[a] In the opposite figure :**

ABCD is a quadrilateral in which $\overline{AC} \cap \overline{BD} = \{M\}$, DA = DC

, m (∠ ADM) = 30° , m (∠ AMB) = 80°

, m (∠ ABD) = 50°

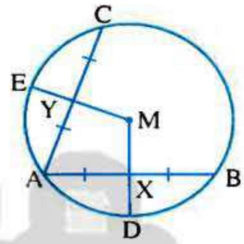
Prove that : The figure ABCD is a cyclic quadrilateral.



[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two chords equal in length in the circle M , X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , $\overrightarrow{MX}$ intersects the circle M at D , $\overrightarrow{MY}$ intersects the circle M at E

Prove that : XD = YE



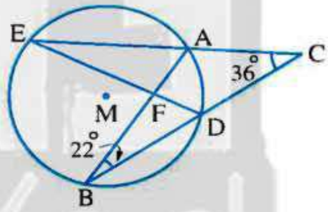
5 **[a] In the opposite figure :**

$\overrightarrow{EA} \cap \overrightarrow{BD} = \{C\}$

, m (∠ C) = 36°

, m (∠ ABD) = 22°

Find with the proof : m $(\overset{\frown}{BE})$



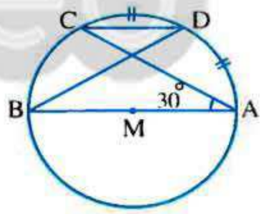
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, m (∠ CAB) = 30° , m $(\overset{\frown}{AD})$ = m $(\overset{\frown}{DC})$

1 **Find with the proof :** m (∠ CDB)

2 **Prove that :** $\overline{DC}$ // $\overline{AB}$



8 El-Dakahlia Governorate

Answer the following questions : ***(Calculator is permitted)***

1 **[a] Choose the correct answer from the given ones :**

1 A circle with greatest chord with length = 12 cm. , then the circumference of the circle = ... cm.

(a) 12 π (b) 6 π (c) 24 π (d) 10 π

2 M and N are two circles whose radii lengths are 6 cm. , 8 cm. and MN = 14 cm. , then the two circles are ....

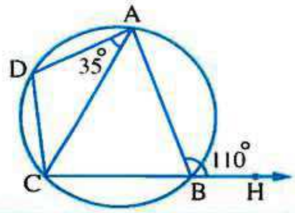
(a) intersecting. (b) distant.

(c) one inside the other. (d) touching externally.

3 The inscribed angle drawn in a semicircle is .... angle.

(a) an acute (b) a straight (c) a right (d) an obtuse

[b] In the opposite figure :



m (∠ ABH) = 110°

, m (∠ CAD) = 35°

Prove that : m $(\widehat{CD})$ = m $(\widehat{AD})$

2 **[a] Choose the correct answer from the given ones :**

1 A chord is of length 8 cm. in a circle of diameter length 10 cm. , then the chord is at .... from the center of the circle.

(a) 2 cm. (b) 4 cm. (c) 3 cm. (d) 6 cm.

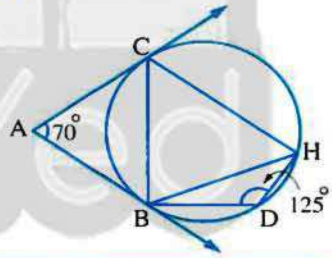
2 The number of common tangents of two circles touching internally is ....

(a) 1 (b) 3 (c) 2 (d) 0

3 ABCD is a cyclic quadrilateral , m (∠ A) = 2 m (∠ C) , then m (∠ A) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

[b] In the opposite figure :



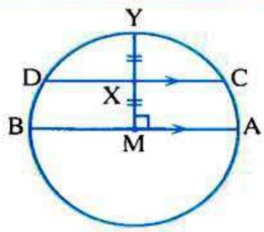
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m (∠ A) = 70° , m (∠ D) = 125°

1 **Find :** m (∠ ABC)

2 **Prove that :** CB = BH

3 **[a] In the opposite figure :**



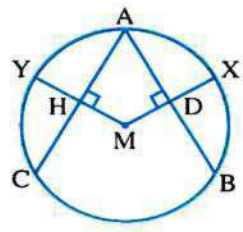
$\overline{AB}$ is a diameter of the circle M , $\overline{CD}$ // $\overline{AB}$

, X is the midpoint of $\overline{MY}$

, $\overline{MY}$ ⊥ $\overline{AB}$

Find : m $(\widehat{AC})$, m $(\widehat{CY})$

[b] In the opposite figure :



$\overline{AB}$, $\overline{AC}$ are two equal chords in the circle M

, $\overrightarrow{MD}$ ⊥ $\overline{AB}$ and cuts the circle at X

, $\overrightarrow{MH}$ ⊥ $\overline{AC}$ and cuts the circle at Y

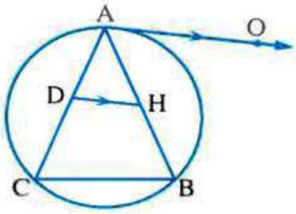
Prove that : XD = HY

4 **[a] In the opposite figure :**

$\overrightarrow{AO}$ is a tangent to the circle at A

, $\overrightarrow{AO}$ // $\overline{DH}$

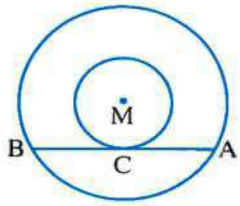
Prove that : DHBC is a cyclic quadrilateral.



[b] In the opposite figure :

$\overline{AB}$ is a chord in the greater circle M and touches the smaller circle at C , if AB = 14 cm.

, find the area of the part included between the two circles.



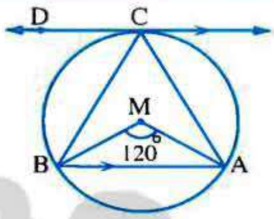
5 **[a] In the opposite figure :**

The circle M passes through the vertices of the triangle ABC

, m (∠ AMB) = 120°

, $\overleftrightarrow{CD}$ is a tangent to the circle M at C , $\overleftrightarrow{CD}$ // $\overline{AB}$

Prove that : Δ ABC is equilateral.

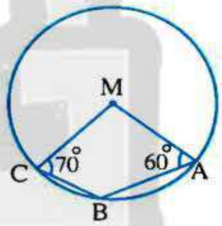


[b] In the opposite figure :

m (∠ MAB) = 60°

, m (∠ MCB) = 70°

Find : m (∠ AMC)



9 Ismailia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The least number of acute angles at any triangle equals ....

(a) zero (b) 1 (c) 2 (d) 3

2 The measure of the central angle drawn in $\frac{1}{3}$ circle equals ....°

(a) 240 (b) 120 (c) 60 (d) 30

3 ABC is a triangle in which : $(AC)^2 = (AB)^2 + (BC)^2 + 5$, then ∠ B is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

4 Which of the following figures is a cyclic quadrilateral ?

(a) The square. (b) The rhombus.

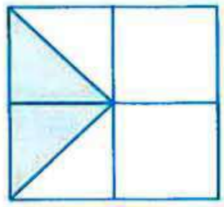
(c) The parallelogram. (d) The trapezium.

5 If AB = 8 cm. , then the length of the radius of the smallest circle can be drawn passing through the two points A and B equals .... cm.

(a) 1 (b) 2 (c) 3 (d) 4

6 **In the opposite figure :**

A square consists of congruent squares , then the area of the shaded part = ... the figure area.



(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$

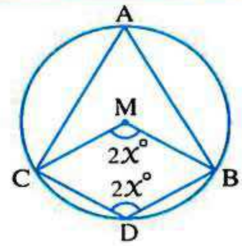
2 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords of the circle M

, $D \in \widehat{BC}$

, m (∠ BMC) = m (∠ BDC) = $(2x)^\circ$

Find with proof : m (∠ A)

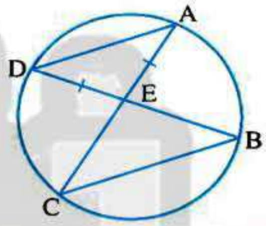


[b] **In the opposite figure :**

$\overline{AC} \cap \overline{BD} = \{E\}$

, EA = ED

Prove that : EB = EC



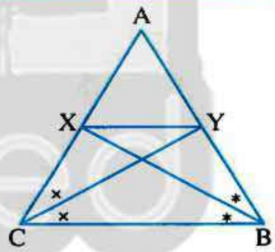
3 [a] **In the opposite figure :**

ABC is a triangle in which AB = AC

, $\overrightarrow{BX}$ bisects ∠ ABC and intersects $\overline{AC}$ at X

, $\overrightarrow{CY}$ bisects ∠ ACB and intersects $\overline{AB}$ at Y

Prove that : BCXY is a cyclic quadrilateral.



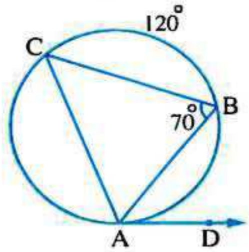
[b] **In the opposite figure :**

$\overrightarrow{AD}$ is a tangent to the circle at A

, m (∠ B) = 70°

, m $(\widehat{BC})$ = 120°

Find : m (∠ DAB)

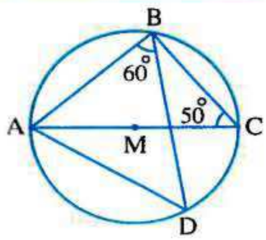


4 [a] **In the opposite figure :**

$\overline{AC}$ is a diameter of the circle M

, m (∠ C) = 50° , m (∠ ABD) = 60°

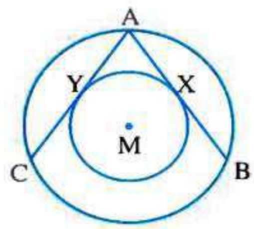
Find : m (∠ CBD) , m (∠ BAD)



[b] In the opposite figure :

Two concentric circles at M , $\overline{AB}$ and $\overline{AC}$ are two chords in the greater circle and two tangents to the smaller circle at X and Y respectively.

Prove that : AB = AC



5 **[a] In the opposite figure :**

Two circles are touching externally at C

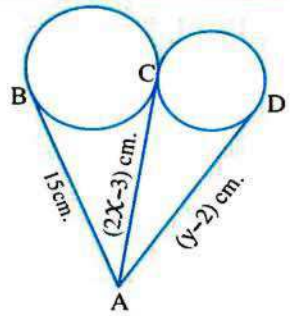
, $\overline{AD}$ is a tangent-segment to the smaller circle at D

, $\overline{AB}$ is a tangent-segment to the greater circle at B

If AD = (y – 2) cm.

, AC = (2 x – 3) cm. , AB = 15 cm.

Find with proof : The value of each of x and y

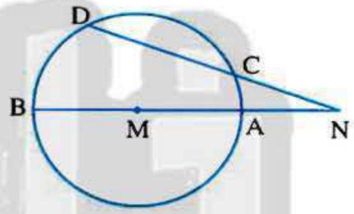


[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{BA} \cap \overrightarrow{DC} = \{N\}$

Prove that : NB > ND



10 Suez Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The inscribed angle drawn in a semicircle is ....

(a) reflex. (b) right. (c) obtuse. (d) acute.

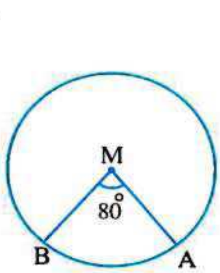
2 **In the opposite figure :**

If M is a circle , m (∠ AMB) = 80°

then m ($\widehat{AB}$) = ...°

(a) 40 (b) 80

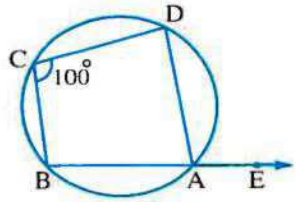
(c) 160 (d) 90



3 If the two circles M , N are touching externally , the length of the radius of one of them is 3 cm. , MN = 8 cm. , then the length of the radius of the other circle is .... cm.

(a) 5 (b) 6 (c) 11 (d) 16

4 In the opposite figure :



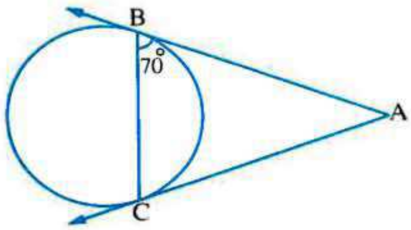
$E \in \overrightarrow{BA}$, m (∠ C) = 100°

, then m (∠ DAE) = ...°

(a) 80 (b) 60

(c) 100 (d) 200

5 In the opposite figure :



$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C , m (∠ ABC) = 70°

, then m (∠ A) = ...°

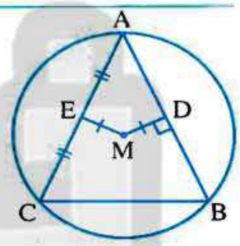
(a) 80 (b) 70

(c) 60 (d) 40

6 The area of the circle = ...

(a) 2 π r (b) πr^2 (c) $2 \pi r^2$ (d) π r

2 [a] In the opposite figure :



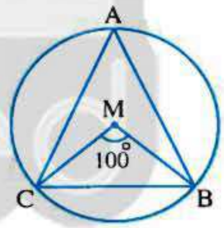
If M is a circle , $\overline{MD} \perp \overline{AB}$

, E is the midpoint of $\overline{AC}$

, MD = ME

, **prove that :** AB = AC

[b] In the opposite figure :

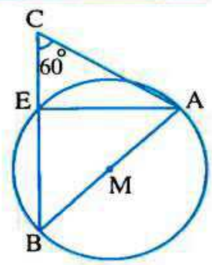


If M is a circle , m (∠ BMC) = 100°

, **find :** 1 m (∠ A)

2 m (∠ MBC)

3 [a] In the opposite figure :



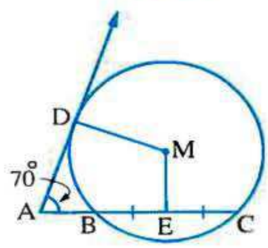
$\overline{AB}$ is a diameter of the circle M

, $C \in \overrightarrow{BE}$, m (∠ ACE) = 60°

Find : 1 m (∠ AEB)

2 m (∠ CAE)

[b] In the opposite figure :



$\overrightarrow{AD}$ is a tangent to the circle M

, $\overrightarrow{AC}$ intersects the circle M at B , C

, E is the midpoint of $\overline{BC}$, m (∠ A) = 70°

Find : m (∠ DME)

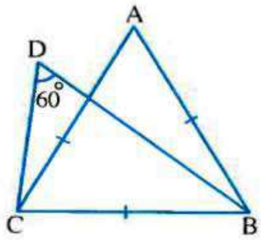
4 **[a]** State two cases of the cyclic quadrilateral.

[b] In the opposite figure :

ABC is an equilateral triangle

, m (∠ D) = 60°

Prove that : ABCD is a cyclic quadrilateral.

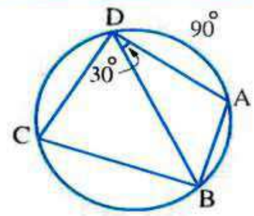


5 **[a] In the opposite figure :**

In the circle , m (∠ ADB) = 30°

, m $(\overset{\frown}{AD})$ = 90°

Find : **1** m $(\overset{\frown}{AB})$ **2** m (∠ DCB)



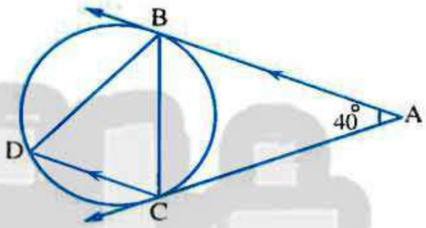
[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, $\overrightarrow{AB}$ // $\overline{CD}$, m (∠ A) = 40°

1 **Find :** m (∠ ABC)

2 **Prove that :** BC = BD



11 Port Said Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 M and N are two intersecting circles. The two radii lengths are 3 cm. and 5 cm. respectively , then MN ∈ ....

(a)]8 , ∞[(b)]2 , ∞[(c)]0 , 2[(d)]2 , 8[

2 If the straight line L is a tangent to the circle M of diameter length 10 cm. , then the distance between L and the center of the circle equals .... cm.

(a) 3 (b) 4 (c) 5 (d) 10

3 The longest chord in the circle is called a ....

(a) chord. (b) diameter. (c) tangent. (d) radius.

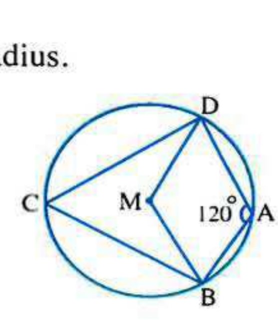
4 **In the opposite figure :**

If m (∠ A) = 120°

, then m (∠ DMB) = ...

(a) 180° (b) 120°

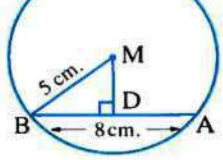
(c) 90° (d) 60°



5 The ratio between the measure of the central angle and the measure of the inscribed angle subtended by the same arc is ....

(a) 4 : 2 (b) 2 : 4 (c) 3 : 2 (d) 2 : 3

6 **In the opposite figure :**



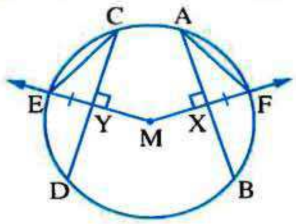
AB = 8 cm. , MB = 5 cm.

, then MD = ...

(a) 5 cm. (b) 3 cm.

(c) 4 cm. (d) 2 cm.

2 [a] **In the opposite figure :**



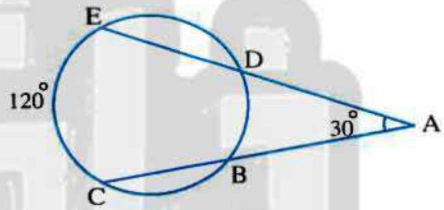
$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle at F

, $\overrightarrow{MY} \perp \overline{CD}$ and intersects the circle at E , FX = EY

Prove that : 1 AB = CD 2 AF = CE

[b] **In the opposite figure :**



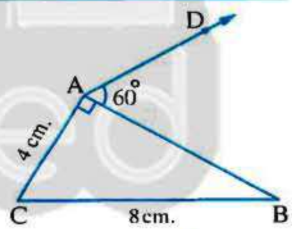
$\overrightarrow{ED} \cap \overrightarrow{CB} = \{A\}$

, m $(\overset{\frown}{CE})$ = 120°

, m (∠ A) = 30°

Find : m $(\overset{\frown}{BD})$

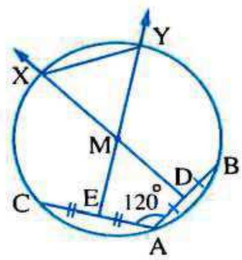
3 [a] **Using the given data , prove that :**



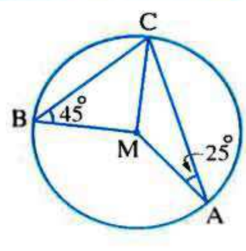
$\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC

[b] **Using the given data , prove that :**

The triangle XYM is an equilateral triangle.



4 [a] **In the opposite figure :**



A circle with center M

, m (∠ MAC) = 25°

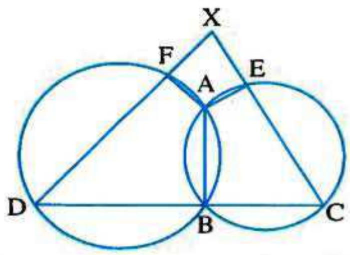
, m (∠ MBC) = 45°

Find : m (∠ AMB)

[b] In the opposite figure :

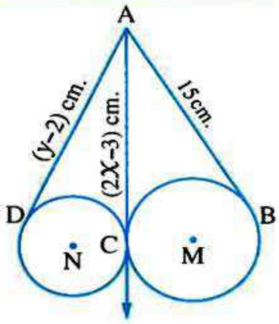
Two intersecting circles at A and B , $\overline{CD}$ passes through the point B and intersects the two circles at C and D , $\overrightarrow{CE} \cap \overrightarrow{DF} = \{X\}$

Prove that : The figure AFXE is a cyclic quadrilateral.



5 **[a] Using the given data , find :**

The values of the symbols x and y

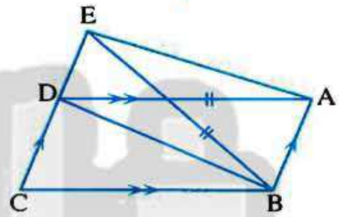


[b] In the opposite figure :

ABCD is a parallelogram

, $E \in \overrightarrow{CD}$ where BE = AD

Prove that : The figure ABDE is a cyclic quadrilateral.



12 Damietta Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer :**

1. The length of the projection of a line segment on a given straight line .... the length of the line segment.

(a) > (b) ≤ (c) ≥ (d) <

2. The number of symmetry axes of any circle is ....

(a) zero (b) 1 (c) 2 (d) an infinite number

3. If a square is of side length 6 cm. , then the square of its diagonal length is .... cm^2

(a) 36 (b) 12 (c) 72 (d) $6\sqrt{2}$

4. If the straight line L is a tangent to the circle M of diameter length 10 cm. , then the distance between L and the center of the circle equals .... cm.

(a) 3 (b) 5 (c) 6 (d) 10

5. If M , N are two touching circles internally , their radii lengths are 7 cm. , 10 cm. , then MN = ... cm.

(a) 3 (b) 17 (c) 7 (d) 10

6 If $\Delta XYZ \sim \Delta ABC$, m ($\angle$ Y) = 60° and m ($\angle$ C) = 40° , then m ($\angle$ X) = ...°

(a) 40 (b) 80 (c) 100 (d) 120

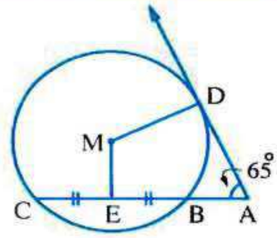
2 **[a] In the opposite figure :**

$\overrightarrow{AD}$ is a tangent to the circle M , $\overline{AC}$ intersects the circle at B , C

, E is the midpoint of $\overline{BC}$

, m ($\angle$ A) = 65°

Find : m ($\angle$ DME)



[b] If the length of $\overline{AB}$ = 6 cm. , draw a circle of radius length 4 cm. that passes through A , B

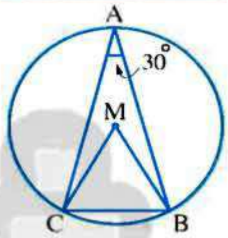
How many circles can be drawn ? (Don't remove the arcs).

3 **[a] In the opposite figure :**

A circle M , m ($\angle$ A) = 30°

1 **Find :** m ($\angle$ BMC)

2 **Prove that :** MBC is an equilateral triangle.



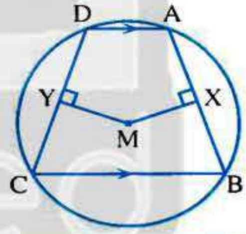
[b] In the opposite figure :

A circle M , $\overline{AD}$ // $\overline{BC}$

, $\overline{MX} \perp \overline{AB}$

, $\overline{MY} \perp \overline{DC}$

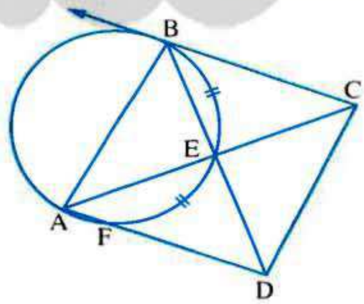
Prove that : MX = MY



4 **[a] In the opposite figure :**

$\overrightarrow{CB}$ is a tangent , m ($\widehat{BE}$) = m ($\widehat{EF}$)

Prove that : ABCD is a cyclic quadrilateral.



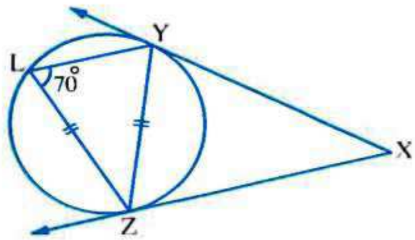
[b] In the opposite figure :

$\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle at Y , Z

, YZ = LZ , m ($\angle$ L) = 70°

1 **Find with proof :** m ($\angle$ X)

2 **Prove that :** $\overline{XZ}$ // $\overline{YL}$



5 **[a]** ABCD is a parallelogram in which AC = BC

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle circumscribed about the triangle ABC

[b] In the opposite figure :

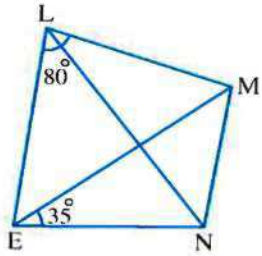
LMNE is a cyclic quadrilateral , m (∠ MEN) = 35°

, m (∠ MLE) = 80°

Find with proof :

1 m (∠ MLN)

2 m (∠ EMN)



13 Kafr El-Sheikh Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The triangle contains two .... angles at least.

(a) acute (b) obtuse (c) right (d) reflex

2 ABCD is a rhombus in which m (∠ ACB) = 32° , then m (∠ D) = ...

(a) 32° (b) 64° (c) 116° (d) 26°

3 A tangent to a circle of diameter length 6 cm. is at a distance of .... cm. from its center.

(a) 6 (b) 12 (c) 3 (d) 2

4 If M , N are two touching circles internally their radii lengths are 8 cm. , 3 cm. , then MN = ... cm.

(a) 3 (b) 5 (c) 7 (d) 11

5 The triangle whose side lengths are 5 cm. , 7 cm. and 8 cm. is .... triangle.

(a) obtuse-angled. (b) acute-angled. (c) right-angled. (d) equilateral.

6 The number of common tangents to two touching circles externally is ....

(a) 0 (b) 1 (c) 2 (d) 3

2 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{BC}$ are two chords in the circle M

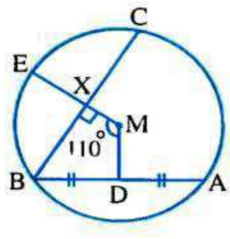
, which has radius length of 10 cm.

, $\overrightarrow{MX} \perp \overline{BC}$ intersecting $\overline{BC}$ at X and intersecting the circle M at E

, D is the midpoint of $\overline{AB}$, BC = 16 cm.

, m (∠ DMX) = 110°

Find : 1 The length of $\overline{XE}$ 2 m (∠ ABC)



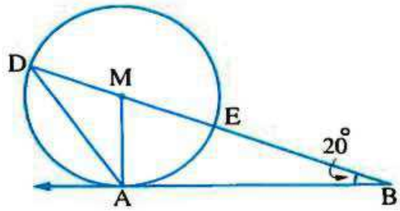
[b] In the opposite figure :

B is a point outside the circle M

, $\overrightarrow{BA}$ is a tangent to the circle M at A

, $\overrightarrow{BM}$ intersects the circle at E and D , m (∠ B) = 20°

Find with proof : m (∠ ADB)

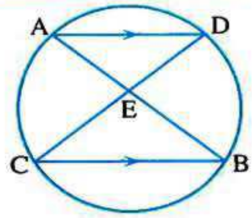


3 **[a] In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{E\}$

, $\overline{AD} \,//\, \overline{CB}$

Prove that : EA = ED



[b] In the opposite figure :

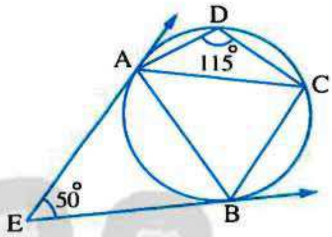
$\overrightarrow{EA}$ and $\overrightarrow{EB}$ are two tangents to the circle at A , B

, m (∠ AEB) = 50°

, m (∠ ADC) = 115°

Prove that :

$\overleftrightarrow{AC}$ is a tangent to the circle passing through the points A , B and E

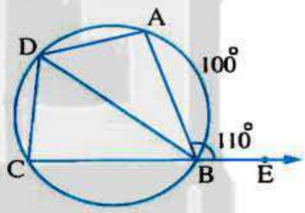


4 **[a] In the opposite figure :**

E $\in \overrightarrow{CB}$, m $(\widehat{AB})$ = 100°

, m (∠ ABE) = 110°

Find with proof : m (∠ BDC)



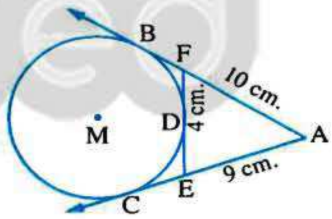
[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M at B , C

, $\overline{FE}$ is a tangent-segment at D , DF = 4 cm.

, AF = 10 cm. , AE = 9 cm.

Find with proof : The length of $\overline{EC}$



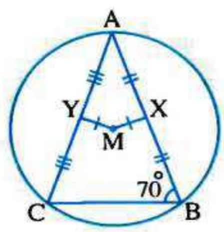
5 **[a] In the opposite figure :**

ABC is an inscribed triangle inside the circle M

, MX = MY , X and Y are the midpoints of $\overline{AB}$

, $\overline{AC}$ respectively , m (∠ B) = 70°

Find with proof : m (∠ A)



[b] ABC is an inscribed triangle in a circle where AB > AC and D $\in \overline{AB}$ where AC = AD , $\overrightarrow{AE}$ bisects ∠ A and intersects $\overline{BC}$ at E and intersects the circle at F

Prove that : BDEF is a cyclic quadrilateral.

14 El-Beheira Governorate

Answer the following questions : ***(Calculator is permitted)***

1 **Choose the correct answer from the given ones :**

1 M and N are two intersecting circles , their radii lengths are 3 cm. and 5 cm. , then MN ∈ ....

(a)]8 , ∞[(b)]2 , ∞[(c)]0 , 2[(d)]2 , 8[

2 ABCD is a cyclic quadrilateral , m (∠ A) = 70° , then m (∠ C) equals ....

(a) 25° (b) 20° (c) 110° (d) 100°

3 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 130° (b) 90° (c) 50° (d) 180°

4 The slope of the straight line $3x + 2y = 1$ is ....

(a) $\frac{2}{3}$ (b) $-\frac{3}{2}$ (c) $-\frac{2}{3}$ (d) $\frac{3}{2}$

5 The measurement of any angle of the regular hexagon is ....

(a) 90° (b) 108° (c) 120° (d) 135°

6 In Δ ABC , if $(AB)^2 = (AC)^2 + (BC)^2$, then ∠ B is ....

(a) acute. (b) obtuse. (c) right. (d) reflex.

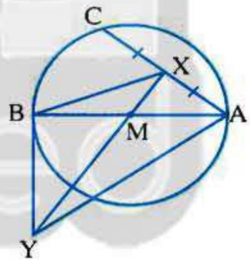
2 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, X is the midpoint of $\overline{AC}$ and $\overrightarrow{XM}$ intersects

the tangent to the circle at B in Y

Prove that : The figure AXBY is a cyclic quadrilateral.



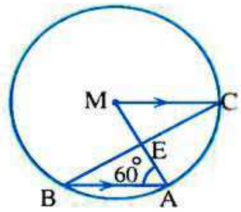
[b] In the opposite figure :

$\overline{AB}$ is a chord in the circle M

, $\overline{CM}$ // $\overline{AB}$, $\overline{BC} \cap \overline{AM}$ = {E}

, m (∠ A) = 60°

Find : m (∠ B)



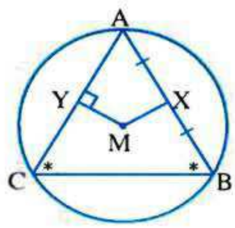
3 **[a] In the opposite figure :**

The triangle ABC is inscribed in the circle M

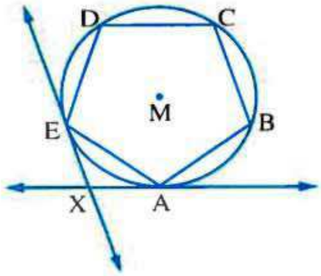
, in which : m (∠ B) = m (∠ C)

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY



[b] In the opposite figure :



ABCDE is a regular pentagon inscribed in a circle M

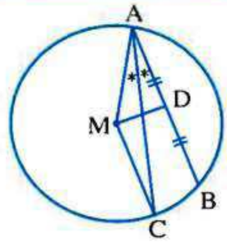
, $\overleftrightarrow{AX}$ is a tangent to the circle at A

, $\overleftrightarrow{EX}$ is a tangent to the circle at E

where $\overleftrightarrow{AX} \cap \overleftrightarrow{EX} = \{X\}$

Find : 1 m $(\widehat{AE})$ 2 m (∠ AXE)

4 **[a] In the opposite figure :**



$\overline{AB}$ is a chord in the circle M

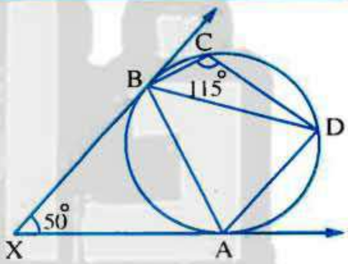
, $\overrightarrow{AC}$ bisects ∠ BAM and intersects the circle M at C

If D is the midpoint of $\overline{AB}$

, prove that : $\overline{DM} \perp \overline{CM}$

[b] $\overline{AB}$ is a diameter in the circle M , $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are two tangents to the circle M , $\overrightarrow{CM}$ intersects the circle M at X and Y and intersects $\overleftrightarrow{BD}$ at E **Prove that :** CX = YE

5 **[a] In the opposite figure :**



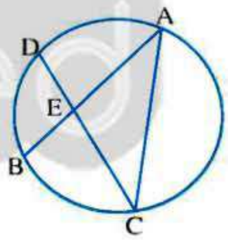
$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

, m (∠ AXB) = 50° , m (∠ DCB) = 115°

Prove that : 1 $\overrightarrow{AB}$ bisects ∠ DAX

2 BD = BA

[b] In the opposite figure :



$\overline{AB}$ and $\overline{CD}$ are two equal chords in length in the circle

, $\overline{AB} \cap \overline{CD} = \{E\}$

Prove that : The triangle ACE is an isosceles triangle.

15 El-Fayoum Governorate

Answer the following questions : (Using calculators is allowed)

1 **Choose the correct answer :**

1 If M is a circle of diameter length 8 cm. , the straight line L is far from the centre M of the circle by 4 cm. , then the straight line L is ....

(a) a secant to the circle in two points. (b) outside the circle.

(c) a tangent to the circle. (d) an axis of symmetry of the circle.

2 If m_1 , m_2 are the slopes of two perpendicular straight lines , then ....

(a) $m_1 = m_2$ (b) $m_1 \times m_2 = -1$ (c) $m_1 \times m_2 = 1$ (d) $m_1 + m_2 = -1$

3 The centre of the circle that passes through the vertices of the triangle is the intersection point of ....

(a) the bisectors of its interior angles. (b) the bisectors of its exterior angles.

(c) its altitudes. (d) the axes of its sides.

4 ABC is a right-angled triangle at B , m (∠ C) = 30° , AC = 12 cm. , then AB = ... cm.

(a) 24 (b) $12\sqrt{3}$ (c) $6\sqrt{3}$ (d) 6

5 Which of the following figures is a cyclic quadrilateral ?

(a) The rectangle. (b) The trapezium. (c) The rhombus. (d) The parallelogram.

6 A trapezium in which the lengths of the two parallel bases are 4 cm. and 12 cm. and its height is 9 cm. , then its area = ... cm^2.

(a) 25 (b) 36 (c) 72 (d) 144

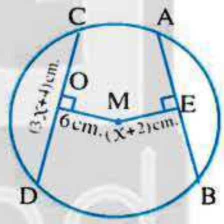
2 [a] **In the opposite figure :**

AB = CD , MO = 6 cm.

, ME = $(X + 2)$ cm.

, CD = $(3X + 4)$ cm.

Find : The value of X , CD



[b] ABC is a triangle drawn inside a circle M , m (∠ AMB) = 90° , m (∠ BMC) = 130°

Find : The measures of the angles of Δ ABC

3 [a] A is a point outside the circle M , $\overrightarrow{AB}$ is a tangent to the circle at B , $\overrightarrow{AM}$ intersects the circle M at C and D respectively , m (∠ A) = 40°

Find with proof : m (∠ BDC)

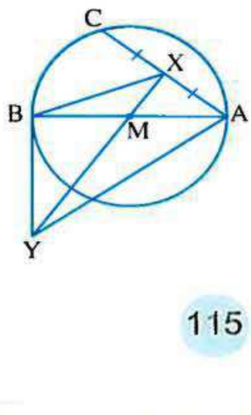
[b] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, X is the midpoint of $\overline{AC}$

and $\overrightarrow{XM}$ intersects the tangent to the circle at B at Y

Prove that : The figure AXBY is a cyclic quadrilateral.



4 **[a] In the opposite figure :**

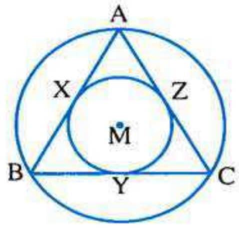
Two concentric circles with centre M

, the radii lengths of them are 4 cm. and 2 cm.

, Δ ABC is an inscribed triangle inside the greater circle

, and its sides touch the smaller circle at X , Y , Z

Prove that : Δ ABC is an equilateral triangle , and calculate its area.



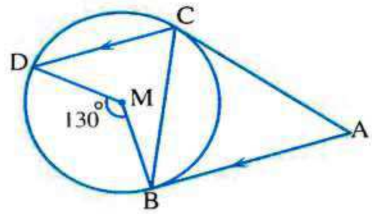
[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB}$ // $\overline{CD}$

, m (∠ BMD) = 130°

Prove that : $\overrightarrow{CB}$ bisects ∠ ACD



5 **[a]** Δ ABC is a triangle inscribed in a circle , $\overleftrightarrow{AD}$ is a tangent to the circle at A , X ∈ $\overline{AB}$ and Y ∈ $\overline{AC}$, where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

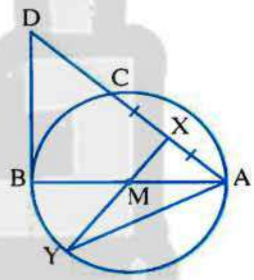
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M ,

X is the midpoint of $\overline{AC}$, $\overrightarrow{BD}$ is a tangent to the circle at B , $\overrightarrow{XM}$ intersects the circle at Y

Prove that : **1** XMBD is a cyclic quadrilateral.

2 m (∠ BAY) = $\frac{1}{2}$ m (∠ D)



16 Beni Suef Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The symmetry axis of the common chord $\overline{AB}$ of the two intersecting circles M , N is ....

(a) $\overleftrightarrow{MA}$ (b) $\overleftrightarrow{MB}$ (c) $\overleftrightarrow{MN}$ (d) $\overleftrightarrow{NA}$

2 ABC is a triangle in which : $(AC)^2 > (AB)^2 + (BC)^2$, then ∠ B is ....

(a) acute. (b) obtuse. (c) right. (d) straight.

3 In the cyclic quadrilateral , each two opposite angles are ....

(a) equal in measure. (b) complementary.

(c) supplementary. (d) alternate.

4 The area of a triangle is 35 cm^2 and its height is 7 cm. , then the length of its base equals .... cm.

(a) 5 (b) 7 (c) 10 (d) 20

5 The measure of the inscribed angle which is drawn in a semicircle equals ....

(a) 45° (b) 90° (c) 120° (d) 180°

6 The area of a square is 100 cm^2 , then its perimeter = ... cm.

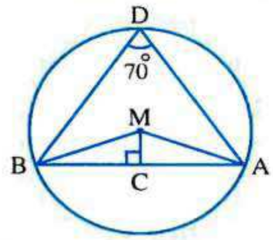
(a) 10 (b) 30 (c) 40 (d) 50

2 [a] **In the opposite figure :**

$\overline{AB}$ is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$, m (∠ ADB) = 70°

Find : m (∠ AMC)

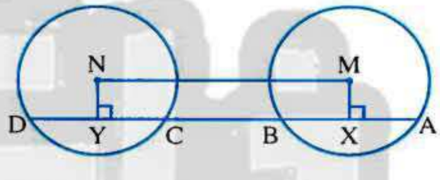


[b] **In the opposite figure :**

M and N are two congruent circles

, AB = CD , $\overline{MX} \perp \overline{AB}$ and $\overline{NY} \perp \overline{CD}$

Prove that : The figure MXYN is a rectangle.



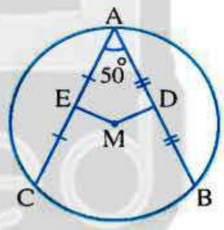
3 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords

in the circle M , D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$ and m (∠ BAC) = 50°

Find : m (∠ DME)



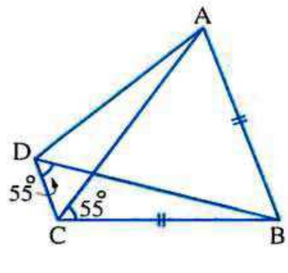
[b] **In the opposite figure :**

AB = BC

, m (∠ ACB) = 55°

and m (∠ BDC) = 55°

Prove that : The figure ABCD is a cyclic quadrilateral.

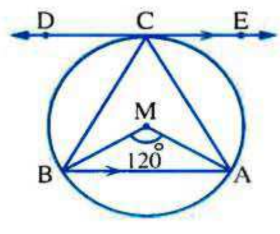


4 [a] **In the opposite figure :**

$\overleftrightarrow{ED}$ is a tangent to the circle M at C

, $\overleftrightarrow{ED}$ // $\overline{AB}$ and m (∠ AMB) = 120°

Prove that : The triangle CAB is an equilateral triangle.

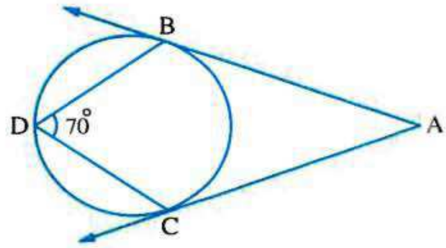


[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ BDC) = 70°

Find : m (∠ A)

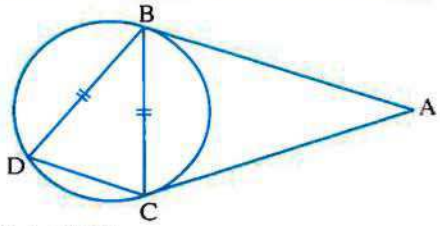


5 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C , BC = BD

Prove that :

$\overleftrightarrow{BD}$ is a tangent to the circle passing through the vertices of Δ ABC

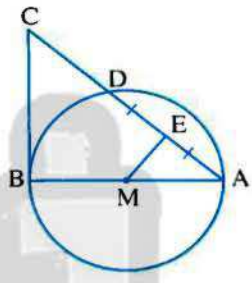


[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overleftrightarrow{BC}$ is a tangent to the circle at B and E is the midpoint of $\overline{AD}$

Prove that : The figure EMBC is a cyclic quadrilateral.



17 El-Menia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 It is possible to draw a circle passing through the vertices of a ....

(a) rhombus. (b) rectangle. (c) right trapezium. (d) parallelogram.

2 The inscribed angle drawn in a semicircle is ....

(a) acute. (b) obtuse. (c) straight. (d) right.

3 The number of rectangles in the opposite figure is ....

(a) 3 (b) 6 (c) 7 (d) 10

4 If the perimeter of a square is 20 cm. , then its surface area is .... cm^2.

(a) 20 (b) 25 (c) 50 (d) 100

5 The measure of the exterior angle of an equilateral triangle equals .... °

(a) 60 (b) 108 (c) 120 (d) 135

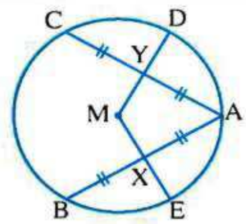
6 If ABCD is a cyclic quadrilateral , 2 m (∠ A) = 120° , then m (∠ C) = …………… °

(a) 120 (b) 45 (c) 60 (d) 90

2 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$ and Y is the midpoint of $\overline{AC}$

Prove that : XE = YD



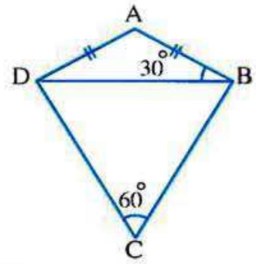
[b] **In the opposite figure :**

ABCD is a quadrilateral , AB = AD

, m (∠ ABD) = 30°

, m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.



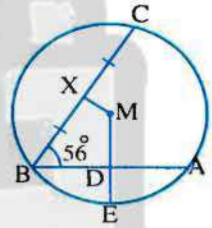
3 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{BC}$ are two chords in the circle M which has radius length of 5 cm. , $\overline{MD} \perp \overline{AB}$

, X is the midpoint of $\overline{BC}$

, AB = 8 cm. , m (∠ B) = 56°

Find : 1 m (∠ DMX) 2 The length of $\overline{DE}$



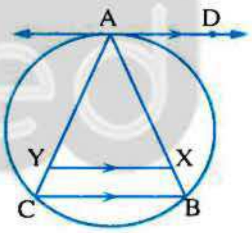
[b] **In the opposite figure :**

$\overleftrightarrow{AD}$ is a tangent to the circle at A

, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

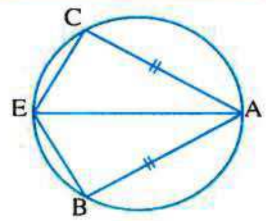


4 [a] **In the opposite figure :**

AB = AC

, E ∈ $\overset{\frown}{BC}$

Prove that : m (∠ AEB) = m (∠ AEC)



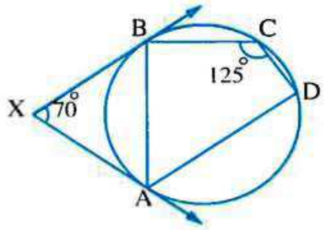
[b] **In the opposite figure :**

$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

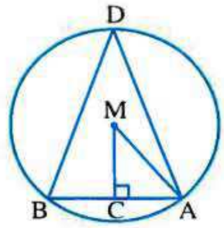
, m (∠ AXB) = 70°

, m (∠ DCB) = 125°

Prove that : m (∠ DAB) = m (∠ XAB)



5 **[a] In the opposite figure :**



$\overline{AB}$ is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$

Prove that : m (∠ AMC) = m (∠ ADB)

[b] ABC is an inscribed triangle in a circle M where AB > AC and D ∈ $\overline{AB}$ where AC = AD , $\overrightarrow{AE}$ bisects ∠ A and intersects $\overline{BC}$ at E and intersects the circle at F

Prove that : BDEF is a cyclic quadrilateral.

18 Assiut Governorate

Answer the following questions : (Calculator is permitted)

1 **Choose the correct answer :**

1. XYZ is a triangle in which : D is the midpoint of $\overline{XY}$, E is the midpoint of $\overline{XZ}$, then DE = …………… YZ

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 2

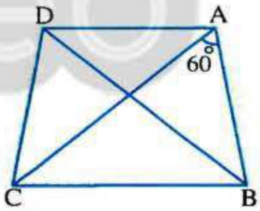
2. The diameter is a …………… passing through the center of the circle.

(a) straight line (b) ray (c) tangent (d) chord

3. If the circumference of a circle is 18 π cm. , then its radius length = …………… cm.

(a) 7 (b) 9 (c) 3 (d) 6

4. **In the opposite figure :**



ABCD is a cyclic quadrilateral

, m (∠ BAC) = 60°

, then m (∠ BDC) = ……………

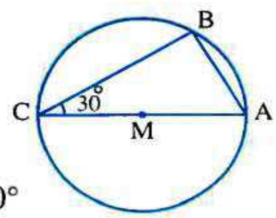
(a) 300° (b) 120°

(c) 60° (d) 30°

5. The area of the triangle which the length of its base is 9 cm. , its height is 12 cm. equals …………… cm^2.

(a) 48 (b) 24 (c) 36 (d) 54

6. **In the opposite figure :**



$\overline{AC}$ is a diameter in the circle M

, m (∠ C) = 30°

, then m (∠ A) = ……………

(a) 120° (b) 60° (c) 90° (d) 40°

2 [a] **In the opposite figure :**

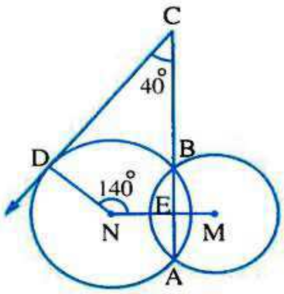
M and N are two intersecting circles at A and B

, $C \in \overleftrightarrow{AB}$, $\overrightarrow{AC} \cap \overline{MN} = \{E\}$

, D $\in$ the circle N , m ($\angle$ DNM) = 140°

and m ($\angle$ C) = 40°

Prove that : $\overrightarrow{CD}$ is a tangent to the circle N at D



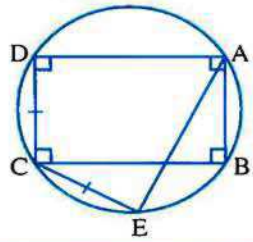
[b] **In the opposite figure :**

ABCD is a rectangle inscribed in a circle

, the chord $\overline{CE}$ is drawn

where CE = CD

Prove that : AE = BC



3 [a] State two cases of the cyclic quadrilateral.

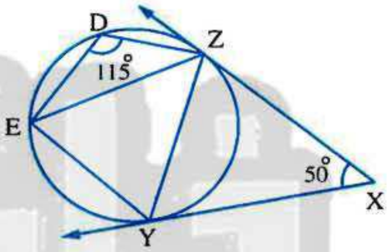
[b] **In the opposite figure :**

$\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle at Y , Z

, m ($\angle$ D) = 115°

and m ($\angle$ X) = 50°

Prove that : ZE = ZY



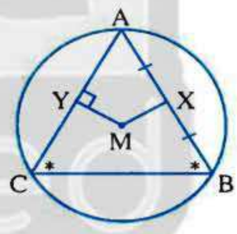
4 [a] **In the opposite figure :**

ABC is a triangle inscribed in the circle M

, in which m ($\angle$ B) = m ($\angle$ C)

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

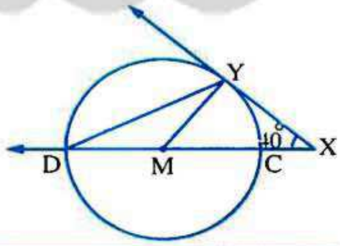
Prove that : MX = MY



[b] **In the opposite figure :**

X is a point outside the circle M , $\overrightarrow{XY}$ is a tangent to the circle at Y , $\overrightarrow{XM}$ intersects the circle M at C and D respectively , m ($\angle$ X) = 40°

Find : m ($\angle$ YDC)



5 [a] **In the opposite figure :**

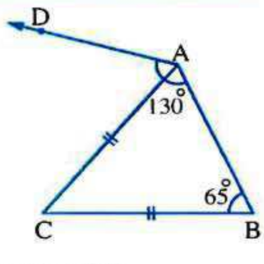
ABC is a triangle , CB = AC

, m ($\angle$ DAB) = 130°

, m ($\angle$ B) = 65°

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC



[b] In the opposite figure :

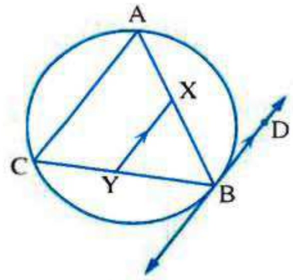
ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, X $\in \overline{AB}$, Y $\in \overline{BC}$

where $\overline{XY}$ // $\overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.



19 Souhag Governorate

Answer the following questions : ***(Calculator is permitted)***

1 Choose the correct answer :

1 If the straight line L is a tangent to the circle M of diameter length 8 cm. , then the distance between L and the center of the circle equals .... cm.

(a) 3 (b) 4 (c) 6 (d) 8

2 The area of the rhombus = ... of the product of the lengths of its diagonals.

(a) 2 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 3

3 The number of symmetry axes of any circle is ....

(a) zero (b) 1 (c) 2 (d) an infinite number.

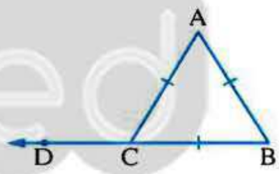
4 **In the opposite figure :**

The triangle ABC is an equilateral triangle

, then m (∠ ACD) = ...°

(a) 45 (b) 60

(c) 120 (d) 135



5 If the lengths of two sides of an isosceles triangle are 2 cm. and (x + 3) cm. , and the length of the third side is 5 cm. , then x = ... cm.

(a) 1 (b) 2 (c) 3 (d) 4

6 If M , N are two touching circles internally , their radii lengths are 5 cm. , 9 cm. , then MN = ... cm.

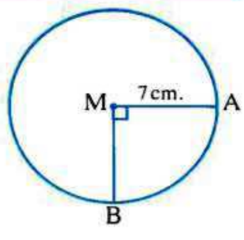
(a) 14 (b) 4 (c) 5 (d) 9

2 [a] In the opposite figure :

M is a circle with radius length 7 cm.

, m (∠ AMB) = 90°

Find : The length of $\widehat{AB}$ $\left(\pi = \frac{22}{7}\right)$



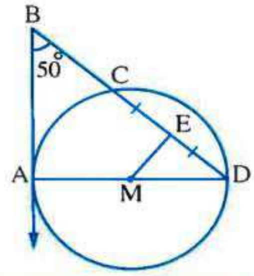
[b] In the opposite figure :

$\overline{AD}$ is a diameter in the circle M

, $\overleftrightarrow{AB}$ is a tangent , m (∠ B) = 50°

, E is the midpoint of $\overline{DC}$

Find : m (∠ EMA)



3 [a] State two cases of the cyclic quadrilateral.

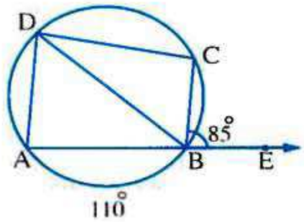
[b] In the opposite figure :

E ∈ $\overrightarrow{AB}$, E ∉ $\overline{AB}$

, m $(\widehat{AB})$ = 110°

, m (∠ CBE) = 85°

Find : m (∠ BDC)

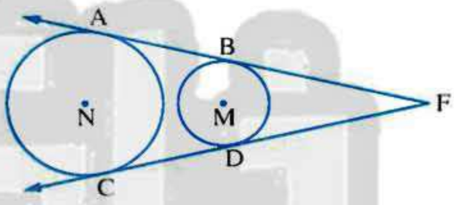


4 **[a] In the opposite figure :**

$\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ are common external tangents

to the two circles M and N , $\overrightarrow{AB} \cap \overrightarrow{CD} = \{F\}$

Prove that : AB = CD



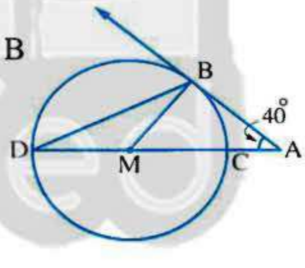
[b] In the opposite figure :

A is a point outside the circle M , $\overrightarrow{AB}$ is a tangent to the circle at B

, $\overrightarrow{AM}$ intersects the circle M at C and D respectively

, m (∠ A) = 40°

Find with proof : m (∠ BDC)

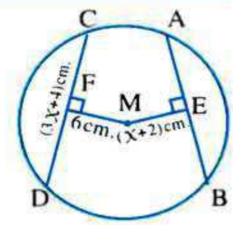


5 **[a] In the opposite figure :**

AB = CD

Find : 1 The value of x

2 The length of $\overline{CD}$



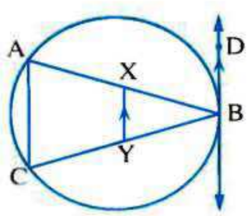
[b] In the opposite figure :

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B , X ∈ $\overline{AB}$

, Y ∈ $\overline{CB}$ where $\overline{YX}$ // $\overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.



20 Qena Governorate

Answer the following questions : *(Calculators are permitted)*

1 Choose the correct answer :

1 The measure of the inscribed angle in a semicircle is .... °

(a) 45 (b) 90 (c) 135 (d) 180

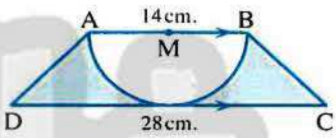
2 The perimeter of a rhombus is 12 cm. , then the length of its side = ... cm.

(a) 3 (b) 4 (c) 6 (d) 8

3 If A and B are two points in the plane , AB = 7 cm. , then the length of the diameter of the smallest circle passing through the two points A and B equals .... cm.

(a) 3 (b) 3.5 (c) 7 (d) 14

4 **In the opposite figure :**



$\overline{AB}$ is a diameter of the circle M , $\overline{CD}$ is a tangent

, AB = 14 cm. , CD = 28 cm.

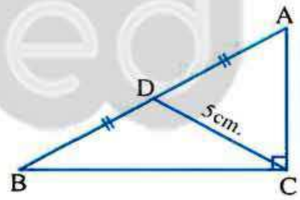
, then the area of the shaded part = ... cm^2

(a) 70 (b) 147 (c) 170 (d) 224

5 It is possible to draw a circle passing through the vertices of a ....

(a) rhombus. (b) rectangle. (c) trapezium. (d) parallelogram.

6 **In the opposite figure :**



Δ ABC is right-angled at C

, $\overline{CD}$ is a median , CD = 5 cm.

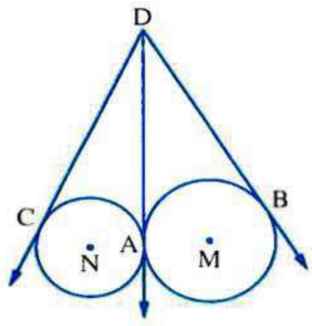
, then AB = ... cm.

(a) 4 (b) 6

(c) 8 (d) 10

2 **[a]** Find the length of the arc and its measure , which is opposite to an inscribed angle of measure 45° in a circle the length of its radius is 7 cm.

[b] In the opposite figure :



M and N are two circles touching externally at A

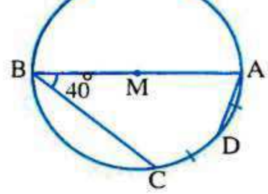
, $\overrightarrow{DA}$ is a common tangent to the circles

, $\overrightarrow{DB}$ is a tangent to the circle M at B

, $\overrightarrow{DC}$ is a tangent to the circle N at C

Prove that : DB = DC

3 **[a] In the opposite figure :**



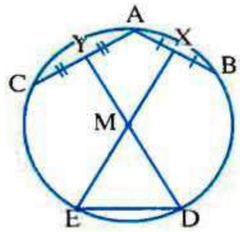
$\overline{AB}$ is a diameter of the circle M

, D is the midpoint of $\widehat{AC}$

, m (∠ ABC) = 40°

Find : 1 m (∠ DAB) 2 m (∠ DCB)

[b] In the opposite figure :



$\overline{AB}$, $\overline{AC}$ are two chords in the circle M

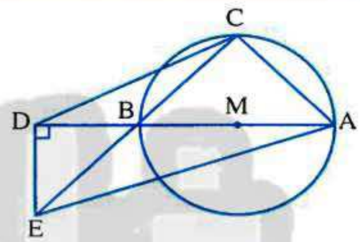
, X and Y are the two midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, $\overrightarrow{YM}$ and $\overrightarrow{XM}$ intersect the circle at D and E

If DE = r where r is the radius length of M

, find by proof : m (∠ BAC)

4 **[a] In the opposite figure :**



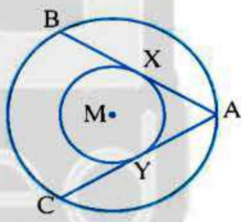
$\overline{AB}$ is a diameter in the circle M

, D ∈ $\overrightarrow{AB}$, D ∉ $\overline{AB}$, $\overrightarrow{DE}$ ⊥ $\overrightarrow{AB}$

, C ∈ $\widehat{AB}$, $\overrightarrow{CB}$ ∩ $\overrightarrow{DE}$ = {E}

Prove that : ACDE is a cyclic quadrilateral

[b] In the opposite figure :

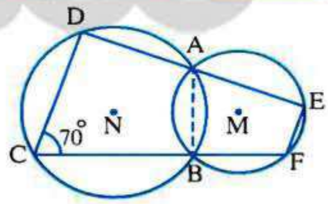


Two concentric circles of center M

, $\overline{AB}$ and $\overline{AC}$ are two chords in the greater circle and tangents to the smaller circle at X and Y respectively.

Prove that : AB = AC

5 **[a] In the opposite figure :**

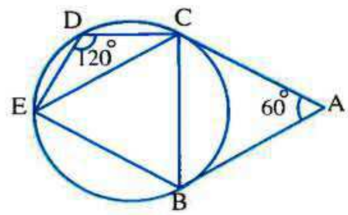


M and N are two intersecting circles at A and B

, $\overleftrightarrow{AD}$ is drawn to intersect the circle M at E and the circle N at D , $\overleftrightarrow{AB}$ is drawn to intersect the circle M at F and the circle N at C , m (∠ BCD) = 70°

1 **Find :** m (∠ EFB) 2 **Prove that :** $\overline{CD}$ // $\overline{EF}$

[b] In the opposite figure :



$\overline{AB}$ and $\overline{AC}$ are tangent-segments to the circle at B and C

, m (∠ BAC) = 60° , m (∠ CDE) = 120°

Prove that : 1 Δ BCE is an equilateral triangle.

2 $\overline{AC}$ // $\overline{BE}$

21 Luxor Governorate

محافظة الأقصر

Answer the following questions :

1 Choose the correct answer :

1 The number of axes of symmetry of the rectangle is ....

(a) 1 (b) 2 (c) 3 (d) 4

2 If M , N are two circles whose radii lengths are r_1 , r_2 and if $r_1 - r_2 < MN < r_1 + r_2$, then the two circles are ....

(a) distant. (b) concentric. (c) intersecting. (d) touching.

3 The length of the median drawn from the vertex of the right angle in the right-angled triangle equals .... the length of the hypotenuse.

(a) quarter (b) twice (c) half (d) three quarters

4 The length of the arc subtending a central angle of measure 60° in a circle whose circumference is 24 cm. equals .... cm.

(a) 4 (b) 8 (c) 12 (d) 16

5 The measure of the exterior angle of the equilateral triangle is ....°

(a) 30 (b) 60 (c) 90 (d) 120

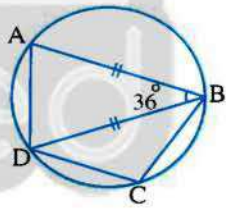
6 **In the opposite figure :**

AB = BD , m (∠ ABD) = 36°

, then m (∠ C) = ... °

(a) 140 (b) 108

(c) 70 (d) 54



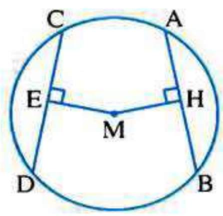
2 [a] In the opposite figure :

AB = CD , $\overline{MH} \perp \overline{AB}$, $\overline{ME} \perp \overline{CD}$

If ME = 6 cm. , MH = (x + 2) cm.

and CD = (3 x + 4) cm.

, find : The value of x and the length of $\overline{AB}$

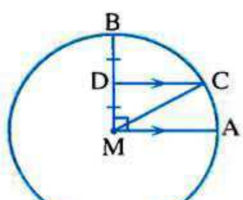


[b] In the opposite figure :

$\overline{AM}$ // $\overline{CD}$

, MD = DB , m (∠ AMB) = 90°

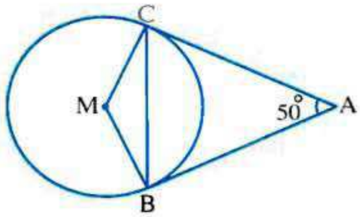
Find : m $(\overset{\frown}{AC})$



3 **[a] In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two tangent segments drawn to the circle from A at B , C respectively , m (∠ A) = 50°

Find : m (∠ ACB) , m (∠ BCM)



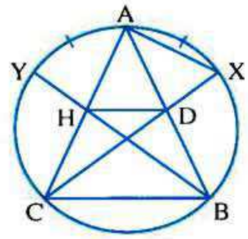
[b] In the opposite figure :

m ($\overset{\frown}{AX}$) = m ($\overset{\frown}{AY}$)

Prove that :

1 DBCH is a cyclic quadrilateral.

2 m (∠ DHB) = m (∠ XAB)



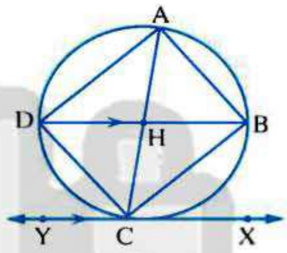
4 **[a]** Draw $\overline{AB}$ of length 3 cm. , then draw a circle passing by the two points A , B whose radius length is 2 cm. How many possible solutions are there ?

[b] In the opposite figure :

$\overline{BD}$ // $\overleftrightarrow{XY}$

Prove that : 1 $\overrightarrow{AC}$ bisects ∠ BAD

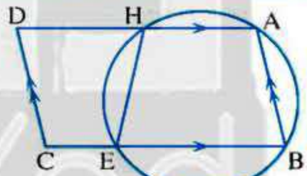
2 $\overleftrightarrow{BC}$ is a tangent to the circle passing by the vertices of Δ ABH



5 **[a] In the opposite figure :**

ABCD is a parallelogram

Prove that : HDCE is a cyclic quadrilateral

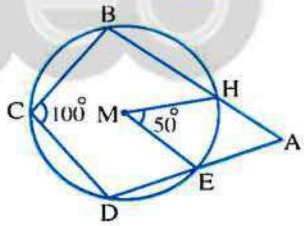


[b] In the opposite figure :

m (∠ M) = 50°

, m (∠ C) = 100°

Find : m (∠ A)



22 Aswan Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer from those given :**

1 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 45° (b) 180° (c) 120° (d) 90°

2 The number of symmetry axes of the isosceles triangle is ....

(a) zero (b) 1 (c) 2 (d) 3

3 The surface of the circle M $\cap$ the surface of the circle N = $\{A\}$ and the radius length of one of them is 3 cm. and MN = 8 cm. , then the radius length of the other circle equals .... cm.

(a) 5 (b) 6 (c) 11 (d) 16

4 The measure of the exterior angle of the equilateral triangle equals ....

(a) 30° (b) 60° (c) 120° (d) 180°

5 The line segment joining the two midpoints of two sides of the triangle is .... the third side.

(a) perpendicular to (b) parallel to (c) bisecting (d) equal to

6 If ABCD is a cyclic quadrilateral , then m (∠ A) + m (∠ C) – 80° = ...

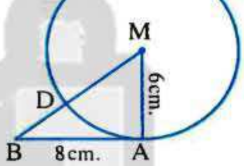
(a) 60° (b) 80° (c) 100° (d) 120°

2 [a] **In the opposite figure :**

$\overleftrightarrow{AB}$ is a tangent to the circle M at A

, MA = 6 cm. , AB = 8 cm.

Find : The length of $\overline{BD}$



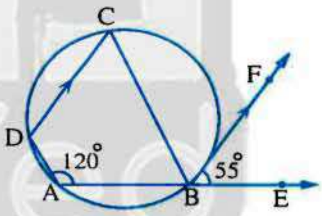
[b] **In the opposite figure :**

ABCD is a cyclic quadrilateral

, $\overrightarrow{BF}$ // $\overline{DC}$, m (∠ BAD) = 120°

, m (∠ EBF) = 55°

Find : m (∠ BCD) , m (∠ ADC)



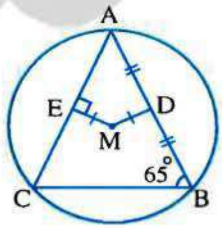
3 [a] **In the opposite figure :**

In the circle M

, MD = ME , D is the midpoint of $\overline{AB}$

, $\overline{ME} \perp \overline{AC}$, m (∠ ABC) = 65°

Find : m (∠ BAC)

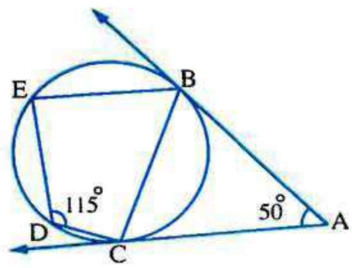


[b] **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 50° , m (∠ CDE) = 115°

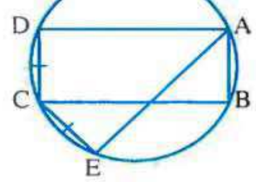
Prove that : $\overrightarrow{BC}$ bisects ∠ ABE



4 **[a] In the opposite figure :**

ABCD is a rectangle inscribed in a circle , the chord $\overline{CE}$ is drawn where CE = CD

Prove that : AE = BC

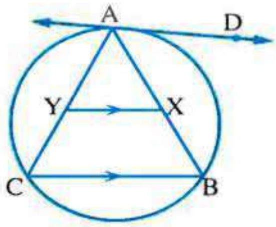


[b] In the opposite figure :

ABC is a triangle inscribed in a circle , $\overleftrightarrow{AD}$ is a tangent to the circle at A , X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$, $\overline{XY}$ // $\overline{BC}$

Prove that :

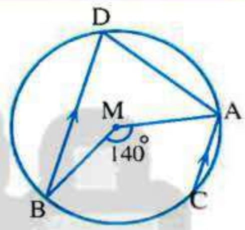
$\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ AXY



5 **[a] In the opposite figure :**

$\overline{AC}$, $\overline{DB}$ are two parallel chords in the circle M , m (∠ AMB) = 140°

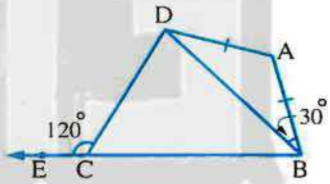
Find : m (∠ D) , m (∠ DAC)



[b] In the opposite figure :

AB = AD , m (∠ ABD) = 30° , m (∠ DCE) = 120°

Prove that : ABCD is a cyclic quadrilateral.



23 New Valley Governorate

إيمان - عمل - تنمية

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 If two polygons are similar and the ratio between the lengths of two corresponding sides is 1 : 3 and the perimeter of the smaller polygon is 15 cm. , then the perimeter of the greater polygon is .... cm.

(a) 30 (b) 45 (c) 60 (d) 75

2 The inscribed angle drawn in a semicircle is ....

(a) acute. (b) obtuse. (c) straight. (d) right.

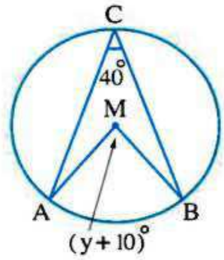
3 ABC is a right-angled triangle at B , $\overline{BD} \perp \overleftrightarrow{AC}$, then the projection of $\overline{BD}$ on $\overleftrightarrow{AC}$ is ....

(a) A (b) B (c) C (d) D

4 A tangent to a circle of diameter length 6 cm. is at a distance of .... cm. from its center.

(a) 6 (b) 12 (c) 3 (d) 2

5 **In the opposite figure :**



If m (∠ AMB) = (y + 10)°

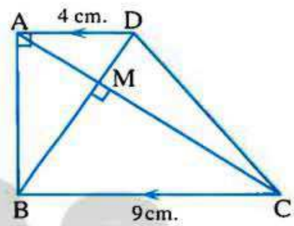
, m (∠ C) = 40°

, then y = ...

(a) 70° (b) 80°

(c) 100° (d) 180°

6 **In the opposite figure :**



$\overline{AD}$ // $\overline{BC}$, m (∠ BAD) = m (∠ BMC) = 90°

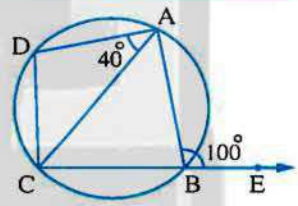
, AD = 4 cm. , BC = 9 cm.

, then the area of the trapezium ABCD = ... cm^2

(a) 26 (b) 39

(c) 52 (d) 65

2 [a] **In the opposite figure :**

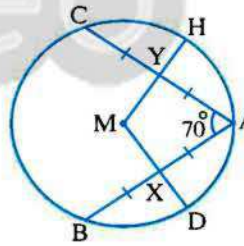


m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{AD}$)

[b] **In the opposite figure :**



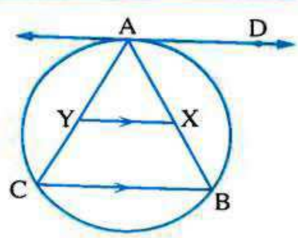
$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$, m (∠ CAB) = 70°

1 **Calculate :** m (∠ DMH)

2 **Prove that :** XD = YH

3 [a] **In the opposite figure :**



ABC is a triangle inscribed in a circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A

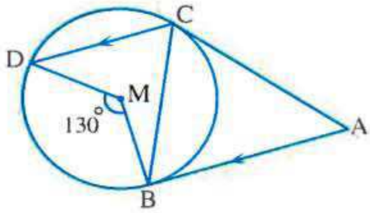
, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB}$ // $\overline{CD}$, m (∠ BMD) = 130°



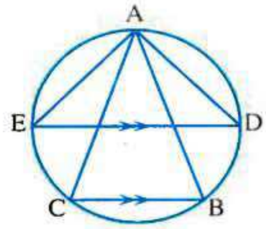
1 **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD

2 **Find :** m (∠ A)

4 **[a] In the opposite figure :**

ABC is an inscribed triangle inside a circle

, $\overline{DE}$ // $\overline{BC}$



Prove that : m (∠ DAC) = m (∠ BAE)

[b] ABC is a triangle inscribed in a circle , X ∈ $\widehat{AB}$, Y ∈ $\widehat{AC}$

where m ($\widehat{AX}$) = m ($\widehat{AY}$) , $\overline{CX}$ ∩ $\overline{AB}$ = {D} , $\overline{BY}$ ∩ $\overline{AC}$ = {E}

Prove that : 1 BCED is a cyclic quadrilateral.

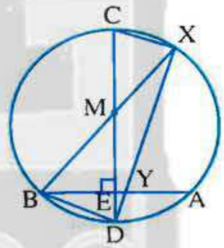
2 m (∠ DEB) = m (∠ XAB)

5 **[a]** State two cases of the cyclic quadrilateral.

[b] In the opposite figure :

$\overline{AB}$ is a chord in the circle M and $\overline{CD}$ is the perpendicular diameter on $\overline{AB}$ and intersects it at E

, $\overrightarrow{BM}$ intersects the circle at X and $\overline{XD}$ ∩ $\overline{AB}$ = {Y}



Prove that : 1 XYEC is a cyclic quadrilateral.

2 m (∠ DYB) = m (∠ DBX)

24 South Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 90° (b) 45° (c) 180° (d) 120°

2 A rhombus whose two diagonals lengths are 6 cm. , 8 cm. , then its area is .... cm^2

(a) 14 (b) 24 (c) 48 (d) 12

3 If ABCD is a cyclic quadrilateral , then m (∠ A) + m (∠ C) – 90° = ...

(a) 180° (b) 100° (c) 90° (d) 120°

4 In the triangle ABC , where $(AB)^2 + (BC)^2 < (AC)^2$, then $\angle$ B is ....

(a) right. (b) acute. (c) straight. (d) obtuse.

5 The sum of measures of the interior angles of the triangle equals ....

(a) 180° (b) 90° (c) 100° (d) 360°

6 The number of axes of symmery of the circle is ....

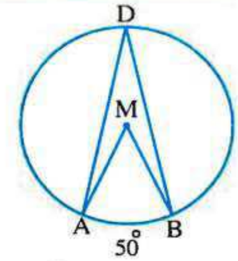
(a) zero (b) an inifinite number

(c) 2 (d) 3

2 **[a] In the opposite figure :**

$m(\widehat{AB}) = 50°$

Find : 1 $m(\angle D)$ 2 $m(\angle AMB)$

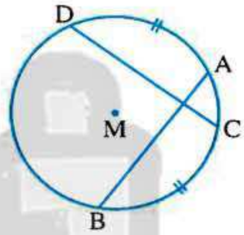


[b] In the opposite figure :

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $m(\widehat{AD}) = m(\widehat{BC})$

Prove that : AB = CD



3 **[a]** If the radius length of the circle M is 5 cm. and the radius length of the circle N is 3 cm. , MN = 8 cm. , show the position of the two circles.

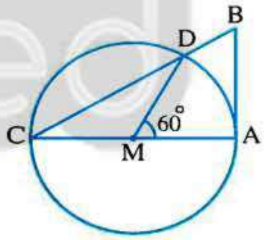
[b] In the opposite figure :

$\overline{AB}$ is a tangent-segment to the circle M

, $\overline{AC}$ is a diameter of it and $m(\angle AMD) = 60°$

1 **Find :** $m(\angle ABC)$

2 **Prove that :** $AB = \frac{1}{2}$ BC



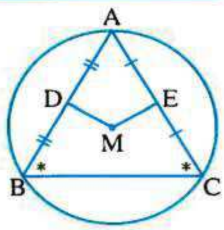
4 **[a] In the opposite figure :**

$m(\angle B) = m(\angle C)$

, D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

Prove that : MD = ME

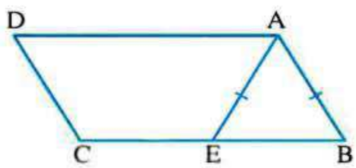


[b] In the opposite figure :

ABCD is a parallelogram

and E $\in \overline{BC}$, such that : AB = AE

Prove that : The figure AECD is a cyclic quadrilateral.



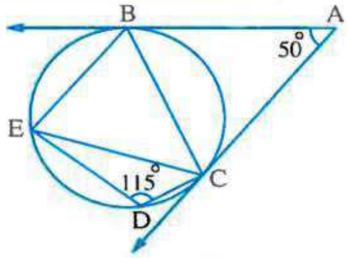
5 **[a] In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 50° , m (∠ EDC) = 115°

Prove that : **1** $\overrightarrow{BC}$ bisects ∠ ABE

2 CB = CE



[b] In the opposite figure :

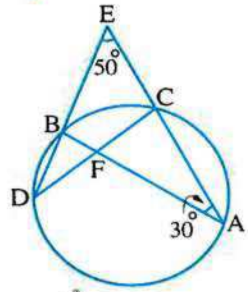
$\overline{AB} \cap \overline{CD} = \{F\}$, $\overrightarrow{AC} \cap \overrightarrow{DB} = \{E\}$

, m (∠ A) = 30°

, m (∠ E) = 50°

Find : **1** m ($\widehat{AD}$)

2 m (∠ AFD)



25 North Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If the surface of the circle M ⋂ the surface of the circle N = {A}

, then M , N are ....

(a) distant. (b) concentric. (c) touching externally. (d) intersecting.

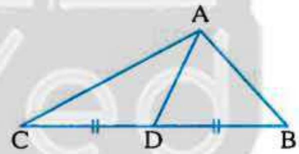
2 **In the opposite figure :**

$\overline{AD}$ is a median in the triangle ABC

, the area of the triangle ABD = 20 cm²

, then the area of the triangle ACD = ... cm²

(a) 20 (b) 40 (c) 60 (d) 80



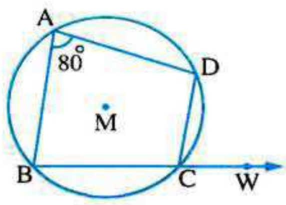
3 **In the opposite figure :**

If m (∠ BAD) = 80°

, then m (∠ DCW) = ...°

(a) 30 (b) 80

(c) 60 (d) 120



4 The area of the square whose diagonal length is 4 cm. equals .... cm²

(a) 4 (b) 8 (c) 16 (d) 16 π

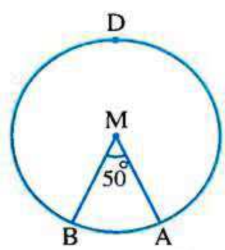
5 **In the opposite figure :**

m (∠ AMB) = 50°

, then m ($\widehat{ADB}$) = ...°

(a) 50 (b) 100

(c) 310 (d) 350



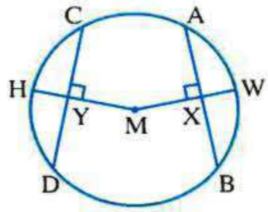
6 A triangle having one symmetry line and its side lengths are 8 , 4 , x cm.
, then x = ...

(a) 2 (b) 4 (c) 8 (d) 12

2 [a] **In the opposite figure :**

If AB = CD
, $\overline{MW} \perp \overline{AB}$
, $\overline{MH} \perp \overline{CD}$

Prove that : WX = HY

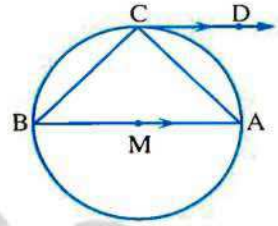


[b] **In the opposite figure :**

$\overrightarrow{CD}$ is a tangent to the circle M at C
, $\overrightarrow{CD}$ // $\overrightarrow{BA}$ and M $\in \overline{AB}$

1 **Prove that :** AC = BC

2 **Find :** m (∠ B)



3 [a] State two cases in which the figure is a cyclic quadrilateral.

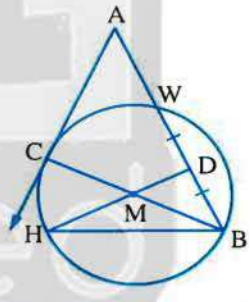
[b] **In the opposite figure :**

$\overline{BC}$ is a diameter in the circle M
, $\overrightarrow{AC}$ is a tangent to the circle M at C
, D is the midpoint of $\overline{BW}$

Prove that :

1 The figure ADMC is a cyclic quadrilateral.

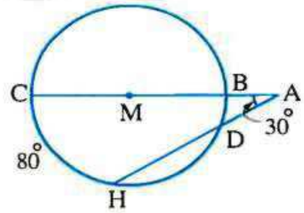
2 m (∠ CBH) = $\frac{1}{2}$ m (∠ A)



4 [a] **In the opposite figure :**

$\overline{BC}$ is a diameter in the circle M
, $\overrightarrow{CA} \cap \overrightarrow{HA} = \{A\}$, m (∠ A) = 30°
and m ($\overset{\frown}{CH}$) = 80°

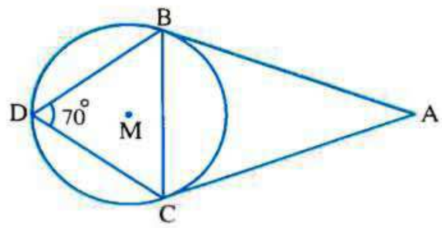
Find : m ($\overset{\frown}{DH}$)



[b] **In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle at B and C
and m (∠ BDC) = 70°

Find : m (∠ BAC)

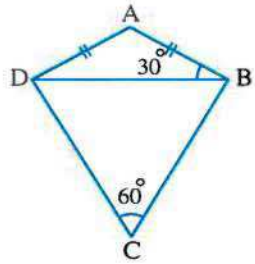


5 [a] **In the opposite figure :**

AB = AD , m (∠ ABD) = 30°

and m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.



[b] By using geometric instruments , draw Δ ABC where

AB = 3 cm. , BC = 4 cm. , AC = 5 cm. , then draw a circle passing through the vertices of Δ ABC

How many circles are there ?

26 Red Sea Governorate

Answer the following questions :

1 **Choose the correct answer from the given answers :**

1 The angle of tangency is included between ....

(a) two chords. (b) two tangents.

(c) a chord and a tangent. (d) a chord and a diameter.

2 The number of symmetry axes of the semicircle is ....

(a) zero (b) 1 (c) 3 (d) an infinite number.

3 A circle of circumference 6 π cm. and a straight line L is at 3 cm. distant from its centre , then L is ....

(a) a tangent. (b) a secant.

(c) outside the circle. (d) a diameter of the circle.

4 The inscribed angle in a semicircle is .... angle.

(a) an acute (b) an obtuse

(c) a straight (d) a right

5 The radius length of the circle whose centre is the point of origin and passes through (– 3 , 4) equals .... length unit.

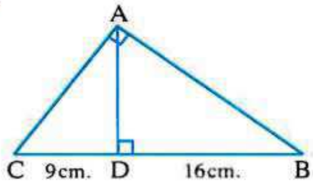
(a) 3 (b) 4 (c) 5 (d) 7

6 **In the opposite figure :**

ABC is a right-angled triangle at A

, $\overline{AD} \perp \overline{BC}$, BD = 16 cm.

, CD = 9 cm. , then AB = ... cm.



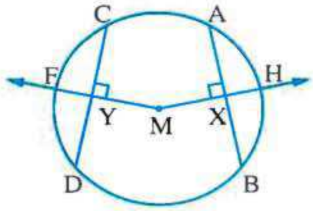
(a) 5 (b) 7 (c) 20 (d) 25

2 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords equal in length in the circle M

, $\overrightarrow{MX} \perp \overline{AB}$, $\overrightarrow{MY} \perp \overline{CD}$

Prove that : HX = FY

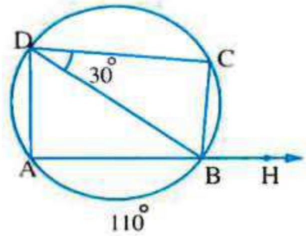


[b] **In the opposite figure :**

$H \in \overrightarrow{AB}$, m $(\widehat{AB}) = 110°$

, m (∠ CDB) = 30°

Find : m (∠ HBC)

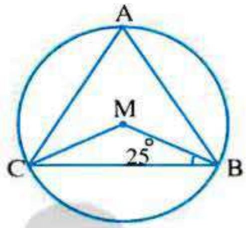


3 [a] **In the opposite figure :**

ABC is a triangle drawn in the circle M

, m (∠ MBC) = 25°

Find : m (∠ BAC)

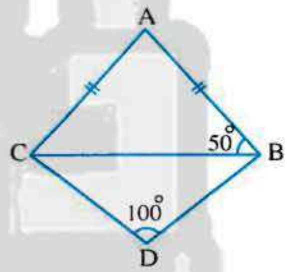


[b] **In the opposite figure :**

AB = AC , m (∠ D) = 100°

, m (∠ ABC) = 50°

Prove that : ABDC is a cyclic quadrilateral.



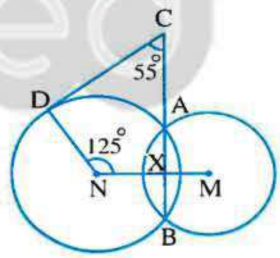
4 [a] **In the opposite figure :**

M and N are two intersecting circles at A and B

, $C \in \overrightarrow{BA}$, D ∈ the circle N , m (∠ MND) = 125°

, m (∠ C) = 55°

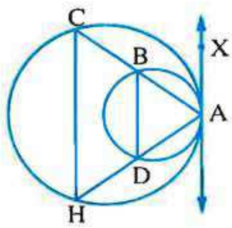
Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle N at D



[b] **In the opposite figure :**

$\overleftrightarrow{AX}$ is a common tangent for the two circles touching internally at A

Prove that : $\overline{BD}$ // $\overline{CH}$



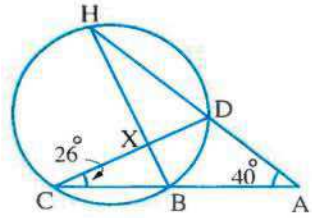
5 **[a] In the opposite figure :**

$\overrightarrow{CB} \cap \overrightarrow{HD} = \{A\}$, m (∠ A) = 40°

, $\overline{CD} \cap \overline{BH} = \{X\}$

, m (∠ DCB) = 26°

Find : m ($\overset{\frown}{CH}$) , m (∠ HXC)



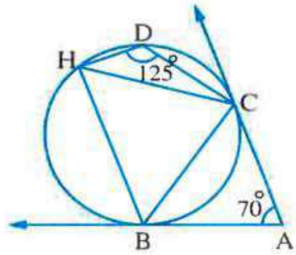
[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 70°

, m (∠ CDH) = 125°

Prove that : CB = CH



27 Matrouh Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 In the cyclic quadrilateral , each two opposite angles are ....

(a) equal in measure. (b) complementary.

(c) supplementary. (d) alternate.

2 A square is of perimeter 20 cm. , then its area equals ....

(a) 50 cm^2. (b) 50 cm. (c) 25 cm^2. (d) 25 cm.

3 Δ ABC is right-angled at B , if BC = 8 cm. , AB = 6 cm. , then sin C = ...

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{5}{3}$ (d) 0.6

4 The ratio between the measure of the central angle and the measure of the inscribed angle subtended by the same arc equals ....

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 1 : 4

5 The measure of the angle of the regular pentagon is equal to ....

(a) 72° (b) 180° (c) 108° (d) 120°

6 A chord with length 8 cm. in a circle with circumference 10 π cm. , then it is distant from its center by ....

(a) 2 cm. (b) 3 cm. (c) 4 cm. (d) 5 cm.

2 **[a]** $\overline{AB}$ and $\overline{AC}$ are two chords equal in length in a circle M , X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively, m (∠ MXY) = 30°

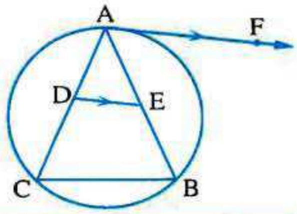
Prove that : Δ MXY is an isosceles triangle.

[b] In the opposite figure :

$\overrightarrow{AF}$ is a tangent to the circle at A

, $\overrightarrow{AF}$ // $\overline{DE}$

Prove that : DEBC is a cyclic quadrilateral.



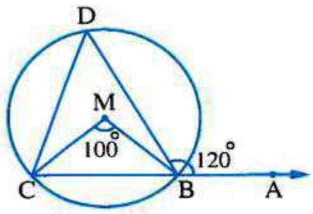
3 **[a] In the opposite figure :**

A circle of center M

, m (∠ BMC) = 100°

, m (∠ ABD) = 120°

Find : m (∠ DCB)



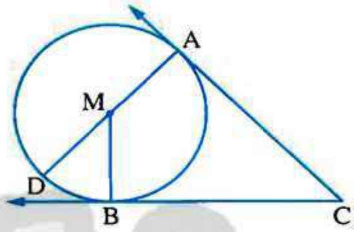
[b] In the opposite figure :

$\overline{AD}$ is a diameter in the circle M

, $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are two tangents to the circle M

, touching it at A and B respectively.

Prove that : m (∠ DMB) = m (∠ ACB)



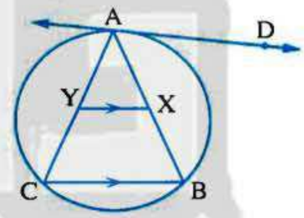
4 **[a] In the opposite figure :**

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A

, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

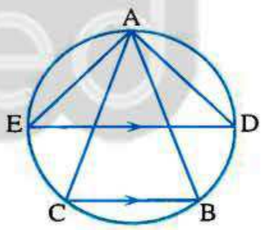


[b] In the opposite figure :

ABC is an inscribed triangle inside a circle

, $\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)



5 **[a] Prove that :** In the same circle , the measures of all inscribed angles subtended by the same arc are equal.

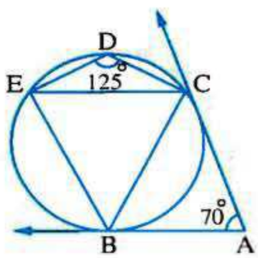
[b] In the opposite figure :

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m (∠ A) = 70° , m (∠ CDE) = 125°

Prove that : 1 CB = CE

2 $\overrightarrow{AC}$ // $\overline{BE}$



Answers of school book examinations in geometry

Model 1

1

1 d 2 a 3 d 4 a 5 b 6 a

2

[a] supplementary , theoratical.

[b] ∵ $\overleftrightarrow{XY} \parallel \overline{BD}$, $\overrightarrow{AB}$ is a transversal

∴ m (∠ DBX) = m (∠ BXY) (alternate angles) (1)

, ∵ m (∠ C) (inscribed) = m (∠ ABD) (tangency) (2)

From (1) and (2) : ∴ m (∠ C) = m (∠ BXY)

∴ AXYC is a cyclic quadrilateral. (Q.E.D.)

3

[a] ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the smaller circle

∴ AB = AC ∴ $2x - 3 = 15$

∴ $2x = 18$ ∴ $x = 9$

, ∵ $\overrightarrow{AB}$, $\overrightarrow{AD}$ are two tangents to the greater circle

∴ AB = AD ∴ y − 2 = 15 ∴ y = 17

[b] ∵ m (∠ BDC) = m (∠ BAC)

(two inscribed angles subtended by $\widehat{BC}$)

∴ m (∠ BDC) = 30°

∵ m ($\widehat{BC}$) = 2 m (∠ BAC) = 60°

, ∵ $\overline{AB}$ is a diameter in the circle M

∴ m ($\widehat{AB}$) = 180°

∴ m ($\widehat{AC}$) = 180° − 60° = 120°

, ∵ D is the midpoint of $\widehat{AC}$

∴ m ($\widehat{AD}$) = $\frac{120°}{2}$ = 60° (First req.)

∴ m (∠ ACD) = $\frac{1}{2}$ m ($\widehat{AD}$) = $\frac{1}{2}$ × 60° = 30°

∴ m (∠ CAB) = m (∠ ACD) but they are alterante

∴ $\overline{AB} \parallel \overline{DC}$ (Second req.)

4

[a] ∵ X is the midpoint of $\overline{AB}$

∴ $\overline{MX} \perp \overline{AB}$ ∴ m (∠ MXA) = 90°

, ∵ Y is the midpoint of $\overline{AC}$

∴ $\overline{MY} \perp \overline{AC}$ ∴ (∠ MYA) = 90°

From the quadrilateral AXMY :

∴ m (∠ DMH) = 360° − (90° + 90° + 70°) = 110° (First req.)

∵ AB = AC , $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

∴ MX = MY , ∵ MD = MH = r

By subtracting : ∴ XD = YH (Second req.)

[b] ∵ m (∠ A) = $\frac{1}{2}$ [m ($\widehat{HC}$) − m ($\widehat{BD}$)]

∴ 30° = $\frac{1}{2}$ [120° − m ($\widehat{BD}$)]

∴ 60° = 120° − m ($\widehat{BD}$) ∴ m ($\widehat{BD}$) = 120° − 60°

∴ m ($\widehat{BD}$) = 60° (First req.)

, ∵ m ($\widehat{BC}$) = m ($\widehat{DH}$) ∴ BC = DH

by adding m ($\widehat{BD}$) to both sides

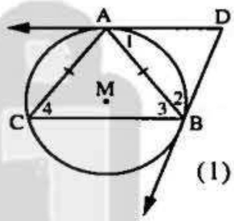
∴ m ($\widehat{CD}$) = m ($\widehat{HB}$) ∴ m (∠ C) = m (∠ H)

In Δ ACH : ∴ AC = AH , ∵ BC = DH

By sbutracting : ∴ AB = AD (Second req.)

5

[a] ∵ $\overline{DA}$ and $\overline{DB}$ are two tangent-segments to the circle M at A and B



∴ DA = DB

∴ m (∠ 1) = m (∠ 2)

∴ m (∠ D)

= 180° − 2 m (∠ 1) (1)

In Δ ABC : ∵ AB = AC

∴ m (∠ 3) = m (∠ 4)

∴ m (∠ BAC) = 180° − 2 m (∠ 4) (2)

, ∵ $\overline{AD}$ is a tangent-segment to the circle

∴ m (∠ 4) (inscribed) = m (∠ 1) (tangency) (3)

From (1) , (2) and (3) : ∴ m (∠ BAC) = m (∠ D)

∴ $\overline{AC}$ is a tangent to the circle passing through the vertices of the Δ ABD (Q.E.D.)

[b] In Δ AMB : ∵ AM = BM = r

∴ m (∠ MBA) = m (∠ MAB) = 20°

∵ C is the midpoint of $\overline{AB}$

∴ $\overline{MC} \perp \overline{AB}$ ∴ m (∠ MCB) = 90°

In Δ BCM : ∴ m (∠ BMC) = 180° − (90° + 20°) = 70°

, ∵ m (∠ BHD) = $\frac{1}{2}$ m (∠ BMD)

(inscribed and central angles subtended by $\widehat{BD}$)

∴ m (∠ BHD) = $\frac{1}{2}$ × 70° = 35° (First req.)

In Δ AMB : ∵ AM = BM = r

, m (∠ MAB) = m (∠ MBA) = 20°

∴ m (∠ AMB) = 180° − (20° + 20°) = 140°

∴ m ($\widehat{ADB}$) = m (< AMB) = 140° (Second req.)

Model 2

1

1 b	2 d	3 b
4 c	5 d	6 b

2

[a] ∵ AB = AC

, $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

∴ MD = ME , ∵ MX = MY = r

∴ DX = EY (Q.E.D.)

[b] In Δ ABD : ∵ AB = AD

∴ m (∠ ABD) = m (∠ ADB) = 30°

∴ m (∠ A) = 180° − 2 × 30° = 120°

, ∵ m (∠ A) + m (∠ C) = 120° + 60° = 180°

∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

3

[a] State by yourself.

[b] ∵ E is the midpoint of $\overset{\frown}{BF}$

∴ m $(\overset{\frown}{FE})$ = m $(\overset{\frown}{BE})$

∴ m (∠ FAE) = m (∠ BAE)

, ∵ m (∠ CBE) (tangency) = m (∠ BAE) (inscribed)

∴ m (∠ DAC) = m (∠ DBC)

and they are drawn on $\overline{DC}$ and on one side of it

∴ ABCD is a cyclic quadrilateral.

4

[a] ∵ $\overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle

∴ AD = AF = 5 cm.

, ∵ $\overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle

∴ BD = BE = 4 cm.

, ∵ $\overline{CE}$, $\overline{CF}$ are two tangent-segments to the circle

∴ CE = CF = 3 cm.

∴ The perimeter of Δ ABC = 5 + 5 + 4 + 4 + 3 + 3 = 24 cm. (The req.)

[b] ∵ $\overleftrightarrow{AF}$ // $\overline{DE}$, $\overrightarrow{AB}$ is a transversal

∴ m (∠ AED) = m (∠ EAF) (alternate angles)

, ∵ m (∠ C) (inscribed) = m (∠ BAF) (tangency)

∴ m (∠ C) = m (∠ AED)

∴ DEBC is a cyclic quadrilateral. (Q.E.D.)

5

∵ BCDE is a cyclic quadrilateral

∴ m (∠ CBE) + m (∠ D) = 180°

∴ m (∠ CBE) = 180° − 125° = 55°

, ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

∴ AB = AC

∴ In Δ ABC : m (∠ ACB) = m (∠ ABC)

$= \frac{180° - 70°}{2} = 55°$

∴ m (∠ CBE) = m (∠ ACB) = 55°

and they are alternate angles

∴ $\overline{AC}$ // $\overline{BE}$

, ∵ m (∠ BEC) (inscribed) = m (∠ ACB) (tangency) = 55°

∴ m (∠ CBE) = m (∠ BEC) = 55°

∴ In Δ CBE : CB = CE

Model examination for the merge students

1

1 diameter
2 perpendicular to this chord
3 equal
4 3
5 infinite
6 180°

2

1 a	2 a	3 d
4 c	5 d	6 c

3

1 ✗	2 ✓	3 ✗
4 ✓	5 ✗	6 ✗

4

1 90°	2 130°	3 40°
4 5	5 30°	6 2 : 1

Answers of governorates' examinations of geometry

1 Cairo

1

1 c | 2 b | 3 a | 4 a | 5 c | 6 d

2

[a] Mention by yourself.

[b] ∵ $\overline{AB}$ is a diameter in the circle M

∴ m (∠ ACB) = 90° (1) (First req.)

, ∵ $\overline{DE} \perp \overline{AD}$

∴ m (∠ ADE) = 90° (2)

From (1) and (2) :

∴ m (∠ ADE) = m (∠ ACE)

but they are drawn on $\overline{AE}$ and on one side of it

∴ The figure ACDE is a cyclic quadrilateral. (Second req.)

3

[a] The measure of the arc = $\frac{1}{3} \times 360° = 120°$ (The req.)

[b] ∵ m (∠ BAC) = $\frac{1}{2}$ m (∠ BMC)

(inscribed and central angles subtended the same arc $\overset{\frown}{BC}$)

∴ m (∠ BAC) = $\frac{1}{2} \times 80° = 40°$

, ∵ AB = AC

∴ m (∠ ABC) = m (∠ ACB) = $\frac{180° - 40°}{2} = 70°$ (First req.)

∵ m ($\overset{\frown}{BC}$) = m (∠ M) = 80°

∴ m ($\overset{\frown}{BC}$ the major) = 360° – 80° = 280° (Second req.)

4

[a] ∵ $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{CB}$

, ∵ The sum of measures of the interior angles of the quadrilateral BDME = 360°

∴ m (∠ DME) = 360° – (70° + 90° + 90°) = 110° (First req.)

∵ MD = ME , $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{CB}$

∴ AB = CB (Second req.)

[b] ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents.

∴ AB = AC

∴ m (∠ ABC) = m (∠ ACB) (1)

, ∵ $\overline{BD}$ // $\overrightarrow{AC}$, $\overleftrightarrow{BC}$ is a transversal.

∴ m (∠ DBC) = m (∠ ACB) (alternate angles) (2)

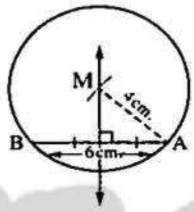
From (1) and (2) :

∴ m (∠ ABC) = m (∠ DBC)

∴ $\overrightarrow{BC}$ bisects ∠ ABD (Q.E.D.)

5

[a]



∵ The radius length of the smallest circle = 3 cm.

[b] ∵ $\overrightarrow{AD}$ is a tangent to the circle at A

∴ m (∠ ABC) (inscribed) = m (∠ CAD) (tangency) = 50°

, ∵ AC = BC

∴ m (∠ BAC) = m (∠ ABC) = 50°

, m (∠ BEC) = m (∠ BAC) = 50°

(two inscribed angles subtended by $\overset{\frown}{BC}$) (First req.)

, ∵ m (∠ BEC) = m (∠ ABC) = 50°

∴ $\overline{BC}$ is a tangent to the circle passing through the vertices of Δ BEO (Second req.)

2 Giza

1

1 d | 2 c | 3 b | 4 b | 5 c | 6 d

2

[a] ∵ m (∠ A) = $\frac{1}{2}$ m (∠ BMD) = $\frac{1}{2} \times 150° = 75°$

(inscribed and central angles subtened by $\overset{\frown}{BD}$)

, ∵ ABCD is a cyclic quadrilateral.

∴ m (∠ C) = 180° – 75° = 105° (The req.)

[b] In ΔABC : $\because m(\angle B) = m(\angle C)$

$\therefore AB = AC$

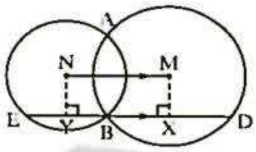
, $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$, $\because \overline{MY} \perp \overline{AC}$

$\therefore MX = MY$ (Q.E.D.)

3

[a] Construction :



Draw $\overline{MX} \perp \overline{BD}$

, $\overline{NY} \perp \overline{BE}$

Proof : $\because \overleftrightarrow{BD} \parallel \overleftrightarrow{MN}$

, $\overline{MX} \perp \overline{BD}$, $\overline{NY} \perp \overline{BE}$

$\therefore \overline{MX} \parallel \overline{NY}$

$\therefore$ The figure MXYN is a rectangle

$\therefore$ X is midpoint of $\overline{BD}$

, Y is midpoint of $\overline{BE}$

$\therefore DE = 2\,XY$, $\because XY = MN$

$\therefore DE = 2\,MN$ (Q.E.D.)

[b] $\because \overline{AB}$ is a tangent to the circle M

$\therefore \overline{MA} \perp \overline{AB}$ $\therefore m(\angle MAB) = 90^\circ$

In ΔMAB : $\because m(\angle ABM) = 30^\circ$, $m(\angle MAB) = 90^\circ$

$\therefore BM = 2\,AM = 2 \times 8 = 16$ cm.

$\therefore (AB)^2 = (BM)^2 - (MA)^2 = (16)^2 - (8)^2 = 192$

$\therefore AB = 8\sqrt{3}$ cm. (First req.)

, $\because AC = \frac{AM \times AB}{BM}$

$\therefore AC = \frac{8 \times 8\sqrt{3}}{16} = 4\sqrt{3}$ cm. (Second req.)

4

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle

$\therefore AB = AC$

$\therefore$ In ΔABC : $m(\angle ABC) = m(\angle ACB)$

$= \frac{180^\circ - 50^\circ}{2} = 65^\circ$

, $\because$ BCDE is a cyclic quadrilateral.

$\therefore m(\angle EBC) + m(\angle D) = 180^\circ$

$\therefore m(\angle EBC) = 180^\circ - 115^\circ = 65^\circ$

$\therefore m(\angle ABC) = m(\angle EBC)$

$\therefore \overrightarrow{BC}$ bisects $\angle ABE$ (Q.E.D.1)

, $\because m(\angle BEC)$ (inscribed)

$= m(\angle ABC)$ (tangency) $= 65^\circ$

$\therefore m(\angle EBC) = m(\angle BEC)$

$\therefore$ In ΔBCE : $CB = CE$ (Q.E.D.2)

[b] $\because \angle ABE$ is an exterior angle of the cyclic quadrilateral ABCD

$\therefore m(\angle D) = m(\angle ABE) = 100^\circ$

In ΔACD :

$\because m(\angle ACD) = 180^\circ - (100^\circ + 40^\circ) = 40^\circ$

$\therefore m(\angle ACD) = m(\angle CAD)$

$\therefore CD = AD$

$\therefore m(\widehat{CD}) = m(\widehat{AD})$ (Q.E.D.)

5

[a] $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB) = 60^\circ$

(inscribed and central angles subtended the same arc $\widehat{AB}$) (1)

$\because \overline{CD} \parallel \overline{AB}$

$\therefore m(\widehat{AC}) = m(\widehat{BC})$

$\therefore AC = BC$ (2)

From (1) and (2) :

$\therefore \Delta CAB$ is an equilateral triangle. (Q.E.D.)

[b] In $\Delta\Delta$ ADE , ACE

$\begin{cases} m(\angle DAE) = m(\angle CAE) \\ AD = AC \\ \overline{AE} \text{ is a common side} \end{cases}$

$\therefore \Delta ADE \equiv \Delta ACE$

$\therefore m(\angle ADE) = m(\angle ACE)$

, $\because m(\angle AFB) = m(\angle ACB)$

(two inscribed angles subtended by $\widehat{AB}$)

$\therefore m(\angle ADE) = m(\angle EFB)$

$\therefore$ DBFE is a cyclic quadrilateral. (Q.E.D.)

3 Alexandria

1

1 b 2 d 3 a 4 b 5 d 6 c

2

[a] $\because \overline{CD}$ is a diameter in a circle M

, AB = 10 cm. , $\overline{MH} \perp \overline{AB}$

$\therefore AH = BH = 5$ cm.

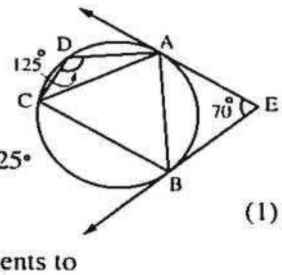
In ΔAHM : $\because m(\angle AMH) = 30^\circ$

, $m(\angle AHM) = 90^\circ$

$\therefore AM = 2\,AH = 10$ cm.

$\therefore CD = 2 \times 10 = 20$ cm. (The req.)

[b] ∵ The figure ABCD is a cyclic quadrilateral.



∴ m (∠ ABC) = 180° − 125° = 55° (1)

∵ $\overrightarrow{EA}$, $\overrightarrow{EB}$ are two tangents to the circle at A and B

∴ EA = EB

∴ m (∠ EAB) = $\frac{180° - 70°}{2}$ = 55°

∵ $\overrightarrow{EA}$ is a tangent to the circle at A

∴ m (∠ EAB) (tangency) = m (∠ ACB) (inscribed) = 55° (2)

From (1) and (2) :

∴ m (∠ ACB) = m (∠ ABC) = 55°

∴ AB = AC (Q.E.D.)

3

[a] ∵ m (∠ A) = $\frac{1}{2}$ [m ($\widehat{HC}$) − m ($\widehat{BD}$)]

∴ 30° = $\frac{1}{2}$ [120° − m ($\widehat{BD}$)]

∴ 60° = 120° − m ($\widehat{BD}$)

∴ m ($\widehat{BD}$) = 120° − 60° = 60° (First req.)

, ∵ m ($\widehat{BC}$) = m ($\widehat{DH}$)

∴ BC = DH

By adding m ($\widehat{BD}$) to both sides.

∴ m ($\widehat{CD}$) = m ($\widehat{HB}$) ∴ m (∠ C) = m (∠ H)

In Δ ACH : ∵ AC = AH

, ∵ BC = DH

By subtracting : AB = AD (Second req.)

[b] In Δ ABD :

∵ AB = AD

∴ m (∠ ABD) = m (∠ ADB) = 30°

∴ m (∠ A) = 180° − 2 × 30° = 120°

∴ m (∠ A) + m (∠ C) = 120° + 60° = 180°

∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

4

[a] ∵ m (∠ ACB) = $\frac{1}{2}$ m (∠ AMB) = 60°

(inscribed and central angles subtended the same arc $\widehat{AB}$) (1)

∵ $\overleftrightarrow{CD}$ // $\overline{AB}$

∴ m ($\widehat{AC}$) = m ($\widehat{BC}$)

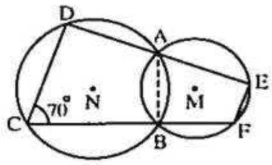
∴ AC = BC (2)

From (1) and (2) :

∴ Δ CAB is equilateral. (Q.E.D.)

[b] **Construction :**

Draw $\overline{AB}$



Proof :

∵ The figure ABCD is a cyclic quadrilateral

∴ m (∠ BAD) = 180° − 70° = 110°

, ∵ The figure ABFE is a cyclic quadrilateral and ∠ BAD is an exteroir angle of it

∴ m (∠ F) = m (∠ BAD) = 110°

∴ m (∠ F) + m (∠ C) = 110° + 70° = 180°

but they are two interior angles on the same side of the transversal $\overline{FC}$

∴ $\overline{CD}$ // $\overline{EF}$ (Q.E.D.)

5

[a] In Δ ABC :

∵ AC = BC

∴ m (∠ BAC) = m (∠ ABC) = 65°

∴ m (∠ CAD) = 130° − 65° = 65°

, ∵ m (∠ B) = m (∠ CAD) = 65°

∴ $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC (Q.E.D.)

[b] ∵ $\overrightarrow{AD}$ is a tangent

∴ $\overline{MD}$ ⊥ $\overline{AD}$ ∴ m (∠ MDA) = 90°

, ∵ H is the midpoint of $\overline{BC}$

∴ $\overline{MH}$ ⊥ $\overline{BC}$ ∴ m (∠ MHA) = 90°

From the quadrilateral ADMH :

∴ m (∠ DMH) = 360° − (56° + 90° + 90°) = 124° (The req.)

4 El-Kalyoubia

1

1 c 2 a 3 d 4 b 5 d 6 c

2

[a] ∵ $\overline{AB} \parallel \overline{CD}$ ∴ m $(\widehat{AC})$ = m $(\widehat{BD})$ = 50°

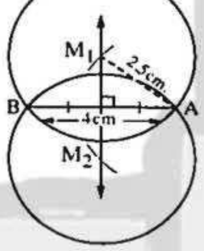
, ∵ m (∠ BED) = $\frac{1}{2}$ m $(\widehat{BD})$

∴ (3 y − 5)° = $\frac{1}{2}$ × 50° = 25°

∴ 3 y = 5° + 25° = 30°

∴ y = 10° (The req.)

[b]



∴ We can draw two circles.

3

[a] ∵ X is a midpoint of $\overline{AB}$

∴ $\overline{MX} \perp \overline{AB}$ ∴ m (∠ MXA) = 90° (1)

, ∵ Y is a midpoint of $\overline{AC}$

∴ $\overline{MY} \perp \overline{AC}$ ∴ m (∠ MYA) = 90° (2)

From (1) and (2) :

∴ m (∠ MXA) = m (∠ MYA)

but they are drawn on $\overline{AM}$ and on one side of it.

∴ AXYM is a cyclic quadrilateral. (Q.E.D.1)

In Δ MAC : ∴ MA = MC = r

∴ m (∠ MCA) = m (∠ MAC)

, ∵ AXYM is a cyclic quadrilateral.

∴ m (∠ MXY) = m (∠ MAY)

∴ m (∠ MXY) = m (∠ MCY) (Q.E.D.2)

[b] ∵ ABCD is a cyclic quadrilateral

∴ m (∠ A) + m (∠ C) = 180°

∴ m (∠ C) = 180° − 120° = 60° (First req.)

∴ m (∠ FBC) = m (∠ C) = 60° (alternate angles)

∴ m (∠ EBC) = 65° + 60° = 125°

, ∵ ABCD is a cyclic quadrilateral.

∴ m (∠ D) = m (∠ EBC) = 125° (Second req.)

4

[a] ∵ The circle M ⋂ The circle N = {A , B}

∴ $\overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$

∴ In Δ ABD :

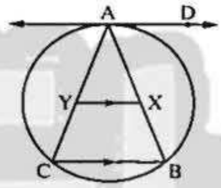
$\overleftrightarrow{DC}$ is the axis of symmetry of $\overline{AB}$

∴ AD = BD

∵ $\overline{MX} \perp \overline{AD}$, $\overline{MY} \perp \overline{BD}$

∴ MX = MY (Q.E.D.)

[b]



∵ $\overleftrightarrow{AD}$ is a tangent to the circle.

∴ m (∠ DAB) (tangency) = m (∠ ACB) (inscribed) (1)

, ∵ $\overleftrightarrow{XY} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{YC}$ is a transversal

∴ m (∠ AYX) = m (∠ ACB) (corresponding angles) (2)

∴ From (1) and (2) : ∴ m (∠ DAB) = m (∠ AYX)

∴ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

5

[a] ∵ $\overline{AC}$ and $\overline{AB}$ are two tangent-segments to the circle M

∴ $\overline{AE} \perp \overline{BC}$ ∴ m (∠ CEM) = 90°

, ∵ $\overline{BD}$ is a diameter in the circle M

∴ m (∠ ECD) = 90°

∴ m (∠ CEM) + m (∠ ECD) = 180°

, but they are two interior angles in the same side of the transversal $\overleftrightarrow{BC}$

∴ $\overline{AM} \parallel \overline{CD}$ (Q.E.D.)

[b] $\because \overline{CM} \parallel \overline{AB}$, $\overleftrightarrow{MA}$ is a transversal.

$\therefore$ m ($\angle$ MAB) = m ($\angle$ AMC) (alternate angles)

, $\because$ m ($\angle$ AMC) = 2 m ($\angle$ B)

(central and inscribed angles subtended by $\widehat{AC}$)

$\therefore$ m ($\angle$ EAB) = 2 m ($\angle$ B)

$\therefore$ m ($\angle$ EAB) > m ($\angle$ B)

From Δ EAB : $\therefore$ BE > AE (Q.E.D.)

5 El-Sharkia

1

1 b 2 a 3 d 4 b 5 d 6 a

2

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$

, $\because$ AB = AC $\therefore$ MX = MY

, $\because$ MD = ME = r $\therefore$ XD = YE (Q.E.D.)

[b] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle.

$\therefore$ AB = AC

$\therefore$ In Δ ABC :

m ($\angle$ ABC) = m ($\angle$ ACB) = $\frac{180° - 70°}{2}$ = 55°

, $\because$ BCDE is a cyclic quadrilateral.

$\therefore$ m ($\angle$ EBC) + m ($\angle$ D) = 180°

$\therefore$ m ($\angle$ EBC) = 180° − 125° = 55°

$\therefore$ m ($\angle$ ABC) = m ($\angle$ EBC)

$\therefore \overrightarrow{BC}$ bisects $\angle$ ABE (Q.E.D.)

3

[a] $\because$ ABDC is a cyclic quadrilateral.

$\therefore$ m ($\angle$ A) + m ($\angle$ D) = 180°

$\therefore$ m ($\angle$ A) = 180° − 140° = 40°

, $\because \overline{AB}$ is a diameter.

$\therefore$ m ($\angle$ ACB) = 90°

In Δ ABC :

$\therefore$ m ($\angle$ ABC) = 180° − (90° + 40°) = 50° (First req.)

, $\because$ m ($\widehat{BD}$) = m ($\widehat{DC}$) $\therefore$ BD = CD

In Δ BCD :

$\therefore$ m ($\angle$ CBD) = m ($\angle$ BCD) = $\frac{180° - 140°}{2}$ = 20°

$\therefore$ m ($\widehat{BD}$) = 2 m ($\angle$ BCD) = 2 × 20° = 40°

, $\because$ m ($\widehat{AB}$) = 180°

$\therefore$ m ($\widehat{ABD}$) = 180° + 40° = 220° (Second req.)

[b] $\because \overleftrightarrow{AD}$ is a tangent to the circle

$\because$ m ($\angle$ DAB) (tangen y)

= m ($\angle$ ACB) (inscribed) (1)

, $\because \overline{XY} \parallel \overline{BC}$, $\overrightarrow{YC}$ is a transversal.

$\therefore$ m ($\angle$ AYX) = m ($\angle$ ACB)

(corresponding angles) (2)

$\therefore$ From (1) and (2) :

$\therefore$ m ($\angle$ DAB) = m ($\angle$ AYX)

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle which passes through the points A , X and Y (Q.E.D.)

4

[a] $\because$ m ($\widehat{BD}$) = 2 m ($\angle$ DCB) = 2 × 25° = 50°

, $\because$ D is midpoint of ($\widehat{AB}$)

$\therefore$ m ($\widehat{AB}$) = 2 × 50° = 100°

$\therefore$ m ($\angle$ AMB) = m ($\widehat{AB}$) = 100° (The req.)

[b] $\because \Delta$ ABC is equilateral.

$\therefore$ m ($\angle$ B) = 60°

$\therefore$ m ($\angle$ D) = m ($\angle$ B) = 60°

(two inscribed angles subtended by $\widehat{AC}$)

, $\because$ AD = DE

$\therefore \Delta$ ADE is an equilateral triangle. (Q.E.D.1)

$\because$ m ($\angle$ DAE) = m ($\angle$ BAC) = 60°

Subtracting $\angle$ BAE from both sides.

$\therefore$ m ($\angle$ DAB) = m ($\angle$ EAC) (Q.E.D.2)

5

[a] $\because \overline{AB}$ is a tangen-tsegment to the circle.

$\therefore \overline{MA} \perp \overline{AB}$ $\therefore$ m ($\angle$ A) = 90°

In Δ MAB : $\because$ tan ($\angle$ B) = $\frac{AM}{AB}$

$\therefore$ tan 30° = $\frac{8}{AB}$

$\therefore$ AB = $\frac{8}{\tan 30°}$ = $8\sqrt{3}$ cm.

In Δ MAB : $\because$ m ($\angle$ AMB) = 180° – (90° + 30°)
= 60°

, $\because$ m ($\angle$ XAB) = $\frac{1}{2}$ m ($\angle$ AMB)

(tangency and central angles)

$\therefore$ m ($\angle$ XAB) = $\frac{1}{2}$ × 60° = 30°

In Δ XAB :

$\because$ m ($\angle$ XAB) = m ($\angle$ XBA)

$\therefore$ Δ XAB is an isosceles triangle. (Second req.)

[b] In $\Delta\Delta$ ADE , ACE :

m ($\angle$ DAE) = m ($\angle$ CAE)
AD = AC
$\overline{AE}$ is a common side

$\therefore$ Δ ADE ≡ Δ ACE

$\therefore$ m ($\angle$ ADE) = m ($\angle$ ACE)

, $\because$ m ($\angle$ AFB) = m ($\angle$ ACB)

(two inscribed angles subtended by $\overset{\frown}{AB}$)

$\therefore$ m ($\angle$ ADE) = m ($\angle$ EFB)

$\therefore$ DBFE is a cyclic quadrilateral. (Q.E.D.)

6 El-Monofia

1

1 c 2 a 3 b 4 b 5 c 6 b

2

[a] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M

$\therefore$ $\overline{AB} \perp \overline{MB}$, $\overline{AC} \perp \overline{MC}$

$\therefore$ m ($\angle$ BAC) = 360° – (90° + 90° + 90°) = 90°

, MB = MC = r

$\therefore$ ABMC is a square. (Q.E.D.)

[b] In Δ AMB : $\because$ AM = MB = r

$\therefore$ m ($\angle$ MAB) = m ($\angle$ ABM)

$\because$ m ($\angle$ CAB) = m ($\angle$ MAB)

$\therefore$ m ($\angle$ CAB) = m ($\angle$ ABM) and they are alternate angles.

$\therefore$ $\overline{AC}$ // $\overline{BM}$

$\because$ D is the midpoint of $\overline{AC}$

$\therefore$ $\overline{MD} \perp \overline{AC}$

, $\because$ $\overline{AC}$ // $\overline{BM}$

$\therefore$ $\overline{DM} \perp \overline{BM}$ (Q.E.D.)

3

[a] $\because$ $\overline{AX}$, $\overline{AZ}$ are two tangent-segments

$\therefore$ AX = AZ = 6 cm. $\because$ AC = 10 cm.

$\therefore$ CZ = 10 – 6 = 4 cm.

$\because$ $\overline{CY}$, $\overline{CZ}$ are two tangent-segments

$\therefore$ CY = CZ = 4 cm.

$\because$ $\overline{BX}$, $\overline{BY}$ are two tangent-segments

$\therefore$ BX = BY

$\because$ The perimeter of Δ ABC = 24 cm.

$\therefore$ BX + BY + 6 + 10 + 4 = 24

$\therefore$ BX + BY = 4 $\therefore$ BX = 2 cm.

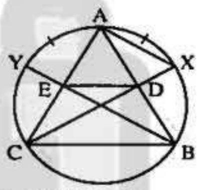
$\therefore$ AB = 6 + 2 = 8 cm. (First req.)

$\because$ $(AC)^2 = (10)^2 = 100$

, $(AB)^2 + (BC)^2 = (8)^2 + (6)^2 = 100 = (AC)^2$

$\therefore$ Δ ABC is a right-angled triangle at B (Second req.)

[b] $\because$ m ($\overset{\frown}{AX}$) = m ($\overset{\frown}{AY}$)



$\therefore$ m ($\angle$ ACX) = m ($\angle$ ABY)

and they are drawn on $\overline{DE}$ and on one side of it

$\therefore$ The figure BCED is a cyclic quadrilateral. (Q.E.D.1)

$\therefore$ m ($\angle$ DEB) = m ($\angle$ DCB)

(drawn on $\overline{DB}$ and on one side of it)

, $\because$ m ($\angle$ XAB) = m ($\angle$ XCB)

(two inscribed angles subtended by $\overset{\frown}{XB}$)

$\therefore$ m ($\angle$ DEB) = m ($\angle$ XAB) (Q.E.D.2)

4

[a] In Δ ABC : $\because$ CA = CB (1)

$\therefore$ m ($\angle$ A) = m ($\angle$ B) $\therefore$ sin A = sin B

$\therefore$ $\frac{XM}{AM} = \frac{YM}{BM}$ $\because$ AM = BM = r

$\therefore$ XM = YM

, $\because$ $\overline{MX} \perp \overline{DA}$, $\overline{MY} \perp \overline{EB}$

$\therefore$ DA = EB (2)

Subtracting (2) from (1) : $\therefore$ CD = CE (Q.E.D.)

[b] $\because$ $\overline{AB}$ is a diameter in the circle M

$\therefore$ m ($\angle$ ACB) = 90°

, $\because$ $\overline{ED} \perp \overline{AB}$

$\therefore$ m ($\angle$ FDA) = 90°

$\therefore$ m (∠ ACF) + m (∠ FDA) = 90° + 90° = 180°

$\therefore$ The figure ADFC is a cyclic quadrilateral. (Q.E.D.1)

$\because$ $\overleftrightarrow{EC}$ is a tangent of the circle M

$\therefore$ m (∠ ECB) (tangency) = m (∠ CAB) (inscribed)

, $\because$ ∠ CFE is an exterior angle of the cyclic quadrilateral ADFC

$\therefore$ m (∠ CAB) = m (∠ CFE)

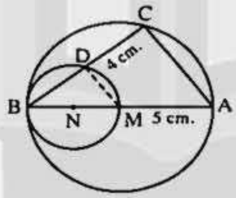
$\therefore$ m (∠ ECF) = m (∠ CFE)

In Δ ECF : $\therefore$ Δ ECF is an isosceles triangle. (Q.E.D.2)

5

[a] Construction :

Draw $\overline{MD}$



Proof :

$\because$ $\overline{BM}$ is a diameter in the circle N

$\therefore$ m (∠ MDB) = 90° $\therefore$ $\overline{MD} \perp \overline{BC}$

$\therefore$ CD = DB = 4 cm.   , $\because$ MB = AM = 5 cm.

In Δ ABC :

$\therefore (AC)^2 = (AB)^2 - (BC)^2 = (10)^2 - (8)^2$

$= 100 - 64 = 36$

$\therefore$ AC = 6 cm. (The req.)

[b] $\because$ $\overleftrightarrow{AD}$ is a tangent to the circle

$\therefore$ m (∠ DAB) (tangency) = m (∠ ACB) (inscribed) (1)

, $\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{YC}$ is a transversal

$\therefore$ m (∠ AYX) = m (∠ ACB) (corresponding angles) (2)

From (1) and (2) :

$\therefore$ m (∠ DAB) = m (∠ AYX)

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

7 El-Gharbia

1

1 b   2 a   3 d   4 c   5 b   6 d

2

[a] $\because$ $\overline{AB}$ // $\overline{CD}$, $\overleftrightarrow{AD}$ is a transversal

$\therefore$ m (∠ ADC) = m (∠ BAD) = 20° (alternate angles)

, $\because$ m (∠ AEC) = m (∠ ADC) = 20° (two inscribed angles subtended by $\overset{\frown}{AC}$)

$\therefore 3x - 7 = 20$ $\therefore 3x = 27$

$\therefore x = 9$ (The req.)

[b] $\because$ $\overline{BD}$ is a tangent-segment to the circle

$\therefore$ m (∠ ABD) = 90°

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore$ $\overline{ME} \perp \overline{AC}$ $\therefore$ m (∠ MED) = 90°

$\therefore$ m (∠ MBD) + m (∠ MED) = 90° + 90° = 180°

$\therefore$ The figure MEDB ia a cyclic quadrilateral. (Q.E.D.1)

$\because$ ∠ BMX is an exterior angle of the cyclic quadrilateral MEDB

$\therefore$ m (∠ D) = m (∠ BMX)

, $\because$ m (∠ BAX) = $\frac{1}{2}$ m (∠ BMX) (inscribed and central angles subtended by $\overset{\frown}{XB}$)

$\therefore$ m (∠ BAX) = $\frac{1}{2}$ m (∠ D) (Q.E.D.2)

3

[a] In Δ ABC : $\because$ m (∠ BAC) = 90°

$\therefore \tan B = \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore$ m (∠ B) = 30°

$\therefore$ m (∠ ABC) = m (∠ DAC) = 30°

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC (Q.E.D.)

[b] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments of the circle

$\therefore$ AB = AC

$\therefore$ m (∠ ABC) = m (∠ ACB) (1)

, $\because$ $\overline{AB}$ // $\overline{CD}$ and $\overleftrightarrow{BC}$ is a transversal

$\therefore$ m (∠ BCD) = m (∠ ABC) (alternate angles) (2)

From (1) and (2) : $\therefore$ m (∠ BCD) = m (∠ ACB)

$\therefore$ $\overrightarrow{CB}$ bisects ∠ ACD (Q.E.D.)

4

[a] ∵ ∠ AMB is an exterior angle of the Δ AMD

∴ m (∠ AMB) = m (∠ ADM) + m (∠ DAM)

∴ 80° = 30° + m (∠ DAM)

∴ m (∠ DAM) = 80° – 30° = 50°

In Δ ADC : ∵ DA = DC

∴ m (∠ DCA) = m (∠ DAC) = 50°

∴ m (∠ ABD) = m (∠ ACD)

and they are drawn on $\overline{AD}$ and on one side of it

∴ The figure ABCD is a cyclic quadrilateral.

(Q.E.D.)

[b] ∵ X is the midpont of $\overline{AB}$

∴ $\overline{MX} \perp \overline{AB}$

, ∵ Y is the midpoint of $\overline{AC}$

∴ $\overline{MY} \perp \overline{AC}$

, ∵ AB = AC ∴ MX = MY

, ∵ MD = ME = r

∴ XD = YE (Q.E.D)

5

[a] ∵ m ($\overset{\frown}{AD}$) = 2 m (∠ ABD) = 2 × 22° = 44°

, ∵ m (∠ C) = $\frac{1}{2}$ [m ($\overset{\frown}{BE}$) – m ($\overset{\frown}{AD}$)]

∵ 36° = $\frac{1}{2}$ [m ($\overset{\frown}{BE}$) – 44°]

∴ 72° = m ($\overset{\frown}{BE}$) – 44°

∴ m ($\overset{\frown}{BE}$) = 116° (The req.)

[b] ∵ m (∠ BDC) = m (∠ BAC)

(two inscribed angles subtended by $\overset{\frown}{BC}$)

∴ m (∠ BDC) = 30° (First req.)

∵ m ($\overset{\frown}{BC}$) = 2 m (∠ BAC) = 60°

, ∵ $\overline{AB}$ is diameter in the circle M

∴ m ($\overset{\frown}{AB}$) = 180°

∴ m ($\overset{\frown}{AC}$) = 180° – 60° = 120°

, ∵ D is the midpoint of $\overset{\frown}{AC}$

∴ m ($\overset{\frown}{AD}$) = $\frac{120°}{2}$ = 60°

∴ m (∠ ACD) = $\frac{1}{2}$ m ($\overset{\frown}{AD}$) = $\frac{1}{2}$ × 60° = 30°

∴ m (∠ BAC) = m (∠ ACD) but they are alternate angles

∴ $\overline{DC}$ // $\overline{AB}$ (Second req.)

8 El-Dakahlia

1

[a] 1 a   2 d   3 c

[b] ∵ ∠ ABH is an exterior angle of the cyclic quadrilateral ABCD

∴ m (∠ ADC) = m (∠ ABH) = 110°

In Δ ACD :

∵ m (∠ ACD) = 180° – (110° + 35°) = 35°

∴ m (∠ CAD) = m (∠ ACD) ∴ CD = AD

∴ m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{AD}$) (Q.E.D.)

2

[a] 1 c   2 a   3 d

[b] ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

∴ AB = AC

∴ m (∠ ABC) = m (∠ ACB) = $\frac{180° - 70°}{2}$ = 55°

(First req.)

∵ BCHD is a cyclic quadrilateral

∴ m (∠ C) + m (∠ D) = 180°

∴ m (∠ C) = 180° – 125° = 55°

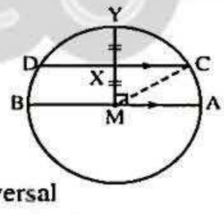
∵ m (∠ BHC) (inscribed)

= m (∠ ABC) (tangency) = 55°

∴ m (∠ BCH) = m (∠ BHC)

In Δ BCH : ∴ CB = BH (Second req.)

3



[a] Construction :

Draw $\overline{MC}$

Proof :

∵ $\overline{CD}$ // $\overline{AB}$, $\overrightarrow{MY}$ is a transversal

∴ m (∠ MXC) + m (∠ XMA) = 180°

∴ m (∠ MXC) = 90°

, ∵ MX = $\frac{1}{2}$ MY , MY = MC

∴ MX = $\frac{1}{2}$ MC ∴ m (∠ MCX) = 30°

∴ m (∠ AMC) = m (∠ MCX) = 30°

(alternate angles)

∴ m ($\overset{\frown}{AC}$) = m (∠ AMC) = 30° (First req.)

, ∵ ($\overset{\frown}{AY}$) = m (∠ AMY) = 90°

∴ m ($\overset{\frown}{CY}$) = 90° – 30° = 60° (Second req.)

[b] $\because AB = AC$

, $\overline{MD} \perp \overline{AB}$, $\overline{MH} \perp \overline{AC}$

$\therefore MD = MH$

, $\because MX = MY = r$ $\therefore XD = HY$ (Q.E.D.)

4

[a] $\because \overline{AO} \parallel \overline{DH}$, $\overline{AH}$ is a transversal

$\therefore m(\angle HAO) = m(\angle AHD)$ (alternate angles) (1)

, $\because m(\angle C)$ (inscribed)

$= m(\angle BAO)$ (tangency) (2)

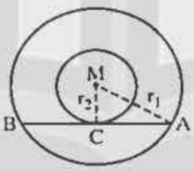
From (1) and (2) :

$\therefore m(\angle C) = m(\angle AHD)$

$\therefore$ DHBC is a cyclic quadrilateral (Q.E.D.)

[b] **Construction :**

Draw $\overline{MA}$, $\overline{MC}$



Proof :

$\because \overline{AB}$ touches the smaller circle at C

$\therefore \overline{MC} \perp \overline{AB}$

, $\because \overline{AB}$ is a chord of the greater circle

, $\overline{MC} \perp \overline{AB}$

$\therefore$ C is the midpoint of $\overline{AB}$

$\therefore AC = \frac{14}{2} = 7$ cm.

$\because \Delta AMC$ is a right-angled at C

$\therefore (AC)^2 = (MA)^2 - (MC)^2$

$\therefore (7)^2 = r_1 - r_2$ $\therefore r_1 - r_2 = 49$

$\therefore$ The area of the part included between the two circles = The area of the greater circle – The area of the smaller circle $= \pi r_1^2 - \pi r_2^2 = \pi (r_1^2 - r_2^2)$

$= \frac{22}{7} \times 49 = 154$ cm^2. (The req.)

5

[a] $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended the same arc $\widehat{AB}$)

$\therefore m(\angle ACB) = \frac{1}{2} \times 120° = 60°$ (1)

$\because \overline{CD} \parallel \overline{AB}$ $\therefore m(\widehat{AC}) = m(\widehat{BC})$

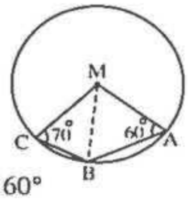
$\therefore AC = BC$ (2)

From (1) and (2) :

$\therefore \Delta ABC$ is equilateral (Q.E.D.)

[b] **Construction :**

Draw $\overline{MB}$



Proof :

In ΔMAB :

$\because MA = MB = r$, $m(\angle MAB) = 60°$

$\therefore \Delta AMB$ is equilateral

$\therefore m(\angle AMB) = 60°$ (1)

In ΔMBC : $\because MB = MC = r$

$\therefore m(\angle MBC) = m(\angle MCB) = 70°$

$\therefore m(\angle CMB) = 180° - (70° + 70°) = 40°$ (2)

From (1) and (2) :

$\therefore m(\angle AMC) = m(\angle AMB) + m(\angle CMB)$

$= 60° + 40° = 100°$ (The req.)

9 Ismailia

1

1 c 2 b 3 c 4 a 5 d 6 b

2

[a] $\because m(\angle A) = \frac{1}{2} m(\angle BMC) = x°$

(inscribed and central angles subtended by $\widehat{BC}$)

, $\because$ The figure ABDC is a cyclic quadrilateral

$\therefore m(\angle A) + m(\angle BDC) = 180°$

$\therefore x + 2x = 180°$ $\therefore 3x = 180°$

$\therefore x = 60°$ $\therefore m(\angle A) = 60°$ (The req.)

[b] $\because m(\angle A) = m(\angle B)$

(two inscribed angles subtended by $\widehat{CD}$)

, $m(\angle C) = m(\angle D)$

(two inscribed angles subtended by $\widehat{AB}$)

$\because EA = ED$ $\therefore m(\angle A) = m(\angle D)$

$\therefore m(\angle C) = m(\angle B)$

$\therefore EB = EC$ (Q.E.D.)

3

[a] In ΔABC :

$\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

, $\because \overrightarrow{BX}$ bisects $(\angle ABC)$, $\overrightarrow{CY}$ bisects $(\angle ACB)$

$\therefore m(\angle XBY) = m(\angle YCX)$

and they are drawn on $\overline{XY}$ and on one side of it

$\therefore$ BCXY is a cyclic quadrilateral (Q.E.D.)

[b] $\because$ m ($\angle$ BAC) = $\frac{1}{2}$ m ($\widehat{BC}$)

$\therefore$ m ($\angle$ BAC) = $\frac{1}{2} \times 120° = 60°$

In Δ ABC : $\therefore$ m ($\angle$ C) = 180° − (70° + 60°) = 50°

$\therefore$ m ($\angle$ DAB) = m ($\angle$ C) = 50°

(inscribed and tangency angles subtended by $\widehat{AB}$)

(The req.)

4

[a] $\because$ $\overline{AC}$ is a diameter of the circle.

$\therefore$ m ($\angle$ ABC) = 90°

, $\because$ m ($\angle$ ABD) = 60°

$\therefore$ m ($\angle$ CBD) = 90° − 60° = 30° (First req.)

, $\because$ m ($\angle$ ADB) = m ($\angle$ C) = 50°

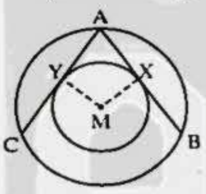
(two inscribed angles subtended by $\widehat{AB}$)

In Δ ABD :

$\therefore$ m ($\angle$ BAD) = 180° − (60° + 50°) = 70°

(Second req.)

[b] Construction :



Draw $\overline{MX}$, $\overline{MY}$

Proof :

In the smaller circle M

$\because$ $\overline{AB}$, $\overline{AC}$ are two tangents

, $\overline{MX}$, $\overline{MY}$ are two radii

$\therefore$ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

, $\because$ MX = MY = r (radii of the smaller circle)

$\therefore$ AB = AC (Q.E.D.)

5

[a] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the greater circle

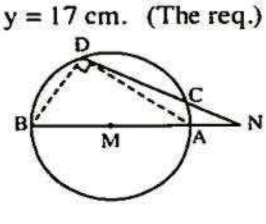
$\therefore$ $2x - 3 = 15$ $\therefore$ $2x = 18$

$\therefore$ $x = 9$ cm.

, $\because$ $\overline{AC}$, $\overline{AD}$ are two tangent-segments to the smaller circle

$\therefore$ y − 2 = 15 $\therefore$ y = 17 cm. (The req.)

[b] Construction :



Draw $\overline{AD}$, $\overline{BD}$

Proof :

$\because$ $\overline{AB}$ is a diameter of the circle

$\therefore$ m ($\angle$ ADB) = 90°

$\therefore$ m ($\angle$ ADB) + m ($\angle$ ADN) > 90°

In Δ NDB : $\therefore$ NB > ND (Q.E.D.)

10 Suez

1

1 b 2 b 3 a 4 c 5 d 6 b

2

[a] $\because$ E is the midpoint of $\overline{AC}$

$\therefore$ $\overline{ME} \perp \overline{AC}$

, $\because$ $\overline{MD} \perp \overline{AB}$, MD = ME

$\therefore$ AB = AC (Q.E.D.)

[b] $\because$ m ($\angle$ A) = $\frac{1}{2}$ m ($\angle$ BMC)

(inscribed and central angles subtended by $\widehat{BC}$)

$\therefore$ m ($\angle$ A) = $\frac{1}{2} \times 100° = 50°$ (First req.)

In Δ MBC : $\because$ MB = MC = r

$\therefore$ m ($\angle$ MBC) = m ($\angle$ MCB)

= $\frac{1}{2}$ (180° − 100°) = 40°

(Second req.)

3

[a] $\because$ $\overline{AB}$ is a diameter of the circle

$\therefore$ m ($\angle$ AEB) = 90° (First req.)

, $\because$ $\angle$ AEB is an exterior angle of Δ AEC

$\therefore$ m ($\angle$ AEB) = m ($\angle$ CAE) + m ($\angle$ ACE)

$\therefore$ m ($\angle$ CAE) = 90° − 60° = 30° (Second req.)

[b] $\because$ $\overleftrightarrow{AD}$ is a tangent to the circle

$\therefore$ $\overline{MD} \perp \overline{AD}$ $\therefore$ m ($\angle$ ADM) = 90°

, $\because$ E is the midpoint of $\overline{BC}$

$\therefore$ $\overline{ME} \perp \overline{BC}$ $\therefore$ m ($\angle$ MEA) = 90°

$\therefore$ In the quadrilateral ADME :

$\therefore$ m ($\angle$ DME) = 360° − (90° + 90° + 70°) = 110°

(The req.)

4

[a] State by yourself.

[b] ∵ ABC is an equilateral triangle

∴ m (∠ A) = 60°

∴ m (∠ D) = m (∠ A) and they are drawn on $\overline{BC}$ and on one side of it

∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] m ($\widehat{AB}$) = 2 m (∠ ADB) = 60° (First req.)

, m (∠ DCB) = $\frac{1}{2}$ [m ($\widehat{AD}$) + m ($\widehat{AB}$)]

= $\frac{1}{2}$ [90° + 60°] = 75° (Second req.)

[b] ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle.

∴ AB = AC

∴ In Δ ABC :

m (∠ ABC) = m (∠ ACB) = $\frac{1}{2}$ (180° – 40°) = 70° (First req.)

∵ $\overrightarrow{AB}$ // $\overline{CD}$, $\overrightarrow{BC}$ is a transversal

∴ m (∠ BCD) = m (∠ ABC) = 70° (1)

(alternate angles)

, ∵ m (∠ BDC) (inscribed)

= m (∠ ABC) (tangency) = 70° (2)

From (1) and (2) :

∴ m (∠ BCD) = m (∠ BDC)

∴ In Δ BCD : BC = BD (Second req.)

11 Port Said

1

1 d   2 c   3 b   4 b   5 a   6 b

2

[a] ∵ MF = ME (lengths of two radii)

, XF = YE ∴ MX = MY

∵ $\overline{MX}$ ⊥ $\overline{AB}$, $\overline{MY}$ ⊥ $\overline{CD}$

∴ AB = CD (Q.E.D.1)

∵ $\overline{MX}$ ⊥ $\overline{AB}$

∴ X is the midpoint of $\overline{AB}$

∴ AX = $\frac{1}{2}$ AB , ∵ $\overline{MY}$ ⊥ $\overline{CD}$

∴ Y is the midpoinf of $\overline{CD}$

∴ CY = $\frac{1}{2}$ CD , ∵ AB = CD

∴ AX = CY

∴ In ΔΔ AXF , CYE

AX = CY
XF = YE
m (∠ AXF) = m (∠ CYE) = 90°

∴ Δ AXF ≡ Δ CYE , AF = CE (Q.E.D.2)

[b] ∵ m (∠ A) = $\frac{1}{2}$ [m ($\widehat{CE}$) – m ($\widehat{BD}$)]

∴ 30° = $\frac{1}{2}$ [120° – m ($\widehat{BD}$)]

∴ 60° = 120° – m ($\widehat{BD}$)

∴ m ($\widehat{BD}$) = 120° – 60° = 60° (The req.)

3

[a] In Δ ABC : ∵ m (∠ BAC) = 90° , AC = $\frac{1}{2}$ BC

∴ m (∠ B) = 30°

∴ m (∠ C) = 180° – (90° + 30°) = 60°

∴ m (∠ C) = m (∠ DAB) = 60°

∴ $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC (Q.E.D.)

[b] ∵ D is the midpoint of $\overline{AB}$

∴ $\overline{MD}$ ⊥ $\overline{AB}$ ∴ m (∠ ADM) = 90°

, ∵ E is the midpoint of $\overline{AC}$

∴ $\overline{ME}$ ⊥ $\overline{AC}$ ∴ m (∠ AEM) = 90°

From the quadrilateral MDAE :

∴ m (∠ DME) = 360° – (90° + 90° + 120°) = 60°

, ∵ m (∠ YMX) = m (∠ DME) = 60° (V.O.A)

, MY = MX = r

∴ Δ XMY is an equilateral triangle. (Q.E.D.)

4

[a] In Δ AMC : ∵ MA = MC = r

∴ m (∠ MCA) = m (∠ MAC) = 25° (1)

In Δ BMC : ∵ MB = MC = r

∴ m (∠ MCB) = m (∠ MBC) = 45° (2)

From (1) and (2) :

∴ m (∠ ACB) = m (∠ MCA) + m (∠ MCB)

∴ m (∠ ACB) = 25° + 45° = 70°

∴ m (∠ AMB) = 2 m (∠ ACB) = 2 × 70° = 140°

(central and inscribed angles subtended by $\widehat{AB}$)

(The req.)

[b] ∵ ABCE is a cyclic quadrilateral

∴ m (∠ XEA) = m (∠ ABC)

, ∵ ABDF is a cyclic quadrilateral

∴ m (∠ XFA) = m (∠ ABD)

, ∵ m (∠ ABC) + m (∠ ABD) = 180°

∴ m (∠ XEA) + m (∠ XFA) = 180°

∴ AFXE is a cyclic quadrilateral. (Q.E.D.)

5

[a] ∵ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the greater circle

∴ AB = AC

∴ $2x - 3 = 15$ ∴ $2x = 18$

∴ $x = 9$ cm.

, ∵ $\overline{AC}$, $\overline{AD}$ are two tangent-segments to the smaller circle

∴ AC = AD ∴ y − 2 = 15

∴ y = 17 cm. (The req.)

[b] ∵ ABCD is a parallelogram

∴ AD = BC , ∵ BE = AD

∴ BC = BE

∴ In Δ BCE : m (∠ C) = m (∠ BEC)

, ∵ m (∠ C) = m (∠ BAD) (from the parallelogram)

∴ m (∠ BAD) = m (∠ BED) and they are drawn on $\overline{BD}$ and on one side of it

∴ The figure ABDE is a cyclic quadrilateral. (Q.E.D.)

12 Damietta

1

1 b	2 d	3 c	4 b	5 a	6 b

2

[a] ∵ $\overleftrightarrow{AD}$ is a tangent

∴ $\overline{MD} \perp \overline{AD}$ ∴ m (∠ MDA) = 90°

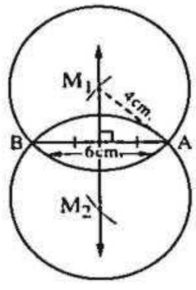
, ∵ E is a midpoint of $\overline{BC}$

∴ $\overline{ME} \perp \overline{BC}$ ∴ m (∠ MEA) = 90°

From the quadrilateral ADME

∴ m (∠ DME) = 360° − (65° + 90° + 90°) = 115° (The req.)

[b]



∴ We can draw two circles.

3

[a] ∵ m (∠ BMC) = 2 m (∠ A)

(central and inscribed angles subtended by $\widehat{BC}$)

∴ m (∠ BMC) = 2 × 30° = 60° (First req.)

In Δ MBC : ∵ MB = MC = r

, m (∠ BMC) = 60°

∴ Δ MBC is equilateral. (Second req.)

[b] ∵ $\overline{AD}$ // $\overline{BC}$

∴ m ($\widehat{AB}$) = m ($\widehat{DC}$) ∴ AB = DC

, ∵ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{DC}$

∴ MX = MY (Q.E.D.)

4

[a] ∵ $\overrightarrow{CB}$ is a tangent

∴ m (∠ BAE) = m (∠ CBE)

(inscribed and tangency angles subtended by $\widehat{BE}$)

, ∵ m ($\widehat{BE}$) = m ($\widehat{EA}$)

∴ m (∠ BAE) = m (∠ EAF)

∴ m (∠ CBD) = m (∠ CAD) and they are drawn on $\overline{CD}$ and on one side of it

∴ ABCD is a cyclic quadrilateral (Q.E.D.)

[b] ∵ m (∠ XYZ) (tangency)

= m (∠ L) (inscribed) = 70°

, ∵ $\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents

∴ XY = XZ

∴ m (∠ XYZ) = m (∠ XZY) = 70°

In Δ XYZ :

∴ m (∠ X) = 180° − 2 × 70° = 40° (First req.)

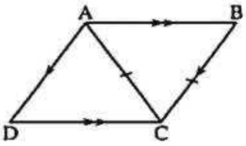
In Δ LZY : ∵ YZ = LZ

∴ m (∠ LYZ) = m (∠ L) = 70°

, ∵ m (∠ LYZ) = m (∠ XZY) and they are alternate angles.

∴ $\overline{XZ}$ // $\overline{YL}$ (Second req.)

5



[a] In ABC :

$\because$ AC = BC

$\therefore$ m (∠ B) = m (∠ CAB) (1)

, $\because$ $\overline{AB}$ // $\overline{CD}$, $\overline{AC}$ is transversal

$\therefore$ m (∠ DCA) = m (∠ CAB) (alternate angles) (2)

From (1) and (2) : $\therefore$ m (∠ DCA) = m (∠ B)

$\therefore$ $\overrightarrow{CD}$ is a tangent to the circle circumscribed about the triangle ABC (Q.E.D.)

[b] $\because$ LMNE is a cyclic quadrilateral

$\therefore$ m (∠ MLN) = m (∠ MEN) = 35° (First req.)

, $\because$ m (∠ ELN) = m (∠ ELM) – m (∠ MLN)

$\therefore$ m (∠ ELN) = 80° – 35° = 45°

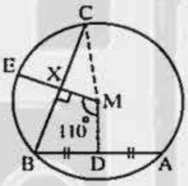
$\therefore$ m (∠ EMN) = m (∠ ELN) = 45° (Second req.)

13 Kafr El-Sheikh

1

1 a 2 c 3 c 4 b 5 b 6 d

2



[a] **Construction :**

Draw $\overline{MC}$

Proof :

$\because$ $\overline{MX} \perp \overline{BC}$

$\therefore$ X is the midpoint of $\overline{BC}$

$\therefore$ XC = 8 cm.

In Δ XMC :

$\because$ m (∠ CXM) = 90° , CM = r = 10 cm.

$\therefore$ MX = $\sqrt{(CM)^2 - (XC)^2} = \sqrt{100 - 64}$
$= \sqrt{36} = 6$ cm.

$\therefore$ XE = 10 – 6 = 4 cm. (First req.)

$\because$ D is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MD} \perp \overline{AB}$

From the quadrilateral BDMX :

$\therefore$ m (∠ ABC) = 360° – (90° + 90° + 110°) = 70° (Second req.)

[b] $\because$ $\overrightarrow{BA}$ is a tangent

$\therefore$ $\overline{MA} \perp \overline{AB}$ $\therefore$ m (∠ BAM) = 90°

In Δ AMB : m (∠ AMB) = 180° – (90° + 20°) = 70°

, $\because$ m (∠ ADE) = $\frac{1}{2}$ m (∠ AME)

(inscribed and central angles subtended by $\widehat{AE}$)

$\therefore$ m (∠ ADB) = $\frac{1}{2}$ × 70° = 35° (The req.)

3

[a] $\because$ $\overline{AD}$ // $\overline{CB}$ $\therefore$ m ($\widehat{BD}$) = m ($\widehat{AC}$)

$\therefore$ m (∠ BAD) = m (∠ CDA)

$\therefore$ In Δ ADE : EA = ED (Q.E.D.)

[b] $\because$ $\overrightarrow{EA}$, $\overrightarrow{EB}$ are two tangents to the circle

$\therefore$ EA = EB

In Δ ABE :

$\therefore$ m (∠ EAB) = m (∠ EBA) = $\frac{180° - 50°}{2}$ = 65°

, $\because$ m (∠ ADC) (inscribed)
= m (∠ CAE) (tangency) = 115°

$\therefore$ m (∠ BAC) = 115° – 65° = 50°

$\because$ m (∠ AEB) = m (∠ BAC)

$\therefore$ $\overleftrightarrow{AC}$ is a tangent to the circle passing through the points A , B and E (Q.E.D.)

4

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m (∠ ADC) = m (∠ ABE) = 110°

, $\because$ m (∠ ADB) = $\frac{1}{2}$ m ($\widehat{AB}$) = $\frac{1}{2}$ × 100° = 50°

$\therefore$ m (∠ BDC) = 110° – 50° = 60° (The req.)

[b] $\because$ $\overrightarrow{FB}$, $\overrightarrow{FD}$ are two tangents to the circle

$\therefore$ BF = DF = 4 cm.

$\therefore$ AB = 10 + 4 = 14 cm.

, $\because$ $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$ are two tangents to the circle

$\therefore$ AC = AB = 14 cm.

$\therefore$ EC = 14 – 9 = 5 cm. (The req.)

5

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$

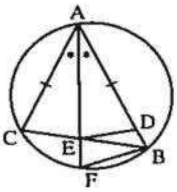
$\therefore$ $\overline{MY} \perp \overline{AC}$, $\because$ MX = MY

$\therefore$ AB = AC

In Δ ABC : $\therefore$ m (∠ C) = m (∠ B) = 70°

$\therefore$ m (∠ A) = 180° – (70° + 70°) = 40° (The req.)

[b] In ΔΔ ADE , ACE



$\begin{cases} AD = AC \\ m(\angle DAE) = m(\angle CAE) \\ \overline{AE} \text{ is a common side} \end{cases}$

∴ Δ ADE ≡ Δ ACE

, m (∠ ADE) = m (∠ ACE)

, ∵ m (∠ AFB) = m (∠ ACB)

(two inscribed angles subtended by $\overset{\frown}{AB}$)

∴ m (∠ AFB) = m (∠ ADE)

∴ BDEF is a cyclic quadrilateral. (Q.E.D.)

14 El-Beheira

1

1 d 2 c 3 b 4 b 5 c 6 a

2

[a] ∵ X is the midpoint of $\overline{AC}$

∴ $\overline{MX} \perp \overline{AC}$ ∴ m (∠ AXY) = 90°

, ∵ $\overleftrightarrow{YB}$ is a tangent to the circle

∴ $\overline{MB} \perp \overline{BY}$ ∴ m (∠ MBY) = 90°

, ∵ m (∠ AXY) = m (∠ ABY) and they are drawn on $\overline{AY}$ and on one side of it

∴ AXBY is a cyclic quadrilateral. (Q.E.D.)

[b] ∵ $\overline{CM}$ // $\overline{AB}$, $\overleftrightarrow{AM}$ is a transversal

∴ m (∠ CMA) = m (∠ A) = 60°

, ∵ m (∠ B) = $\frac{1}{2}$ m (∠ CMA)

(two inscribed angles subtended by $\overset{\frown}{AC}$)

∴ m (∠ B) = $\frac{1}{2} \times 60° = 30°$ (The req.)

3

[a] ∵ m (∠ B) = m (∠ C) ∴ AB = AC

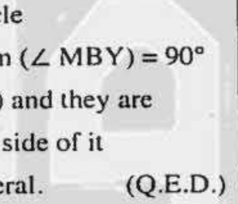
, ∵ X is the midpoint of $\overline{AB}$

∴ $\overline{MX} \perp \overline{AC}$, $\overline{MY} \perp \overline{AC}$

∴ MX = MY (Q.E.D.)

[b] Construction :

Draw $\overline{AM}$, $\overline{ME}$



Proof :

∵ AB = BC = CD = DE = AE

(The properties of the regular pentagon)

∴ m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{DE}$) = m ($\overset{\frown}{AE}$)

∵ measure of the circle = 360°

∴ m ($\overset{\frown}{AE}$) = $\frac{360°}{5}$ = 72° (First req.)

∴ m (∠ AME) = m ($\overset{\frown}{AE}$) = 72°

∵ $\overrightarrow{AX}$ is a tangent to the circle at A

∴ m (∠ MAX) = 90°

similarly m (∠ MEX) = 90°

In the quadrilateral MAXE :

∴ m (∠ AXE) = 360° − (72° + 90° + 90°) = 108° (Second req.)

4

[a] In Δ AMC : ∵ AM = MC = r

∴ m (∠ MAC) = m (∠ ACM)

∵ m (∠ BAC) = m (∠ MAC)

∴ m (∠ BAC) = m (∠ ACM) and they are alternate angles.

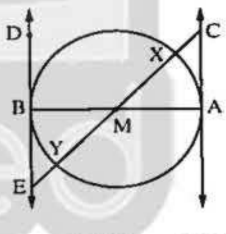
∴ $\overline{AB}$ // $\overline{CM}$

∵ D is the midpoint of $\overline{AB}$

∴ $\overline{MD} \perp \overline{AB}$, ∵ $\overline{AB}$ // $\overline{CM}$

∴ $\overline{DM} \perp \overline{CM}$ (Q.E.D.)

[b] ∵ $\overleftrightarrow{AC}$ is a tangent to the circle M at A



∴ $\overline{MA} \perp \overleftrightarrow{AC}$

∴ m (∠ CAM) = 90°

∵ $\overleftrightarrow{BD}$ is a tangent to the circle M at B

∴ $\overline{MB} \perp \overleftrightarrow{BD}$ ∴ m (∠ EBM) = 90°

In ΔΔ CAM , EBM :

$\begin{cases} m(\angle CAM) = m(\angle EBM) = 90° \\ m(\angle AMC) = m(\angle BME) \text{ (V.O.A.)} \\ MA = MB \text{ (lengths of two radii)} \end{cases}$

∴ The two triangles are congruent and we deduce that CM = EM

, ∵ XM = YM (lengths of two radii)

, by subtracting

∴ CX = YE (Q.E.D.)

5

[a] ∵ $\overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

∴ XA = XB

∴ In Δ ABX

$m(\angle XAB) = m(\angle XBA) = \frac{180° - 50}{2} = 65°$

, ∵ ABCD is a cyclic quadrilateral

$m(\angle BAD) + m(\angle DCB) = 180°$

∴ $m(\angle BAD) = 180° - 115° = 65°$

∴ $m(\angle XAB) = m(\angle BAD)$

∴ $\overrightarrow{AB}$ bisects ∠ DAX (Q.E.D.1)

, ∵ m (∠ ADB) (inscribed)

= m (∠ XAB) (tangency) = 65°

∴ $m(\angle BAD) = m(\angle ADB)$

∴ BD = BA (Q.E.D.2)

[b] ∵ AB = CD

∴ $m(\widehat{AB}) = m(\widehat{CD})$

Subtracting $m(\widehat{BD})$ from both sides

∴ $m(\widehat{AD}) = m(\widehat{BC})$

∴ $m(\angle ACD) = m(\angle BAC)$

∴ In Δ ACE : AE = CE

∴ Δ ACE is an isosceles triangle. (Q.E.D.)

15 El-Fayoum

1

1 c 2 b 3 a 4 d 5 a 6 c

2

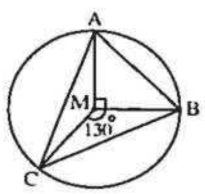
[a] ∵ AB = CD

, $\overline{ME} \perp \overline{AB}$, $\overline{MO} \perp \overline{CD}$

∴ ME = MO ∴ $x + 2 = 6$

∴ $x = 4$ cm. (First req.)

∴ CD = AB = 3 × 4 + 4 = 16 cm. (Second req.)



[b] ∵ $m(\angle C) = \frac{1}{2} m(\angle AMB)$

$= \frac{1}{2} \times 90° = 45°$

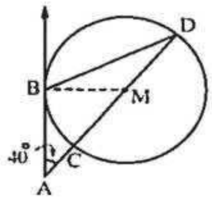
(inscribed and central angles subtended by $\widehat{AB}$)

, ∵ $m(\angle A) = \frac{1}{2} (\angle BMC) = \frac{1}{2} \times 130° = 65°$

(inscribed and central angles subtended by $\widehat{BC}$)

∴ $m(\angle B) = 180° - (45° + 65°) = 70°$ (The req.)

3



[a] Constraction :

Draw $\overline{MB}$

Proof :

∵ $\overleftrightarrow{AB}$ is a tangent to the circle

∴ $\overline{MB} \perp \overline{AB}$

∴ m (∠ MBA) = 90°

In Δ ABM :

$m(\angle BMA) = 180° - (90° + 40°) = 50°$

$m(\angle BDC) = \frac{1}{2} m(\angle BMC) = \frac{1}{2} \times 50° = 25°$

(inscribed and central angles subtended by $\widehat{BC}$)

(The req.)

[b] ∵ X is the midpoint of $\overline{AC}$

∴ $\overline{MX} \perp \overline{AC}$ ∴ m (∠ AXY) = 90°

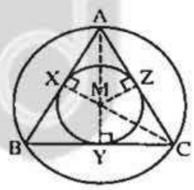
, ∵ $\overleftrightarrow{YB}$ is a tangent to the circle

∴ $\overline{MB} \perp \overline{BY}$ ∴ m (∠ MBY) = 90°

, ∵ m (∠ AXY) = m (∠ ABY) and they are drawn on $\overline{AY}$ and on one side of it

∴ AXBY is a cyclic quadrilateral (Q.E.D.)

4



[a] Construction :

Draw $\overline{XM}$, $\overline{YM}$, $\overline{ZM}$, $\overline{AY}$, $\overline{CM}$

Proof :

∵ $\overline{XM} \perp \overline{AB}$, $\overline{YM} \perp \overline{BC}$, $\overline{ZM} \perp \overline{AC}$

, ∵ XM = YM = ZM = r

∴ AB = BC = AC

∴ Δ ABC is an equilateral triangle (First req.)

In Δ MYC : m (∠ MYC) = 90°

∴ $(YC)^2 = (MC)^2 - (MY)^2 = (4)^2 - (2)^2 = 12$

∴ $YC = 2\sqrt{3}$ cm. ∴ $BC = 4\sqrt{3}$ cm.

∴ The area of Δ ABC $= \frac{1}{2} \times BC \times AY$

$= \frac{1}{2} \times 4\sqrt{3} \times 6$

$= 12\sqrt{3}$ cm². (Second req.)

[b] $\because$ m ($\angle$ BCD) = $\frac{1}{2}$ m ($\angle$ BMD)

(inscribed and central angles subtended by $\widehat{BD}$)

$\therefore$ m ($\angle$ BCD) = $\frac{1}{2}$ × 130° = 65°

$\because$ $\overline{AB}$ // $\overline{CD}$, $\overrightarrow{BC}$ is a transversal

$\therefore$ m ($\angle$ ABC) = m ($\angle$ BCD) = 65°

(alternate angles) (1)

, $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent segments

$\therefore$ AB = AC

$\therefore$ m ($\angle$ ACB) = m ($\angle$ ABC) = 65° (2)

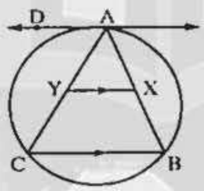
From (1) and (2) :

$\therefore$ m ($\angle$ ACB) = m ($\angle$ BCD) = 65°

$\therefore$ $\overrightarrow{CB}$ bisects $\angle$ ACD (Q.E.D.)

5

[a] $\because$ $\overleftrightarrow{AD}$ is a tangent to the circle



$\therefore$ m ($\angle$ DAC) (tangency)

= m ($\angle$ B) (inscribed) (1)

, $\because$ $\overleftrightarrow{XY}$ // $\overline{BC}$, $\overleftrightarrow{AB}$

is a transversal

$\therefore$ m ($\angle$ AXY) = m ($\angle$ B) (2)

(corresponding angles)

From (1) and (2) : $\therefore$ m ($\angle$ AXY) = m ($\angle$ DAC)

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] $\because$ X is the midpoint of $\overline{AC}$

$\therefore$ $\overline{MX}$ $\perp$ $\overline{AC}$

$\therefore$ m ($\angle$ CXM) = 90°

, $\because$ $\overrightarrow{BD}$ is a tangent to the circle

$\therefore$ $\overline{BD}$ $\perp$ $\overline{AB}$

$\therefore$ m ($\angle$ DBM) = 90°

$\therefore$ m ($\angle$ CXM) + m ($\angle$ DBM) = 180°

$\therefore$ XMBD is a cyclic quadrilateral (Q.E.D.1)

, $\because$ $\angle$ BMY is an exterior angle of the cyclic quadrilateral XMBD

$\therefore$ m ($\angle$ BMY) = m ($\angle$ D) (1)

, $\because$ m ($\angle$ BAY) = $\frac{1}{2}$ m ($\angle$ BMY) (2)

(inscribed and central angles subtended the same arc $\widehat{BY}$)

From (1) and (2) :

$\therefore$ m ($\angle$ BAY) = $\frac{1}{2}$ m ($\angle$ D) (Q.E.D.2)

16 Beni Suef

1

1 c	2 a	3 c	4 c	5 b	6 c

2

[a] $\because$ m ($\angle$ AMB) = 2 m ($\angle$ ADB) = 2 × 70° = 140°

(central and inscribed angles subtended by $\widehat{AB}$)

In Δ ABM : $\because$ $\overline{MC}$ $\perp$ $\overline{AB}$

, MA = MB = r

$\therefore$ m ($\angle$ AMC) = $\frac{1}{2}$ m ($\angle$ AMB) = $\frac{1}{2}$ × 140 = 70

(The req.)

[b] $\because$ AB = CD

, $\overline{MX}$ $\perp$ $\overline{AB}$, $\overline{NY}$ $\perp$ $\overline{CD}$

$\therefore$ MX = NY , $\overline{MX}$ // $\overline{NY}$

$\therefore$ MXYN is a rectangle (Q.E.D.)

3

[a] $\because$ D is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MD}$ $\perp$ $\overline{AB}$ $\therefore$ m ($\angle$ ADM) = 90°

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore$ $\overline{ME}$ $\perp$ $\overline{AC}$ $\therefore$ m ($\angle$ AEM) = 90°

$\therefore$ ADME is a cyclic quadrilateral

$\therefore$ m ($\angle$ DME) = 360° – (90° + 90° + 50°) = 130°

(The req.)

[b] $\because$ AB = BC

$\therefore$ m ($\angle$ BAC) = m ($\angle$ ACB) = 55°

, $\because$ m ($\angle$ BDC) = m ($\angle$ BAC) 55° and they are drawn on $\overline{BC}$ and on one side of it

$\therefore$ ABCD is a cyclic quadrilateral (Q.E.D.)

4

[a] $\because$ m ($\angle$ ACB) = $\frac{1}{2}$ m ($\angle$ AMB)

(inscribed and central angles subtended the same arc $\widehat{AB}$)

$\therefore$ m ($\angle$ ACB) = $\frac{1}{2}$ × 120° = 60° (1)

$\because$ $\overleftrightarrow{ED}$ // $\overline{AB}$

$\therefore$ m ($\widehat{AC}$) = m ($\widehat{BC}$)

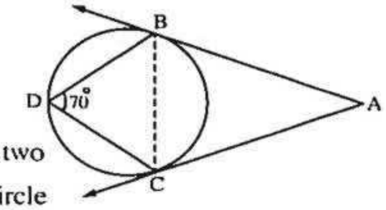
$\therefore$ AC = BC (2)

From (1) and (2) :

$\therefore$ Δ CAB is an equilateral triangle. (Q.E.D.)

[b] Construction :

Draw $\overline{BC}$



Proof :

$\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

, $\because m(\angle ABC)$ (tangency) $= m(\angle BDC)$ (inscribed) $= 70°$

$\therefore$ In ΔABC :

$m(\angle A) = 180° - (70° + 70°) = 40°$ (The req.)

5

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$ (1)

In ΔBCD : $\because BC = BD$

$\therefore m(\angle BDC) = m(\angle BCD)$ (2)

, $\because m(\angle BDC)$ (inscribed) $= m(\angle ABC)$ (tangency) (3)

From (1) , (2) and (3) :

$\therefore m(\angle A) = m(\angle CBD)$

$\therefore \overrightarrow{BD}$ is a tangent to the circle passing through the vertices of ΔABC (Q.E.D.)

[b] $\because \overleftrightarrow{BC}$ is a tangent to the circle

$\therefore \overline{AB} \perp \overline{BC}$

$\therefore m(\angle ABC) = 90°$

, $\because$ E is the midpoint of $\overline{AD}$

$\therefore \overline{ME} \perp \overline{AD}$

$\therefore m(\angle CEM) = 90°$

$\therefore m(\angle ABC) + m(\angle CEM) = 180°$

$\therefore$ EMBC is a cyclic quadrilateral (Q.E.D.)

17 El-Menia

1

1 b   2 d   3 b   4 b   5 c   6 a

2

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$, $\because AB = AC$

$\therefore MX = MY$, $\because ME = MD = r$

$\therefore XE = YD$ (Q.E.D.)

[b] In ΔABC : $\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = 30°$

$\therefore m(\angle A) = 180° - 2 \times 30° = 120°$

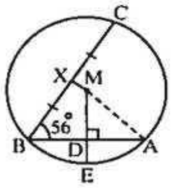
, $\because m(\angle A) + m(\angle C) = 120° + 60° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral (Q.E.D.)

3

[a] Construction :

Draw $\overline{AM}$



Proof :

$\because \overline{MD} \perp \overline{AB}$

$\therefore m(\angle MDB) = 90°$

, $\because$ X is the midpoint of $\overline{BC}$

$\therefore \overline{MX} \perp \overline{BC}$

$\therefore m(\angle MXB) = 90°$

In the quadrilateral MDXB :

$\therefore m(\angle DMX) = 360° - (56° + 90° + 90°) = 124°$ (First req.)

$\because \overline{MD} \perp \overline{AB}$

$\therefore$ D is the midpoint of $\overline{AB}$

$\therefore AD = 4$ cm.

In ΔADM :

$(MD)^2 = (AM)^2 - (AD)^2 = (5)^2 - (4)^2 = 25 - 16 = 9$

$\therefore MD = 3$ cm.

$\therefore DE = 5 - 3 = 2$ cm. (Second req.)

[b] $\because \overrightarrow{AD}$ is a tangent to the circle

$\therefore m(\angle DAB)$ (tangency) $= m(\angle ACB)$ (inscribed) (1)

, $\because \overline{XY} \mathbin{/\!/} \overline{BC}$, $\overleftrightarrow{YC}$ is a transversal

$\therefore m(\angle AYX) = m(\angle ACB)$ (corresponding angles) (2)

$\therefore$ From (1) and (2) :

$\therefore m(\angle DAB) = m(\angle AYX)$

$\therefore \overrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

4

[a] $\because AB = AC$

$\therefore m(\widehat{AB}) = m(\widehat{AC})$

$\therefore m(\angle AEB) = m(\angle AEC)$ (Q.E.D.)

[b] $\because \overrightarrow{XA}, \overrightarrow{XB}$ are two tangents to the circle

$\therefore XA = XB$

$\therefore m(\angle XAB) = m(\angle XBA) = \frac{180° - 70}{2} = 55°$ (1)

$, \because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle DAB) = 180° - 125° = 55°$ (2)

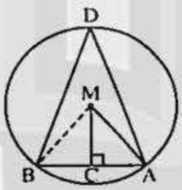
From (1) and (2) :

$\therefore m(\angle DAB) = m(\angle XAB)$ (Q.E.D.)

5

[a] Construction :

Draw $\overline{MB}$



Proof :

$\because MA = MB = r$

$, \overline{MC} \perp \overline{AB}$

$\therefore \overrightarrow{MC}$ bisects $\angle AMB$

$\therefore m(\angle AMC) = \frac{1}{2} m(\angle AMB)$ (1)

$, \because m(\angle ADB) = \frac{1}{2} m(\angle AMB)$ (2)

(inscribed and central angles subtended by $\widehat{AB}$)

$\therefore$ From (1) and (2) :

$\therefore m(\angle AMC) = m(\angle ADB)$ (Q.E.D.)

[b] $\because$ In $\Delta\Delta$ ADE , ACE

$\begin{cases} AD = AC \\ m(\angle DAE) = m(\angle CAE) \\ \overline{AE} \text{ is a common side} \end{cases}$

$\therefore \Delta ADE \equiv \Delta ACE$

$\therefore m(\angle ADE) = m(\angle ACE)$

$, \because m(\angle AFB) = m(\angle ACB)$

(two inscribed angles subtended by $\widehat{AB}$)

$\therefore m(\angle AFB) = m(\angle ADE)$

$\therefore$ BDEF is a cyclic quadrilateral. (Q.E.D.)

18 Assiut

1

1 c 2 d 3 b 4 c 5 d 6 b

2

[a] $\because \overleftrightarrow{MN}$ is the line of centres

$, \overline{AB}$ is the common chord.

$\therefore \overline{AB} \perp \overline{MN}$ $\therefore m(\angle BEN) = 90°$

In the quadrilateral CDNE :

$\therefore m(\angle CDN) = 360° - (140° + 40° + 90°) = 90°$

$\therefore \overline{ND} \perp \overline{CD}$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

[b] $\because$ AB = CD (properties of the rectangle)

$, \because CE = CD$ $\therefore AB = CE$

$\therefore m(\widehat{AB}) = m(\widehat{CE})$ and adding $m(\widehat{BE})$ to both sides.

$\therefore m(\widehat{AE}) = m(\widehat{BC})$

$\therefore AE = BC$ (Q.E.D.)

3

[a] State by yourself.

[b] $\because \overrightarrow{XY}, \overrightarrow{XZ}$ are two tangents to the circle

$\therefore XY = XZ$

$\therefore$ In ΔXYZ :

$m(\angle XYZ) = m(\angle XZY) = \frac{180° - 50°}{2} = 65°$

$, \because$ YZDE is a cyclic quadrilateral

$\therefore m(\angle EYZ) + m(\angle D) = 180°$

$\therefore m(\angle EYZ) = 180° - 115° = 65°$

$, \because m(\angle YEZ)$ (inscribed)

$= m(\angle XYZ)$ (tangency) $= 65°$

$\therefore m(\angle EYZ) = m(\angle YEZ)$

$\therefore$ In Δ YZE : ZE = ZY (Q.E.D.)

4

[a] In ΔABC : $\because m(\angle B) = m(\angle C)$

$\therefore AB = AC$

$, \because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $, \because \overline{MY} \perp \overline{AC}$

$\therefore MX = MY$ (Q.E.D.)

[b] $\because \overleftrightarrow{XY}$ is a tangent to the circle

$\therefore \overline{MY} \perp \overline{XY}$ $\therefore m(\angle XYM) = 90°$

In ΔXYM :

$\therefore m(\angle XMY) = 180° - (90° + 40°) = 50°$

, ∵ m (∠ YDC) = $\frac{1}{2}$ m (∠ YMC)

(inscribed and central angles subtended by $\widehat{YC}$)

∴ m (∠ YDC) = $\frac{1}{2}$ × 50° = 25° (The req.)

5

[a] In Δ ABC : ∵ CB = AC

∴ m (∠ BAC) = m (∠ ABC) = 65°

∴ m (∠ CAD) = 130° − 65° = 65°

, ∵ m (∠ ABC) = m (∠ CAD) = 65°

∴ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC (Q.E.D.)

[b] ∵ $\overleftrightarrow{XY}$ // $\overleftrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

∴ m (∠ DBX) = m (∠ YXB)
(alternate angles) (1)

, ∵ m (∠ C) (inscribed)
= m (∠ ABD) (tangency) (2)

From (1) and (2) :

∴ m (∠ C) = m (∠ YXB)

∴ AXYC is a cyclic quadrilateral. (Q.E.D.)

19 Souhag

1

1 b 2 c 3 d 4 c 5 b 6 b

2

[a] ∵ m (∠ AMB) = 90° ∴ m ($\widehat{AB}$) = 90°

, ∵ r = 7 cm.

∴ The length of $\widehat{AB}$ = $\frac{90°}{360°}$ × 2 × $\frac{22}{7}$ × 7 = 11 cm. (The req.)

[b] ∵ $\overleftrightarrow{AB}$ is a tangent

∴ $\overline{MA}$ ⊥ $\overleftrightarrow{AB}$ ∴ m (∠ MAB) = 90°

, ∵ E is the midpoint of $\overline{DC}$

∴ $\overline{ME}$ ⊥ $\overline{DC}$ ∴ m (∠ MEB) = 90°

From the quadrilateral ABEM :

∴ m (∠ EMA) = 360° − (50° + 90° + 90°) = 130° (The req.)

3

[a] State by yourself.

[b] ∵ ∠ CBE is an exterior angle of the cyclic quadrilateral ABCD

∴ m (∠ ADC) = m (∠ CBE) = 85°

, ∵ m (∠ ADB) (inscribed) = $\frac{1}{2}$ m ($\widehat{AB}$)
= $\frac{1}{2}$ × 110° = 55°

∴ m (∠ BDC) = 85° − 55° = 30° (The req.)

4

[a] ∵ $\overrightarrow{AB}$, $\overrightarrow{CD}$ are two tangents to the circles M , N

In circle M

BF = DF (1)

, in circle N : AF = CF (2)

Subtracting (1) from (2) :

∴ AF − BF = CF − DF

∴ AB = CD (Q.E.D.)

[b] ∵ $\overleftrightarrow{AB}$ is a tangent to the circle

∴ $\overline{MB}$ ⊥ $\overline{AB}$ ∴ m (∠ ABM) = 90°

In Δ ABM :

∴ m (∠ AMB) = 180° − (40° + 90°) = 50°

, ∵ m (∠ BDC) = $\frac{1}{2}$ m (∠ BMC)

(inscribed and central angles subtended by $\widehat{BC}$)

∴ m (∠ BDC) = $\frac{1}{2}$ × 50° = 25° (The req.)

5

[a] ∵ AB = CD , $\overline{ME}$ ⊥ $\overline{AB}$, $\overline{MF}$ ⊥ $\overline{CD}$

∴ ME = MF ∴ x + 2 = 6

∴ x = 4 cm. (First req.)

, $\overline{CD}$ = 3 × 4 + 4 = 16 cm. (Second req.)

[b] ∵ $\overleftrightarrow{XY}$ // $\overleftrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

∴ m (∠ DBX) = m (∠ BXY)
(alternate angles) (1)

, ∵ m (∠ C) (inscribed)
= m (∠ ABD) (tangency) (2)

From (1) and (2) :

∴ m (∠ C) = m (∠ BXY)

∴ AXYC is a cyclic quadrilateral. (Q.E.D.)

20 Qena

1

1 b   2 a   3 c   4 a   5 b   6 d

2

[a] The measure of the arc $= 45° \times 2 = 90°$

, its length $= \frac{90°}{360°} \times 2 \times \frac{22}{7} \times 7$

$= 11$ cm. (The req.)

[b] ∵ $\overrightarrow{DB}$, $\overrightarrow{DA}$ are two tangent to the circle M

∴ DB = DA (1)

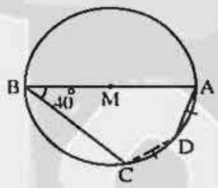
, ∵ $\overrightarrow{DC}$, $\overrightarrow{DA}$ are two tangent to the circle N

∴ DC = DA (2)

From (1) and (2) : ∴ DB = DC (Q.E.D.)

3

[a] Construction :



Draw $\overline{CD}$

Proof :

∵ D is the midpoint of $\overset{\frown}{AC}$

∴ m $(\overset{\frown}{AD})$ = m $(\overset{\frown}{DC}) = 40°$

∵ $\overline{AB}$ is a diameter

∴ m $(\overset{\frown}{BC}) = 180° - (40° + 40°) = 100°$

∴ m (∠ DAB) $= \frac{1}{2}$ m $(\overset{\frown}{BD}) = \frac{1}{2} (100° + 40°)$

$= \frac{1}{2} \times 140°$

$= 70°$ (First req.)

, ∵ m (∠ DCB) $= \frac{1}{2}$ m $(\overset{\frown}{BAD}) = \frac{1}{2} (180° + 40°)$

$= \frac{1}{2} \times 220° = 110°$

(Second req.)

[b] ∵ $\overline{AB}$, $\overline{AC}$ are two chords in the circle.

, X and Y are the two midpoints of $\overline{AB}$ and $\overline{AC}$

∴ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

∴ m (∠ MXA) = 90° , m (∠ MYA) = 90°

In Δ MDE : ∵ DE = MD = ME = r

∴ m (∠ EMD) = 60°

, ∵ m (∠ XMY) = m (∠EMD) = 60° (V.O.A.)

In the quadrilateral AXMY :

∴ m (∠ BAC) = 360° − (90° + 90° + 60°) = 120°

(The req.)

4

[a] ∵ $\overline{AB}$ is a diameter of the circle.

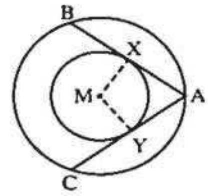
∴ m (∠ ACB) = 90°

∴ m (∠ ACE) = m (∠ ADE)

and they are drawn on $\overline{AE}$ and on one side of it

∴ ACDE is a cyclic quadrilateral. (Q.E.D.)

[b] Construction :



Draw $\overline{MX}$, $\overline{MY}$

Proof :

∵ $\overline{AB}$, $\overline{AC}$ are two tangents to the smaller circle.

∴ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

, ∵ MX = MY = r (radii of the smaller circle)

∴ AB = AC (Q.E.D.)

5

[a] ∵ ABCD is a cyclic quadr quadrilateral ilateral

∴ m (∠ BAD) = 180° − 70° = 110°

, ∵ ABFE is a cyclic quadrilateral and ∠ BAD is exterior of it.

∴ m (∠ EFB) = m (∠ BAD) = 110° (First req.)

∴ m (∠ EFB) + m (∠ BCD) = 110° + 70° = 180°

and they are interior angle in the same side of $\overleftrightarrow{FC}$

∴ $\overline{CD}$ // $\overline{EF}$ (Second req.)

[b] ∵ $\overline{AB}$, $\overline{AC}$ are tangent-segments to the circle

∴ AB = AC

∴ m (∠ ACB) $= \frac{180° - 60}{2} = 60°$ (1)

∴ m (∠ BEC) (inscribed)

= m (∠ ACB) (tangency) = 60° (2)

, ∵ EBCD is cyclic quadrilateral

∴ m (∠ EBC) = 180° − 120° = 60° (3)

∴ From (2) , (3) in Δ EBC :

∴ m (∠ BCE) = 60°

∴ Δ BCE is equilateral (Q.E.D. 1)

From (1) , (3) : ∴ m (∠ ACB) = m (∠ EBC) and they are alternate angles

∴ $\overline{AC}$ // $\overline{BE}$ (Q.E.D. 2)

21 Luxor

1

1 b   2 c   3 c   4 a   5 d   6 b

2

[a] $\because AB = CD$

, $\overline{MH} \perp \overline{AB}$, $\overline{ME} \perp \overline{CD}$

$\therefore MH = ME$ $\therefore x + 2 = 6$

$\therefore x = 4$ cm.   (First req.)

$\therefore AB = CD = 3 \times 4 + 4 = 16$ cm.   (Second req.)

[b] $\because \overrightarrow{AM} // \overrightarrow{CD}$, $\overleftrightarrow{MD}$ is a transversal.

$\therefore m(\angle CDM) + m(\angle AMD) = 180°$

(two interior angles in the same side of the transversal)

$\therefore m(\angle CDM) = 180° - 90° = 90°$

, $\because MD = \frac{1}{2} MB$, $MC = MB = r$

$\therefore MD = \frac{1}{2} MC$ $\therefore m(\angle MCD) = 30°$

, $\because \overrightarrow{AM} // \overrightarrow{CD}$, $\overleftrightarrow{CM}$ is a transversal.

$\therefore m(\angle AMC) = m(\angle MCD) = 30°$

(alternate angles)

$\therefore m(\widehat{AC}) = m(\angle AMC) = 30°$ (The req.)

3

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangent segments

$\therefore AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC) = \frac{180° - 50°}{2} = 65°$

(First req.)

, $\because \overline{MC}$ is a radius $\therefore \overline{MC} \perp \overline{AC}$

$\therefore m(\angle ACM) = 90°$

$\therefore m(\angle BCM) = 90° - 65° = 25°$ (Second req.)

[b] $\because m(\widehat{AX}) = m(\widehat{AY})$

$\therefore m(\angle ACX) = m(\angle ABY)$

, $\because$ They are drawn on $\overline{HD}$ and on one side of it.

$\therefore$ DBCH is a cyclic quadrilateral.   (Q.E.D.1)

$\therefore m(\angle DHB) = m(\angle DCB)$

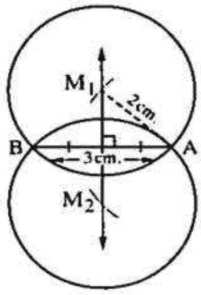
, $\because m(\angle XCB) = m(\angle XAB)$

(two inscribed angles subtended by $\widehat{XB}$)

$\therefore m(\angle DHB) = m(\angle XAB)$ (Q.E.D.2)

4

[a]



$\therefore$ There are two solutions.

[b] $\because \overline{BD} // \overline{XY}$ $\therefore m(\widehat{BC}) = m(\widehat{CD})$

$\therefore m(\angle BAC) = m(\angle DAC)$ (1)

$\therefore \overrightarrow{AC}$ bisects $\angle BAD$ (Q.E.D.1)

, $\because m(\angle CBD) = m(\angle DAC)$ (2)

(inscribed angles subtended by $\widehat{CD}$)

$\therefore m(\angle CBH) = m(\angle BAH)$

$\therefore \overrightarrow{BC}$ is a tangent to the circle passing by the vertices of Δ ABH   (Q.E.D.2)

5

[a] $\because \overline{AB} // \overline{DC}$, $\overline{AD}$ is a transversal to them.

$\therefore m(\angle A) + m(\angle D) = 180°$ (1)

but $\angle$ CEH is an exterior angle of the cyclic quadrilateral ABEH

$\therefore m(\angle CEH) = m(\angle A)$ (2)

From (1) and (2) :

$\therefore m(\angle CEH) + m(\angle D) = 180°$

$\therefore$ HDCE is a cyclic quadrilateral.   (Q.E.D.)

[b] $\because m(\widehat{BD}$ The major$) = 2\ m(\angle BCD)$

$= 2 \times 100° = 200°$

$\therefore m(\widehat{BCD}) = 360° - 200° = 160°$

, $\because m(\widehat{HE}) = m(\angle HME) = 50°$

$\therefore m(\angle A) = \frac{1}{2}[m(\widehat{BCD}) - m(\widehat{HE})]$

$= \frac{1}{2}[160° - 50°] = 55°$ (The req.)

22 Aswan

1

1 d   2 b   3 a   4 c   5 b   6 c

2

[a] ∵ $\overleftrightarrow{AB}$ is a tangent to the circle.

∴ $\overline{MA} \perp \overline{AB}$ ∴ m (∠ MAB) = 90°

In Δ ABM :

∵ $(BM)^2 = (AB)^2 + (AM)^2 = (8)^2 + (6)^2 = 100$

∴ BM = 10 cm.

, ∵ MA = MD = 6 cm.

∴ BD = 10 − 6 = 4 cm. (The req.)

[b] ∵ ABCD is a cyclic quadrilateral.

∴ m (∠ BCD) + m (∠ BAD) = 180°

∴ m (∠ BCD) = 180° − 120° = 60° (First req.)

, ∵ $\overrightarrow{BF}$ // $\overline{DC}$, $\overline{BC}$ is a transversal.

∴ m (∠ CBF) = m (∠ BCD) = 60° (alternate angles)

∴ m (∠ CBE) = 60° + 55° = 115°

, ∵ ∠ CBE is an exterior angle of a cyclic quadrilateral.

∴ m (∠ ADC) = m (∠ CBE) = 115° (Second req.)

3

[a] ∵ D is midpoint of $\overline{AB}$

∴ $\overline{MD} \perp \overline{AB}$

, ∵ $\overline{ME} \perp \overline{AC}$, MD = ME

∴ AB = AC

∴ In Δ ABC : m (∠ ACB) = m (∠ ABC) = 65°

∴ m (∠ BAC) = 180° − (65° + 65°) = 50° (The req.)

[b] ∵ $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$ are two tangents to the circle

∴ AB = AC

∴ In Δ ABC :

$m(\angle ABC) = m(\angle ACB) = \frac{180° - 50}{2} = 65°$

, ∵ BCDE is a cyclic quadrilateral

∴ m (∠ EBC) + m (∠ D) = 180°

∴ m (∠ EBC) = 180° − 115° = 65°

∴ m (∠ ABC) = m (∠ EBC)

∴ $\overrightarrow{BC}$ bisects ∠ ABE (Q.E.D.)

4

[a] ∵ AB = CD (properties of the rectangle)

, ∵ CE = CD ∴ AB = CE

∴ m $(\widehat{AB})$ = m $(\widehat{CE})$ and adding m $(\widehat{BE})$ to both sides

∴ m $(\widehat{AE})$ = m $(\widehat{BC})$

∴ AE = BC (Q.E.D.)

[b] ∵ $\overleftrightarrow{AD}$ is a tangent to the circle.

∴ m (∠ DAB) (tangency) = m (∠ ACB) (inscribed) (1)

, ∵ $\overleftrightarrow{XY}$ // $\overline{BC}$, $\overleftrightarrow{YC}$ is a transversal.

∴ m (∠ AYX) = m (∠ ACB) (corresponding angles) (2)

From (1) and (2) :

∴ m (∠ DAB) = m (∠ AYX)

∴ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ AXY (Q.E.D.)

5

[a] ∵ m (∠ D) = $\frac{1}{2}$ m (∠ AMB)

(inscribed and central angles substanded by $\widehat{AB}$)

∴ m (∠ D) = $\frac{1}{2}$ × 140° = 70° (First req.)

, ∵ $\overline{AC}$ // $\overline{DB}$, $\overleftrightarrow{AD}$ is transversal

∴ m (∠ DAC) + m (∠ D) = 180°

(two interior angles in the same side of the transversal)

∴ m (∠ DAC) = 180° − 70° = 110° (Second req.)

[b] In Δ ABD : ∵ AB = AD

∴ m (∠ BDA) = m (∠ ABD) = 30°

∴ m (∠ A) = 180° − (30° + 30°) = 120°

, ∵ m (∠ DCE) = m (∠ A) = 120°

∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

23 New valley

1

1 b	2 d	3 d	4 c	5 a	6 b

2

[a] ∵ ABCD is cyclic quadrilateral.

∴ m (∠ ADC) = m (∠ ABE) = 100°

In Δ ACD :

∴ m (∠ ACD) = 180° − (100° + 40°) = 40°

∴ m (∠ CAD) = m (∠ ACD)

∴ m $(\widehat{CD})$ = m $(\widehat{AD})$ (Q.E.D.)

[b] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle AXM) = 90°$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\therefore m(\angle AYM) = 90°$

From the quadrilateral AXMY :

$m(\angle DMH) = 360° - (90° + 90° + 70°) = 110°$

(First req.)

, $\because AB = AC$ $\therefore MX = MY$

, $\because MD = MH = r$ $\therefore XD = YH$

(Second req.)

3

[a] $\because \overleftrightarrow{AD}$ is a tangent to the circle.

$\therefore m(\angle DAB)$ (tangency)

$= m(\angle ACB)$ (inscribed) (1)

, $\because \overleftrightarrow{XY} \parallel \overline{BC}$, $\overleftrightarrow{YC}$ is a transversal.

$\therefore m(\angle AYX) = m(\angle ACB)$

(corresponding angles) (2)

From (1) and (2) :

$\therefore m(\angle DAB) = m(\angle AYX)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] $\because m(\angle BCD) = \frac{1}{2} m(\angle BMD)$

(inscribed and central angles subteneded by $\overset{\frown}{BD}$)

$\therefore m(\angle BCD) = \frac{1}{2} \times 130° = 65°$

$\because \overline{AB} \parallel \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal.

$\therefore m(\angle ABC) = m(\angle BCD) = 65°$

(alternate angles) (1)

, $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC) = 65°$ (2)

From (1) and (2) :

$\therefore m(\angle ACB) = m(\angle BCD) = 65°$

$\therefore \overrightarrow{CB}$ bisects $\angle ACD$ (First req.)

In Δ ABC :

$m(\angle A) = 180° - (65° + 65°) = 50°$ (Second req.)

4

[a] $\because \overline{DE} \parallel \overline{BC}$

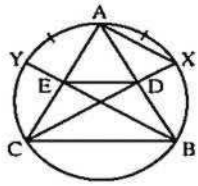
$\therefore m(\overset{\frown}{DB}) = m(\overset{\frown}{EC})$

adding $m(\overset{\frown}{BC})$ to both sides.

$\therefore m(\overset{\frown}{DC}) = m(\overset{\frown}{EB})$

$\therefore m(\angle DAC) = m(\angle BAE)$ (Q.E.D.)

[b] $\because m(\overset{\frown}{AX}) = m(\overset{\frown}{AY})$



$\therefore m(\angle ACX) = m(\angle ABY)$

and they are drawn on $\overline{ED}$ and on one side of it.

$\therefore$ BCED is a cyclic quadrilateral. (Q.E.D. 1)

, $\because m(\angle DEB) = m(\angle DCB)$

, $\because m(\angle XCB) = m(\angle XAB)$

(two inscribed angles subtended by $\overset{\frown}{XB}$)

$\therefore m(\angle DEB) = m(\angle XAB)$ (Q.E.D. 2)

5

[a] State by yourself.

[b] $\because \overline{CD}$ is a diameter in the circle.

$\therefore m(\angle CXD) = 90°$

, $\because \overline{CD} \perp \overline{AB}$

$\therefore m(\angle BEC) = 90°$

$\therefore m(\angle CXD) = m(\angle BEC)$

, $\angle BEC$ is an exterior angle of the figure XYEC

$\therefore$ XYEC is a cyclic quadrilateral. (Q.E.D. 1)

$\therefore m(\angle DYB) = m(\angle XCD)$ (1)

, $\because m(\angle DBX) = m(\angle XCD)$ (2)

(two inscribed angles subtended by $\overset{\frown}{XD}$)

From (1) and (2) :

$\therefore m(\angle DYB) = m(\angle DBX)$ (Q.E.D. 2)

24 South Sinai

1

1 a   2 b   3 c   4 d   5 a   6 b

2

[a] $\because m(\overset{\frown}{AB}) = 50°$

$\therefore m(\angle D) = \frac{1}{2} m(\overset{\frown}{AB}) = \frac{1}{2} \times 50° = 25°$

(First req.)

, $m(\angle AMB) = m(\overset{\frown}{AB}) = 50°$ (Second req.)

[b] $\because m(\overset{\frown}{BC}) = m(\overset{\frown}{AD})$

adding $m(\overset{\frown}{AC})$ to both sides

$\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$ $\therefore AB = CD$ (Q.E.D.)

3

[a] ∵ $r_1 = 5$ cm. , $r_2 = 3$ cm.

, ∵ $r_1 + r_2 = 5 + 3 = 8$ cm.

∴ $r_1 + r_2 = MN$

∴ The two circles are touching externally.

[b] ∵ $\overline{AB}$ is a tangent-segment to the circle.

, $\overline{AC}$ is a diameter of it.

∴ $\overline{AB} \perp \overline{AC}$

∴ m (∠ BAC) = 90° (1)

, ∵ m (∠ ACD) = $\frac{1}{2}$ m (∠ AMD)

(inscribed and central angles subtended by $\overset{\frown}{AD}$)

∴ m (∠ ACD) = $\frac{1}{2}$ × 60° = 30° (2)

In Δ ABC :

m (∠ ABC) = 180° – (90° + 30°) = 60° (First req.)

From (1) and (2) :

∴ AB = $\frac{1}{2}$ BC (Second req.)

4

[a] In Δ ABC : ∵ m (∠ B) = m (∠ C)

∴ AB = AC

, ∵ D is midpoint of $\overline{AB}$ ∴ $\overline{MD} \perp \overline{AB}$

, ∵ E is midpoint of $\overline{AC}$ ∴ $\overline{ME} \perp \overline{AC}$

∴ MD = ME (Q.E.D.)

[b] In Δ ABE : ∵ AB = AE

∴ m (∠ AEB) = m (∠ B)

, ∵ m (∠ D) = m (∠ B)

(properties of parallelogram)

∴ m (∠ AEB) = m (∠ D)

∴ The figure AECD is a cyclic quadrilateral.

(Q.E.D.)

5

[a] ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle.

∴ AB = AC

∴ In Δ ABC :

m (∠ ABC) = m (∠ ACB) = $\frac{180° - 50°}{2}$ = 65°

, ∵ BCDE is a cyclic quadrilateral.

∴ m (∠ EBC) + m (∠ D) = 180°

∴ m (∠ EBC) = 180° – 115° = 65°

∴ m (∠ ABC) = m (∠ EBC)

∴ $\overrightarrow{BC}$ bisects ∠ ABE (Q.E.D. 1)

, ∵ m (∠ BEC) (inscribed)

= m (∠ ABC) (tangency) = 65°

∴ m (∠ EBC) = m (∠ BEC)

∴ In Δ BCE : CB = CE (Q.E.D. 2)

[b] ∵ m ($\overset{\frown}{BC}$) = 2 m (∠ A) = 2 × 30° = 60°

, ∵ m (∠ E) = $\frac{1}{2}$ [m ($\overset{\frown}{AD}$) – m ($\overset{\frown}{BC}$)]

∴ 50° = $\frac{1}{2}$ [m ($\overset{\frown}{AD}$) – 60°]

∴ 100° = m ($\overset{\frown}{AD}$) – 60°

∴ m ($\overset{\frown}{AD}$) = 160° (First req.)

, ∵ m (∠ AFD) = $\frac{1}{2}$ [m ($\overset{\frown}{AD}$) + m ($\overset{\frown}{BC}$)]

∴ m (∠ AFD) = $\frac{1}{2}$ [160° + 60°] = 110°

(Second req.)

25 North Sinai

1

1 c 2 a 3 b 4 b 5 c 6 c

2

[a] ∵ AB = CD , $\overline{MW} \perp \overline{AB}$, $\overline{MH} \perp \overline{CD}$

∴ MX = MY

, ∵ MW = MH = r

∴ WX = HY (Q.E.D.)

[b] ∵ $\overrightarrow{CD}$ // $\overline{BA}$ ∴ m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{BC}$)

∴ AC = BC (First req.)

, ∵ $\overline{AB}$ is a diameter of the circle

∴ m (∠ ACB) = 90°

In Δ ABC : ∴ m (∠ B) = m (∠ A) = $\frac{180° - 90°}{2}$ = 45°

(Second req.)

3

[a] State by yourself.

[b] ∵ D is the midpoint of $\overline{BW}$

∴ $\overline{MD} \perp \overline{BW}$

∴ m (∠ WDM) = 90°

, ∵ $\overrightarrow{AC}$ is a tangent to the circle

∴ $\overline{AC} \perp \overline{BC}$ ∴ m (∠ ACM) = 90°

∴ m (∠ WDM) + m (∠ ACM) = 180°

∴ The figure ADMC is a cyclic quadrilateral. (Q.E.D. 1)

, ∵ ∠ CMH is an exterior angle of the cyclic quadrilateral ADMC

∴ m (∠ CMH) = m (∠ A) (1)

, ∵ m (∠ CBH) = $\frac{1}{2}$ m (∠ CMH) (2)

(inscribed and central angles subtended by $\overset{\frown}{BC}$)

From (1) and (2) :

∴ m (∠ CBH) = $\frac{1}{2}$ m (∠ A) (Q.E.D. 2)

4

[a] ∵ m (∠ A) = $\frac{1}{2}$ [m ($\overset{\frown}{CH}$) – m ($\overset{\frown}{BD}$)]

∴ 30° = $\frac{1}{2}$ [80° – m ($\overset{\frown}{BD}$)]

∴ 60° = 80° – m ($\overset{\frown}{BD}$)

∴ m ($\overset{\frown}{BD}$) = 80° – 60° = 20°

, ∵ $\overline{BC}$ is a diameter in the circle

∴ m ($\overset{\frown}{BC}$) = 180°

∴ m ($\overset{\frown}{DH}$) = 360° – [180° + 20° + 80°] = 80° (The req.)

[b] ∵ m (∠ BDC) (inscribed)

= m (∠ ABC) (tangency) = 70°

, ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents

∴ AB = AC

∴ m (∠ ABC) = m (∠ ACB) = 70°

In Δ ABC :

∴ m (∠ BAC) = 180° – (70° + 70°) = 40° (The req.)

5

[a] In Δ ABD : ∵ AB = AD

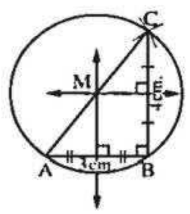
∴ m (∠ ABD) = m (∠ ADB) = 30°

∴ m (∠ A) = 180° – 2 × 30° = 120°

∴ m (∠ A) + m (∠ C) = 120° + 60° = 180°

∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b]



We can draw one circle only.

26 Red Sea

1

1 c 2 b 3 a 4 d 5 c 6 c

2

[a] ∵ AB = CD , $\overline{MX}$ ⊥ $\overline{AB}$, $\overline{MY}$ ⊥ $\overline{CD}$

∴ MX = MY

, ∵ MH = MF = r ∴ HX = FY (Q.E.D.)

[b] ∵ m (∠ ADB) = $\frac{1}{2}$ m ($\overset{\frown}{AB}$) = $\frac{1}{2}$ × 110° = 55°

, ∵ ABCD is a cyclic quadrilateral.

∴ m (∠ HBC) = m (∠ CDB) + m (∠ ADB)

= 30° + 55° = 85° (The req.)

3

[a] In Δ BMC : ∵ MB = MC = r

∴ m (∠ MCB) = m (∠ MBC) = 25°

∴ m (∠ BMC) = 180° – (25° + 25°) = 130°

, ∵ m (∠ BAC) = $\frac{1}{2}$ m (∠ BMC)

(inscribed and central angles subtended by $\overset{\frown}{BC}$)

∴ m (∠ BAC) = $\frac{1}{2}$ × 130° = 65° (The req.)

[b] In Δ ABC : ∵ AB = AC

∴ m (∠ ACB) = m (∠ ABC) = 50°

∴ m (∠ A) = 180° – (50° + 50°) = 80°

, ∵ m (∠ A) + m (∠ D) = 80° + 100° = 180°

∴ ABDC is a cyclic quadrilateral . (Q.E.D.)

4

[a] ∵ $\overleftrightarrow{MN}$ is the line of centres

, $\overline{AB}$ is the common chord

∴ $\overline{AB}$ ⊥ $\overline{MN}$ ∴ m (∠ AXN) = 90°

∵ The sum of the measures of the interior angles of the quadrilateral CDNX = 360°

∴ m (∠ CDN) = 360° – (125° + 55° + 90°) = 90°

∴ $\overline{ND}$ ⊥ $\overline{CD}$

∴ $\overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

[b] ∵ $\overleftrightarrow{AX}$ is a common tangent for two circles

∴ m (∠ BDA) (inscribed)

= m (∠ BAX) (tangency)

, m (∠ CHA) (inscribed)
= m (∠ CAX) (tangency)
∴ m (∠ BDA) = m (∠ CHA)
and they are corresponding angles
∴ $\overline{BD}$ // $\overline{CH}$ (Q.E.D.)

5

[a] ∵ m ($\widehat{BD}$) = 2 m (∠ C)
∴ m ($\widehat{BD}$) = 2 × 26° = 52°
∵ m (∠ A) = $\frac{1}{2}$ [m ($\widehat{CH}$) – m ($\widehat{BD}$)]
∴ 40° = $\frac{1}{2}$ [m ($\widehat{CH}$) – 52°]
∴ 80° = m ($\widehat{CH}$) – 52°
∴ m ($\widehat{CH}$) = 80° + 52° = 132° (First req.)
∴ m (∠ HXC) = $\frac{1}{2}$ [m ($\widehat{CH}$) + m ($\widehat{BD}$)]
= $\frac{1}{2}$ [132° + 52°] = 92°
(Second req.)

[b] ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle
∴ AB = AC
∴ m (∠ ABC) = m (∠ ACB) = $\frac{180° - 70°}{2}$ = 55°
∴ m (∠ BHC) (inscribed)
= m (∠ ABC) (tangency) = 55°
, ∵ BCDH is a cyclic quadrilateral.
∴ m (∠ CBH) + m (∠ CDH) = 180°
∴ m (∠ CBH) = 180° – 125° = 55°
In Δ BCH : ∴ m (∠ BHC) = m (∠ CBH)
∴ CB = CH (Q.E.D.)

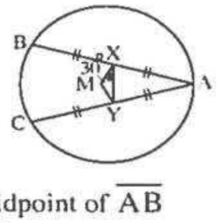
27 Matrouh

1

1 c 2 c 3 d 4 b 5 c 6 b

2

[a]



∵ X is the midpoint of $\overline{AB}$
∴ $\overline{MX}$ ⊥ $\overline{AB}$
, ∵ Y is the midpoint of $\overline{AC}$
∴ $\overline{MY}$ ⊥ $\overline{AC}$
, ∵ AB = AC ∴ MX = MY
∴ Δ MXY is an isosceles triangle. (Q.E.D.)

[b] ∵ $\overleftrightarrow{AF}$ // $\overline{DE}$, $\overleftrightarrow{AB}$ is a transversal.
∴ m (∠ BAF) = m (∠ AED)
(alternate angles) (1)
, ∵ m (∠ C) (inscribed)
= m (∠ BAF) (tangency) (2)
From (1) and (2) : ∴ m (∠ C) = m (∠ AED)
∴ DEBC is a cyclic quadrilateral. (Q.E.D.)

3

[a] ∵ m (∠ D) = $\frac{1}{2}$ m (∠ M)
(inscribed and central angles subtended by $\widehat{BC}$)
∴ m (∠ D) = $\frac{1}{2}$ × 100° = 50°
, ∵ ∠ ABD is an exterior angle of Δ BCD
∴ m (∠ ABD) = m (∠ BDC) + m (∠ DCB)
∴ m (∠ DCB) = 120° – 50° = 70° (The req.)

[b] ∵ $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are two tangents to the circle.
∴ $\overline{MA}$ ⊥ $\overline{AC}$ ∴ m (∠ MAC) = 90°
, $\overline{MB}$ ⊥ $\overline{BC}$
∴ m (∠ MBC) = 90°
∴ m (∠ MAC) + m (∠ MBC) = 180°
∴ ACBM is a cyclic quadrilateral.
, ∵ ∠ DMB is an exterior angle of it
∴ m (∠ DMB) = m (∠ ACB) (Q.E.D.)

4

[a] ∵ $\overleftrightarrow{AD}$ is a tangent to the circle.
∴ m (∠ DAB) (tangency)
= m (∠ ACB) (inscribed) (1)
, ∵ $\overleftrightarrow{XY}$ // $\overline{BC}$, $\overleftrightarrow{YC}$ is a transversal.
∴ m (∠ AYX) = m (∠ ACB)
(corresponding angles) (2)
From (1) and (2) : ∴ m (∠ DAB) = m (∠ AYX)
∴ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] ∵ $\overline{DE}$ // $\overline{BC}$
∴ m ($\widehat{DB}$) = m ($\widehat{EC}$) adding m ($\widehat{BC}$) to both sides
∴ m ($\widehat{DC}$) = m ($\widehat{EB}$)
∴ m (∠ DAC) = m (∠ BAE) (Q.E.D.)

5

[a] Prove by yourself.

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180° - 70°}{2} = 55°$

$\therefore m(\angle CEB)$ (inscribed)

$= m(\angle CBA)$ (tangency) $= 55°$

, $\because$ BCDE is a cyclic quadrilateral

$\therefore m(\angle CBE) + m(\angle CDE) = 180°$

$\therefore m(\angle CBE) = 180° - 125° = 55°$

In Δ EBC : $\therefore m(\angle CEB) = m(\angle CBE)$

$\therefore CB = CE$ (Q.E.D.1)

, $\because m(\angle ACB) = m(\angle CBE) = 55°$

and they are alternate angles

$\therefore \overrightarrow{AC} \parallel \overline{BE}$ (Q.E.D.2)

Governorates' Examinations

1 Giza Governorate

Answer the following questions :

1 Choose the correct answer :

(1) The measure of the inscribed angle is .... the measure of the central angle , subtended by the same arc.

(a) half (b) third (c) quarter (d) double

(2) It is possible to draw a circle passing through the vertices of a ....

(a) trapezium. (b) parallelogram. (c) rectangle. (d) rhombus.

(3) The centre of the inscribed circle of any triangle is the point of intersection of its ....

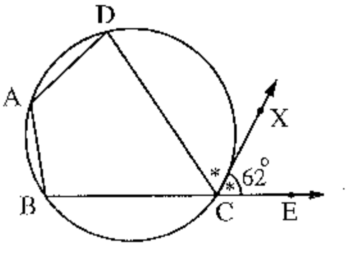
(a) altitudes. (b) medians.

(c) axes of symmetry of its sides. (d) bisectors of its interior angles.

(4) If the two circles M and N are touching internally , the radius length of one of them = 3 cm. and MN = 8 cm. , then the radius length of the other circle = ... cm.

(a) 12 (b) 11 (c) 6 (d) 5

(5) **In the opposite figure :**



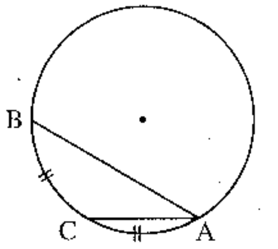
If $E \in \overrightarrow{BC}$, $\overrightarrow{CX}$ bisects ∠ DCE

, m (∠ XCE) = 62°

, then m (∠ A) = ...

(a) 62° (b) 118° (c) 56° (d) 124°

(6) **In the opposite figure :**



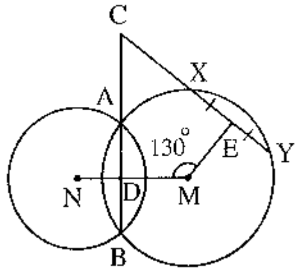
If C is the midpoint of $\overset{\frown}{AB}$

, then AB .... 2 AC

(a) < (b) > (c) ≥ (d) =

---

2 [a] In the opposite figure :



If E is the midpoint of $\overline{XY}$

, m (∠ EMN) = 130°

, then find : m (∠ C)

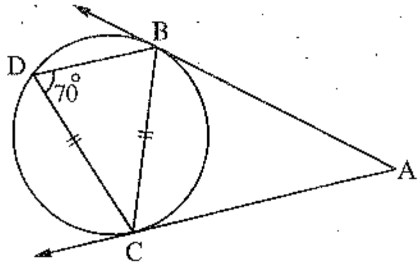
[b] In the opposite figure :

If $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m (∠ D) = 70° , CB = CD

(1) **Find :** m (∠ A)

(2) **Prove that :** $\overline{BD}$ // $\overline{AC}$

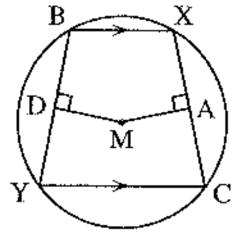


3 **[a] In the opposite figure :**

$\overline{XB}$ // $\overline{CY}$, $\overline{MA} \perp \overline{XC}$

, $\overline{MD} \perp \overline{BY}$

Prove that : MA = MD



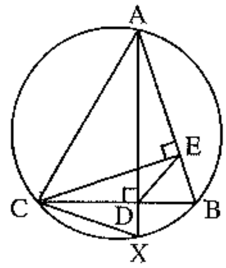
[b] In the opposite figure :

$\overline{CE} \perp \overline{AB}$, $\overrightarrow{AD} \perp \overline{BC}$ and intersects the circle at X

Prove that :

(1) AEDC is a cyclic quadrilateral.

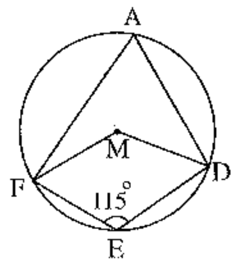
(2) $\overrightarrow{CB}$ bisects ∠ ECX



4 **[a] In the opposite figure :**

If m (∠ DEF) = 115°

, **then find :** m (∠ DMF)



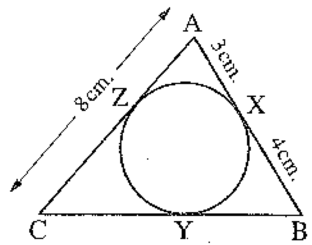
[b] In the opposite figure :

Inscribed circle of the triangle ABC touches

its sides at X , Y and Z

If AX = 3 cm. , XB = 4 cm. , AC = 8 cm.

Find : The length of $\overline{BC}$

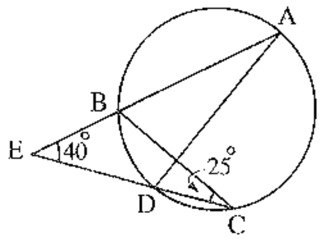


5 **[a] In the opposite figure :**

$\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, m (∠ C) = 25°

, m (∠ E) = 40°

Find : m (∠ ADC)



[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

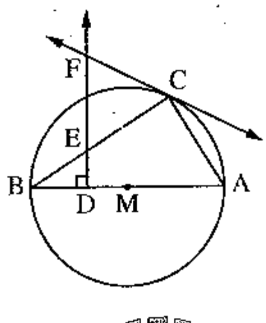
, $\overleftrightarrow{CF}$ is a tangent to the circle at C

, $\overrightarrow{DF} \perp \overline{AB}$ and intersects $\overline{BC}$ at E

Prove that :

(1) ADEC is a cyclic quadrilateral.

(2) Δ FCE is an isosceles triangle.



2 Alexandria Governorate

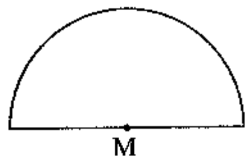
Answer the following questions :

1 Choose the correct answer from those given :

(1) The two opposite angles in the cyclic quadrilateral are ....

(a) equal. (b) supplementary. (c) complementary. (d) alternate.

(2) The opposite figure represents a semicircle its centre is M and its radius length is r length unit, then the area of the opposite figure = ... square units.



(a) $2\pi r$ (b) πr (c) πr^2 (d) $\frac{\pi r^2}{2}$

(3) In a regular hexagon, the measure of the angle of its vertex equals ....

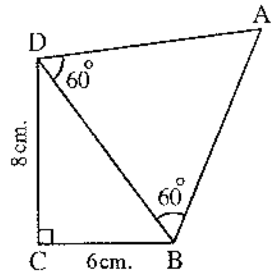
(a) 60° (b) 108° (c) 120° (d) 135°

(4) If $\overline{AB}$ is a line segment, then the number of circles can be drawn passing through A and B equals ....

(a) 1 (b) 2 (c) 3 (d) an infinite number.

(5) **In the opposite figure :**

The length of $\overline{AB}$ = ... cm.



(a) $10\sqrt{3}$ (b) 10

(c) 5 (d) $5\sqrt{3}$

(6) The inscribed angle which is opposite to the minor arc in a circle is ....

(a) acute. (b) right. (c) obtuse. (d) reflex.

---

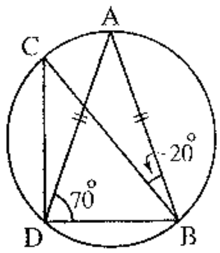
2 [a] In the opposite figure :

AB = AD

, m (∠ ABC) = 20°

, m (∠ ADB) = 70°

Find : m (∠ C), m (∠ BDC)



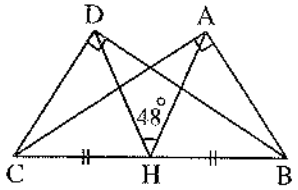
[b] In the opposite figure :

m (∠ BAC) = m (∠ BDC) = 90°

, H is the midpoint of $\overline{BC}$ and m (∠ AHD) = 48°

(1) **Prove that :** ABCD is a cyclic quadrilateral.

(2) **Find :** m (∠ ABD)



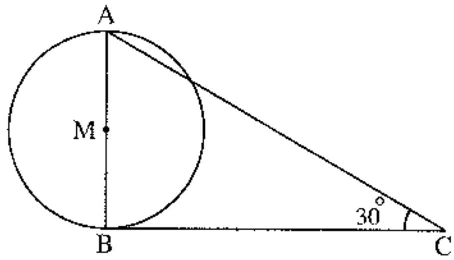
3 **[a] In the opposite figure :**

A circle M of circumference 44 cm.

, $\overline{AB}$ is a diameter , $\overline{BC}$ is a tangent at B

and m (∠ ACB) = 30°

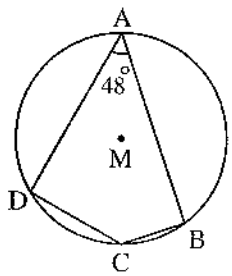
Find : The length of $\overline{BC}$ $\left(\pi = \frac{22}{7}\right)$



[b] In the opposite figure :

If M is a circle , m (∠ A) = 48°

Find : m ($\overset{\frown}{BD}$ the major)



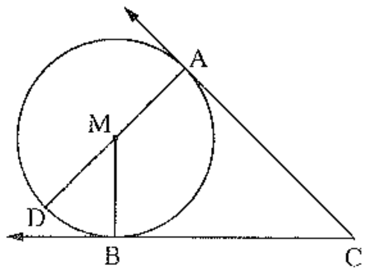
4 **[a] In the opposite figure :**

$\overline{AD}$ is a diameter in a circle M

, $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are two tangents to the circle M ,

touch it at A and B respectively.

Prove that : m (∠ DMB) = m (∠ ACB)



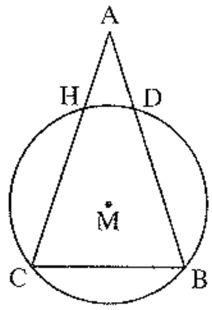
[b] In the opposite figure :

ABC is a triangle in which AB = AC

, $\overline{BC}$ is a chord in the circle M

, if $\overline{AB}$ and $\overline{AC}$ cut the circle at D and H respectively.

Prove that : m ($\overset{\frown}{DB}$) = m ($\overset{\frown}{HC}$)

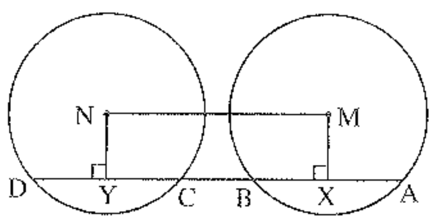


5 **[a] In the opposite figure :**

M and N are two congruent circles

, AB = CD

Prove that : The figure MXYN is a rectangle.



[b] ABCD is a quadrilateral inscribed in a circle , H is a point outside the circle and $\overrightarrow{HA}$ and $\overrightarrow{HB}$ are two tangents to the circle at A and B , if m (∠ AHB) = 70° and m (∠ ADC) = 125° , **prove that :**

(1) AB = AC

(2) $\overleftrightarrow{AC}$ is a tangent to the circle passing through the points A , B and H

3 El-Kalyoubia Governorate

Answer the following questions :

1 Choose the correct answer :

(1) If the area of the circle is 9 π cm^2 , then its radius length = ... cm.

(a) 9 (b) 2 (c) (– 3) (d) 3

(2) The number of symmetric axes of a square = ...

(a) 1 (b) 2 (c) 3 (d) 4

(3) If M is a circle of a diameter length equals 14 cm. , MA = (2 X + 3) cm. where A lies on the circle , then X = ...

(a) 5 (b) 3 (c) 2 (d) 1

(4) The raito between the measure of the inscribed angle and the measure of the central angle subtended by the same arc = ...

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3

(5) If ABCD is a cyclic quadrilateral and m (∠ B) = $\frac{1}{2}$ m (∠ D) , then m (∠ B) = ...

(a) 90° (b) 60° (c) 120° (d) 180°

(6) If the figure ABCD ~ the figure XYZL , then m (∠ B) = m (∠ ....)

(a) X (b) Y (c) Z (d) L

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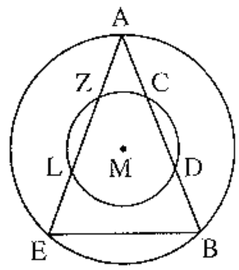
2 [a] In the opposite figure :

Two concentric circles at M

, m (∠ ABE) = m (∠ AEB)

Prove that : CD = ZL

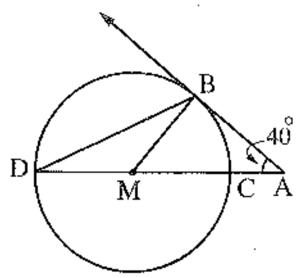


[b] In the opposite figure :

$\overrightarrow{AB}$ is a tangent to the circle M

, m (∠ A) = 40°

Find with proof : m (∠ BDC)



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3 [a] Using your geometric tools , draw $\overline{AB}$ with a length of 4 cm. , then draw a circle passing through the two points A and B whose radius length is 3 cm.

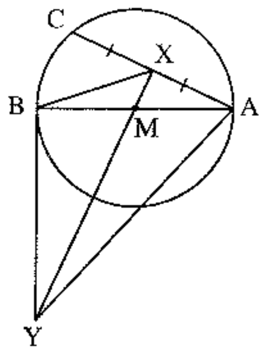
What are the possible solutions ? (Don't remove the arcs)

[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, X is the midpoint of $\overline{AC}$ and $\overrightarrow{XM}$ intersecting the tangnet of the circle at B in Y

Prove that : The figure AXBY is a cyclic quadrilateral.



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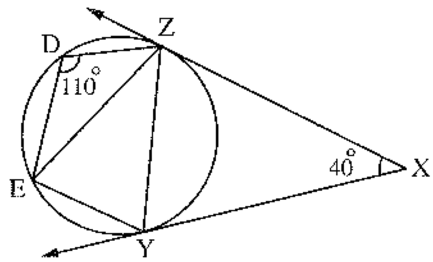
4 [a] **In the opposite figure :**

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle at the two points Y and Z , m (∠ X) = 40°

, m (∠ D) = 110°

Prove that : m (∠ ZYE) = m (∠ ZEY)

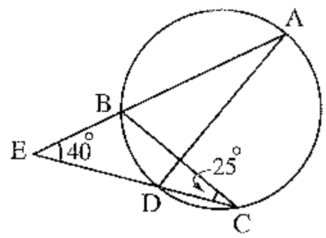


[b] In the opposite figure :

m (∠ E) = 40° , m (∠ C) = 25°

Find with proof :

(1) m (∠ ADC) (2) m ($\overset{\frown}{AC}$)



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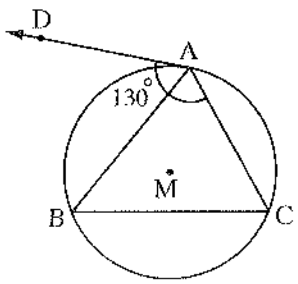
5 [a] **In the opposite figure :**

$\overrightarrow{AD}$ is the tangent to the circle M at A

, m (∠ DAC) = 130°

Find with proof : m (∠ B)



[b] ABCD is a quadrilateral drawn in a circle , E ∈ $\overrightarrow{AB}$, E ∉ $\overline{AB}$

, m ($\overset{\frown}{AB}$) = 110° , m (∠ CBE) = 85°

Find with proof : m (∠ BDC)

4 El-Sharkia Governorate

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

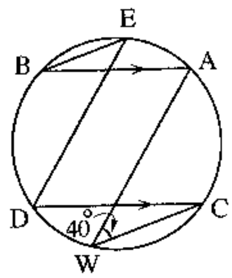
(1) The two tangents which are drawn from the two endpoints of a diameter of a circle are ....

(a) parallel. (b) perpendicular. (c) coincide. (d) intersecting.

(2) The number of the axes of symmetry of the semicircle .... the number of the axes of symmetry of the isosceles triangle.

(a) > (b) < (c) = (d) ≥

(3) **In the opposite figure :**



$\overline{AB} \mathbin{//} \overline{CD}$, m (∠ AWC) = 40° ,

then m (∠ DEB) = ...

(a) 50° (b) 40°

(c) 30° (d) 45°

(4) A circle , its radius length $(2X + 6)$ cm. and the straight line L is at distance $(X + 2)$ cm. from its centre where $X > 0$, then L is ....

(a) outside the circle. (b) a tangent to the circle.

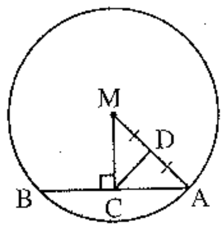
(c) a secant to the circle. (d) passing through the centre.

(5) If the straight line $\overleftrightarrow{AB} \cap$ the circle M = {A , B}

, then $\overleftrightarrow{AB} \cap$ the surface of the circle M = ...

(a) {A , B} (b) $\overline{AB}$ (c) $\overrightarrow{AB}$ (d) $\overrightarrow{BA}$

(6) **In the opposite figure :**



CD = 3 cm. , $\overline{MC} \perp \overline{AB}$

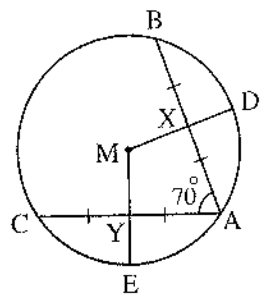
, D is the midpoint of $\overline{MA}$

then the area of the circle M = ... π cm².

(a) 3 (b) 6 (c) 9 (d) 36

---

2 [a] **In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two chords equal in length at the circle M

, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$, m (∠ A) = 70°

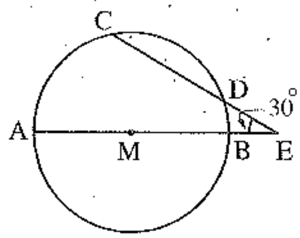
(1) **Find :** m (∠ DME) (2) **Prove that :** XD = YE

[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, m (∠ E) = 30° , m ($\overset{\frown}{AC}$) = 80°

Find : m ($\overset{\frown}{CD}$)



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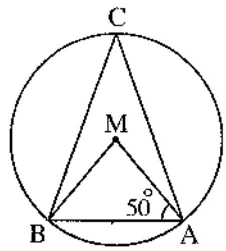
3 **[a] Complete :** The measure of the inscribed angle equals .... the measure of the central angle .... by the same arc.

[b] In the opposite figure :

M is a circle , m (∠ MAB) = 50°

Find : m (∠ C)



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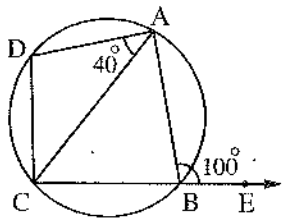
4 **[a] In the opposite figure :**

m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : Δ DAC is an isosceles triangle.

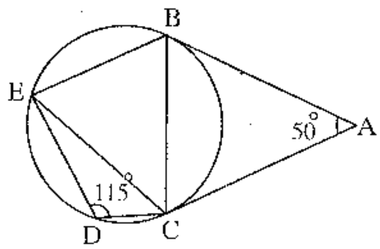


[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C

, m (∠ A) = 50° , m (∠ D) = 115°

Prove that : (1) $\overrightarrow{BC}$ bisects ∠ ABE (2) CB = CE



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5 **[a] Complete :** The measure of the inscribed angle in a semicircle equals ....°

[b] In the opposite figure :

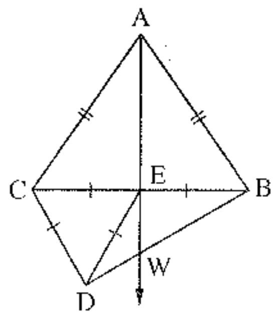
ABC and DCE are two equilateral triangles

, E is the midpoint of $\overline{BC}$, $\overrightarrow{AE} \cap \overline{BD} = \{W\}$

(1) **Prove that :** $\overline{AC}$ is a tangent-segment to the circle which passes through the vertices of Δ CED

(2) **Prove that :** CDWE is a cyclic quadrilateral.

(3) **Find :** The centre of the circle which passes through the vertices of the quadrilateral CDWE

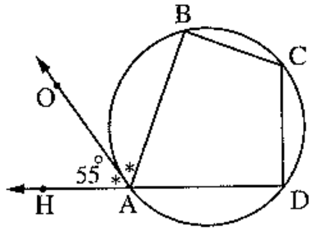


5 El-Monofia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer :

(1) **In the opposite figure :**



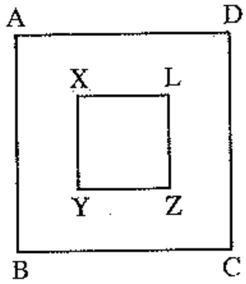
$H \in \overrightarrow{DA}$, $\overrightarrow{AO}$ bisects $\angle$ HAB

, m ($\angle$ HAO) = 55°

, then m ($\angle$ C) = ...

(a) 55° (b) 75° (c) 110° (d) 125°

(2) **In the opposite figure :**



If the side length of the square ABCD = 7 cm.

and the side length of the square XYZL = 3 cm.

, then the area of the shaded part = ... cm^2.

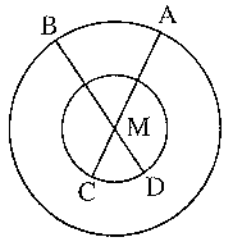
(a) (7 – 3) (b) 4 (7 – 3)

(c) $(7 - 3)^2$ (d) $(7^2 - 3^2)$

(3) If $\overleftrightarrow{AB} \cap$ the circle M = {A , B} , then $\overleftrightarrow{AB} \cap$ the surface of the circle M = ...

(a) $\overleftrightarrow{AB}$ (b) $\overline{AB}$ (c) {A , B} (d) $\overrightarrow{AB}$

(4) **In the opposite figure :**



Two concentric circles with centre M

, the radii lengths of them are 6 cm. and 3 cm.

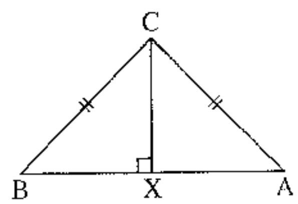
, if m ($\overset{\frown}{AB}$) = 60° , then m ($\overset{\frown}{DC}$) = ...

(a) 60° (b) 30° (c) 120° (d) 40°

(5) If $\overline{MA}$ and $\overline{MB}$ are two perpendicular radii in a circle M and the area of triangle AMB = 8 cm^2. , then the length of radius of this circle = ...

(a) 8 cm. (b) 16 cm. (c) 4 cm. (d) 2 cm.

(6) **In the opposite figure :**



CA = CB , $\overline{CX} \perp \overline{AB}$

, AB = 2 CX

, then m ($\angle$ A) = ...

(a) 30° (b) 60° (c) 90° (d) 45°

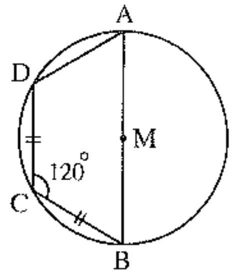
2 [a] **In the opposite figure :**

ABCD is a quadrilateral inscribed in the circle M

, M ∈ $\overline{AB}$, CB = CD

, m (∠ BCD) = 120°

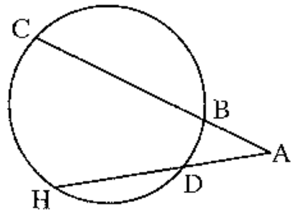
Find : (1) m (∠ A) (2) m (∠ D)



[b] **In the opposite figure :**

If m ($\overset{\frown}{HC}$) = 100° , m ($\overset{\frown}{BD}$) = 30°

Find : m (∠ A)



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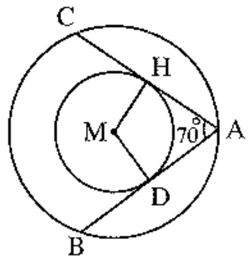
3 [a] **In the opposite figure :**

Two concentric circles at M

, $\overline{AB}$ and $\overline{AC}$ are two tangents to the smaller circle

, m (∠ A) = 70°

(1) **Find :** m (∠ DMH) (2) **Prove that :** AB = AC

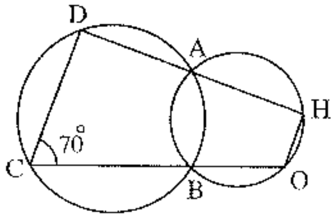


[b] **In the opposite figure :**

Two intersecting circles at A and B , m (∠ C) = 70°

(1) **Find :** m (∠ O)

(2) **Prove that :** $\overline{CD}$ // $\overline{HO}$



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4 [a] $\overline{AB}$ is a diameter in the circle M , $\overline{AC}$ is a chord such that m (∠ BAC) = 30°

, draw $\overline{BC}$ and draw $\overline{MD}$ ⊥ $\overline{AC}$ and cut it at D

(1) **Prove that :** $\overline{MD}$ // $\overline{BC}$

(2) **Porve that :** The length $\overline{BC}$ = the length of the radius of this circle.

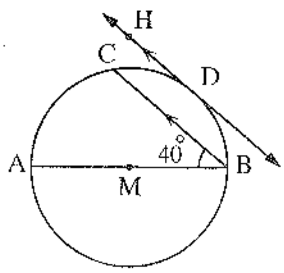
[b] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m (∠ B) = 40° , $\overleftrightarrow{DH}$ is a tangent at D

, $\overleftrightarrow{DH}$ // $\overline{BC}$

Find : m ($\overset{\frown}{DC}$)



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5 [a] If circle with radius length 5 cm. , A is a point in its plane where MA = (2 X − 3) cm.

Find the value of X if A is located outside the circle.

[b] In the opposite figure :

$\overline{AB}$ is a diameter of the circle M , H is a midpoint of a chord $\overline{AC}$

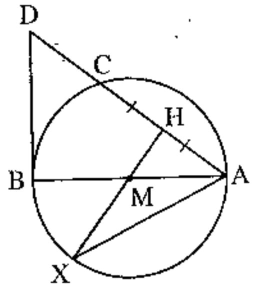
, $\overline{BD}$ is a tangent to the circle at B

, $\overrightarrow{HM}$ cuts the circle at X , **porve that :**

(1) MHDB is a cyclic quadrilateral.

(2) m (∠ BAX) = $\frac{1}{2}$ m (∠ D)

(3) $\overleftrightarrow{AB}$ is a tangent to the circle passing through the points B , C and D



6 El-Gharbia Governorate

Answer the following questions :

1 Choose the correct answer from those given :

(1) If the length of a diameter of a circle is 8 cm. and the straight line L at a distance of 4 cm. from its centre , then L is ....

(a) a secant to the circle at two points. (b) lying outside the circle.

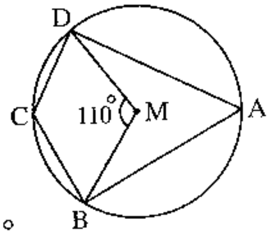
(c) a tangent to the circle. (d) an axis of symmetry to the circle.

(2) **In the opposite figure :**

If M is the centre of the circle

, m (∠ BMD) = 110°

, then m (∠ C) = ...

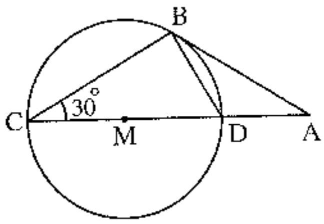


(a) 70° (b) 110° (c) 125° (d) 55°

(3) **In the opposite figure :**

$\overline{AB}$ is a tangent of the circle M

, then m (∠ ABC) = ...



(a) 120° (b) 110°

(c) 90° (d) 30°

(4) The centre of the inscribed circle of any triangle is the intersection point ....

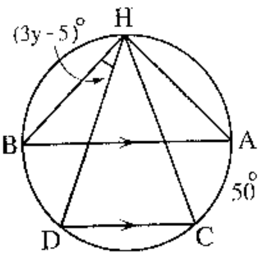
(a) its medians. (b) its heights.

(c) the symmetric axes of its sides. (d) bisectors of its interior angles.

(5) **In the opposite figure :**

m ($\overset{\frown}{AC}$) = 50° , $\overline{AB}$ // $\overline{CD}$

, then the value of y = ...



(a) 5° (b) 10°

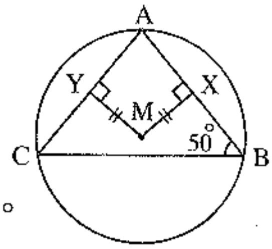
(c) 15° (d) 20°

(6) **In the opposite figure :**

MX = MY , m (∠ B) = 50°

, then m (∠ A) = ...

(a) 50° (b) 60° (c) 70° (d) 80°



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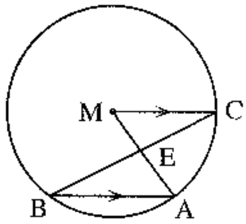
2 [a] **In the opposite figure :**

$\overline{AB}$ is a chord in the circle M

, $\overline{CM}$ // $\overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$

Prove that : BE > AE



[b] **In the opposite figure :**

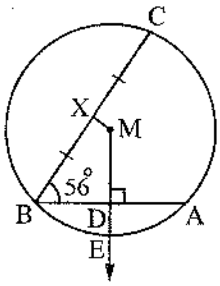
$\overline{AB}$ and $\overline{BC}$ are two chords in the circle M

, its radius of length 5 cm. , $\overrightarrow{MD} \perp \overline{AB}$ and cuts $\overline{AB}$

at D and cuts the circle at E , X is midpoint of $\overline{BC}$

, AB = 8 cm. and m (∠ ABC) = 56°

Find : (1) m (∠ DMX) (2) The length of $\overline{DE}$



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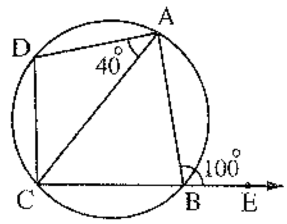
3 [a] **In the opposite figure :**

m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : m $(\overset{\frown}{CD})$ = m $(\overset{\frown}{AD})$



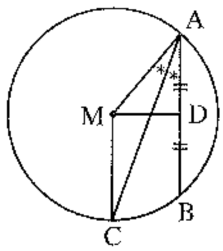
[b] **In the opposite figure :**

$\overline{AB}$ is a chord in the circle M

, $\overrightarrow{AC}$ bisects ∠ BAM and cuts the circle M at C

, D is midpoint of $\overline{AB}$

Prove that : $\overline{DM} \perp \overline{CM}$



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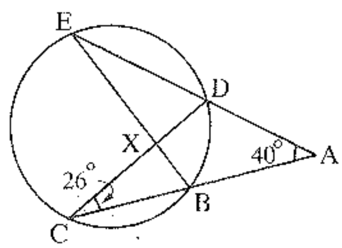
4 [a] **In the opposite figure :**

$\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$, m (∠ A) = 40°

, $\overline{DC} \cap \overline{BE} = \{X\}$, m (∠ BCD) = 26°

Find : (1) m $(\overset{\frown}{CE})$ (2) m (∠ EXC)

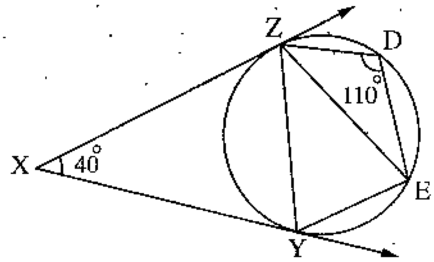


[b] In the opposite figure :

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle from the point X , m (∠ X) = 40°

, m (∠ D) = 110°

Prove that : m $(\widehat{ZDE})$ = m $(\widehat{ZY})$



5 **[a] In the opposite figure :**

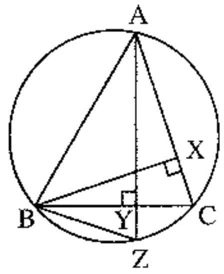
ABC is a triangle drawn in a circle

, $\overline{BX} \perp \overline{AC}$, $\overrightarrow{AY} \perp \overline{BC}$ cuts it at Y and cuts the circle at Z

Prove that :

(1) ABYX is a cyclic quadrilateral.

(2) $\overrightarrow{BC}$ bisects ∠ XBZ



[b] In the opposite figure :

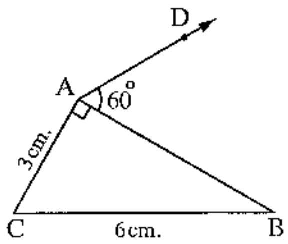
ABC is a right-angled triangle at A

, AC = 3 cm. , BC = 6 cm.

, m (∠ BAD) = 60°

Prove that :

$\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC



7 El-Dakahlia Governorate

Answer the following questions : (Calculator is allowed)

1 **[a] Choose the correct answer from the given answers :**

(1) M and N are two circles of radii lengths 9 cm. , 4 cm. , MN = 5 cm.
, then the two circles are ....

(a) intersecting. (b) touching internally.

(c) touching externally. (d) distant.

(2) The centres of all circles passing through the points A and B lie on ....

(a) $\overline{AB}$ (b) midpoint of $\overline{AB}$

(c) the symmetry axis of $\overline{AB}$ (d) the perpendicular to $\overline{AB}$ from B

(3) The measure of the inscribed angle which is drawn in a semicircle equals ....

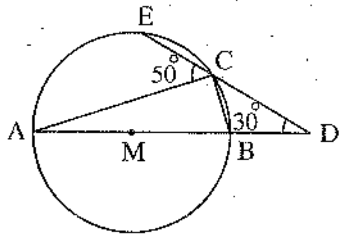
(a) 180° (b) 90° (c) 45° (d) 100°

[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, m (∠ D) = 30° , m (∠ ACE) = 50°

Find by proof : m (∠ CBA)



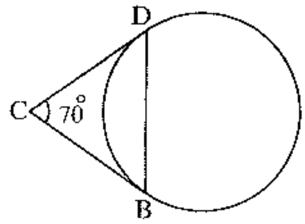
2 **[a] Choose the correct answer from the given answers :**

(1) **In the opposite figure :**

$\overline{CB}$ and $\overline{CD}$ are two tangent-segments at B and D

, m (∠ C) = 70°

, then m ($\overset{\frown}{DB}$ the minor) = ...



(a) 180° (b) 90° (c) 100° (d) 110°

(2) $\overline{AB}$ and $\overline{CD}$ are two equal chords in length in the circle M , X and Y are the two midpoints of $\overline{AB}$ and $\overline{CD}$ respectively , MX = 3 cm. , then MY = ... cm.

(a) 3 (b) 6 (c) $\frac{3}{2}$ (d) 4

(3) The length of the arc which represents $\frac{1}{4}$ of the circle equals ....

(a) 4 π r (b) 2 π r (c) π r (d) $\frac{1}{2}$ π r

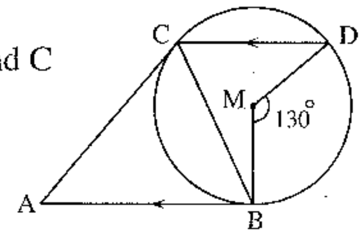
[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M at B and C

, $\overline{AB}$ // $\overline{CD}$, m (∠ BMD) = 130°

(1) **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD

(2) **Find by proof :** m (∠ A)



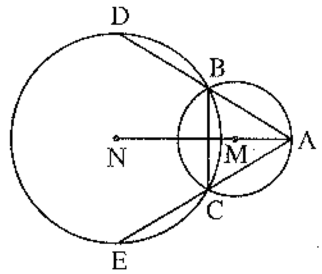
3 [a] Using the geometric tools , draw $\overline{AB}$ with length 6 cm. , then draw $\overrightarrow{AC}$ where m (∠ CAB) = 60° , draw the circle that passes through the points A , B and its centre lies on $\overrightarrow{AC}$ and calculate the length of its radius (Don't reomve the arcs).

[b] In the opposite figure :

M and N are two intersecting circles at B and C

, A ∈ $\overleftrightarrow{MN}$

Prove that : BD = CE



4 [a] **In the opposite figure :**

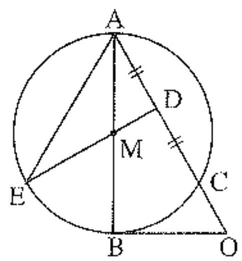
$\overline{OB}$ is a tangent-segment to the circle M at B

, $\overline{AB}$ is a diameter , D is the midpoint of $\overline{AC}$

Prove that :

(1) DOBM is a cyclic quadrilateral.

(2) m (∠ AOB) = 2 m (∠ BAE)



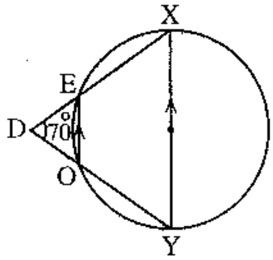
[b] In the opposite figure :

$\overline{XY}$ is a diameter in the circle

, $\overline{EO}$ is a chord in it , where $\overline{XY} \parallel \overline{EO}$

, m (∠ D) = 70°

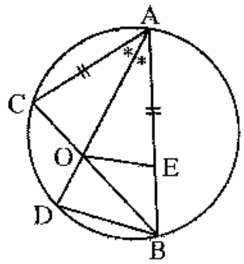
Find : m $(\overset{\frown}{EX})$



5 **[a] In the opposite figure :**

AE = AC , $\overrightarrow{AD}$ bisects ∠ BAC

Prove that : EBDO is a cyclic quadrilateral.



[b] $\overline{AB}$ is a diameter in a circle , $\overline{AC}$ is a chord in it , m (∠ CAB) = 30°

, draw $\overrightarrow{AC}$ to cut the tangent to the circle at B at D.

Prove that : $\overrightarrow{BA}$ touches the circle passing through the vertices of the triangle BCD

8 Ismailia Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

(1) A circle its radius length is 5 cm. , then its circumference = ... cm.

(a) 5 π (b) 7 π (c) 10 π (d) 25 π

(2) We can draw a circle passes through the vertices of ....

(a) rectangle. (b) rhombus. (c) trapezium. (d) parallelogram.

(3) The number of axes of symmetry of the circle = ...

(a) one axis. (b) two axes.

(c) three axes. (d) an infinite number of axes.

(4) M is a circle with radius length r , $\overleftrightarrow{MA} \perp$ straight line L where $\overleftrightarrow{MA} \cap L = \{A\}$

If MA > r , then L is ....

(a) a tangent to the circle. (b) a diameter in the circle.

(c) outside the circle. (d) a secant to the circle.

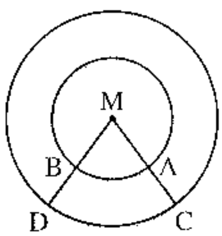
(5) **In the opposite figure :**

Two concentric circles.

If the lengths of their radii are 2 cm. and 5 cm.

, then $\frac{m(\overset{\frown}{AB})}{m(\overset{\frown}{CD})}$ = ...

(a) $\frac{2}{5}$ (b) 1 (c) $\frac{2}{3}$ (d) $\frac{3}{5}$

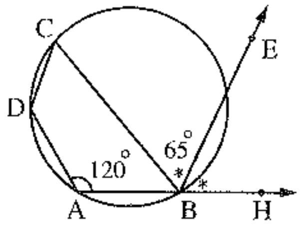


(6) The sum of measures of the interior angles of the quadrilateral = ...

(a) 90° (b) 180° (c) 270° (d) 360°

---

2 **[a] In the opposite figure :**



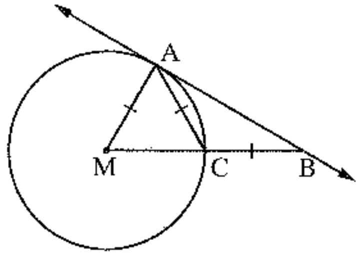
ABCD is a cyclic quadrilateral in which

m (∠ A) = 120° , $\overrightarrow{BE}$ bisects ∠ HBC

, m (∠ EBC) = 65°

Find with proof : (1) m (∠ C) (2) m (∠ D)

[b] In the opposite figure :



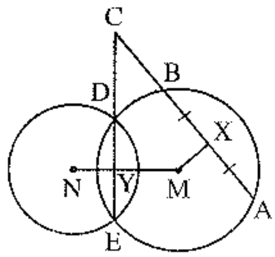
M is a circle , AM = AC = BC

Prove that :

$\overleftrightarrow{AB}$ is a tangent to the circle at A

---

3 **[a] In opposite figure :**

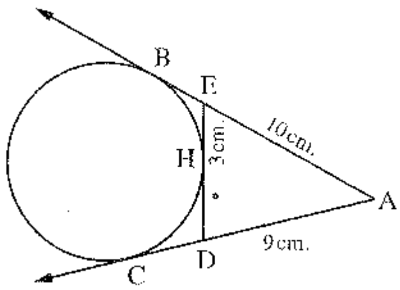


X is the midpoint of $\overline{AB}$, $\overline{MN} \cap \overline{EC} = \{Y\}$

(1) **Prove that :** CXMY is a cyclic quadrilateral.

(2) **Find :** The centre of the circle which passes through the vertices of the figure CXMY

[b] In the opposite figure :



$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to a circle

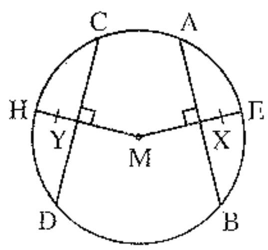
, $\overline{ED}$ is a tangent to the circle at H such that AE = 10 cm.

, EH = 3 cm. , AD = 9 cm.

Find : The length of $\overline{ED}$

---

4 **[a] In the opposite figure :**



$\overline{ME} \perp \overline{AB}$, $\overline{MH} \perp \overline{CD}$

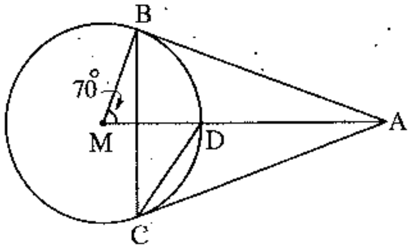
, EX = YH

Prove that : AB = CD

[b] Using geometric tools. Draw $\overline{AB}$ its length is 6 cm. , then draw a circle passing through the two points A , B and its radius length is 3 cm.

How many circles can be drawn ?

5 **[a] In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two tangent-segments drawn from A

, m (∠ AMB) = 70°

Find : (1) m (∠ ABC) (2) m (∠ ACD)

[b] $\overline{AB}$ and $\overline{CD}$ are two equal chords in length in a circle

, $\overline{AB} \cap \overline{CD} = \{E\}$, m ($\overset{\frown}{AD}$) = 50°

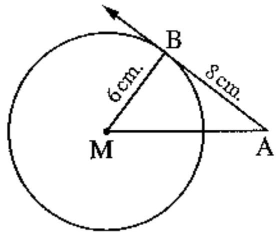
(1) **Prove that :** m ($\overset{\frown}{AD}$) = m ($\overset{\frown}{BC}$) (2) **Find :** m (∠ AED)

9 Suez Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

(1) **In the opposite figure :**



$\overrightarrow{AB}$ is a tangent to the circle M

, MB = 6 cm. , AB = 8 cm.

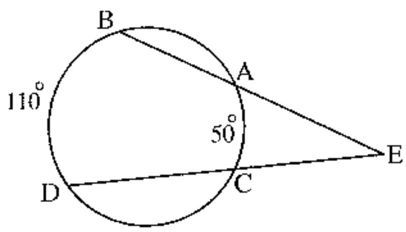
, then AM = ... cm.

(a) 5 (b) 10 (c) 12 (d) 13

(2) If the two circles M and N are touching externally, the radius length of one of them is 5 cm. , and MN = 9 cm. , then the radius length of the other circle equals .... cm.

(a) 4 (b) 5 (c) 9 (d) 14

(3) **In the opposite figure :**



If m ($\overset{\frown}{AC}$) = 50° , m ($\overset{\frown}{BD}$) = 110°

, then m (∠ E) = ...°

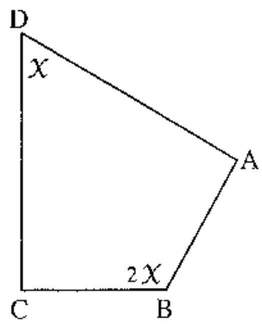
(a) 60 (b) 50

(c) 40 (d) 30

(4) A circle can be drawn passing the vertices of a ....

(a) rhombus. (b) rectangle. (c) trapezoid. (d) parallelogram.

(5) **In the opposite figure :**



ABCD is a cyclic quadrilateral , m (∠ D) = X° , (∠ B) = 2 X°

, then X = ...

(a) 120° (b) 100°

(c) 60° (d) 50°

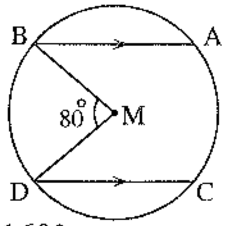
(6) **In the opposite figure :**

In a circle M , $\overline{AB} // \overline{CD}$

, m (∠ BMD) = 80°

, then m $(\overset{\frown}{AC})$ = ...

(a) 20° (b) 40° (c) 80° (d) 160°



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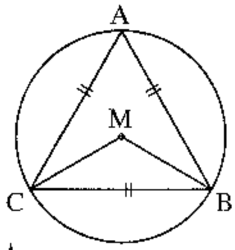
2 [a] **In the opposite figure :**

ABC is an equilateral triangle drawn inside a circle M

Find : (1) m (∠ BAC) (2) m (∠ BMC)



[b] **In the opposite figure :**

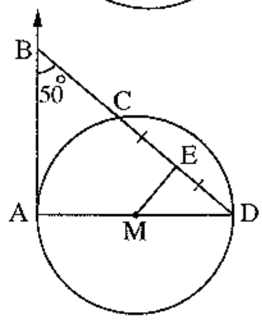
$\overline{AD}$ is a diameter of the circle M

, $\overrightarrow{AB}$ is a tangent touches it at A

, m (∠ ABC) = 50°

, E is the midpoint of $\overline{DC}$

Find with proof : m (∠ AME)



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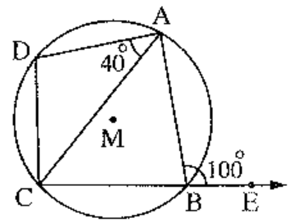
3 [a] **In the opposite figure :**

m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : ADC is an isosceles triangle.

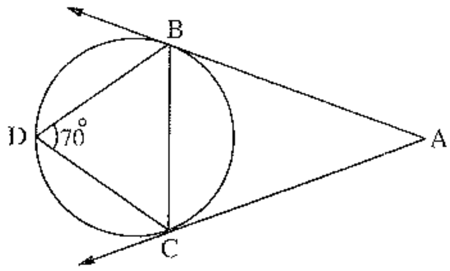


[b] **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m (∠ D) = 70°

Find : (1) m (∠ ABC) (2) m (∠ A)



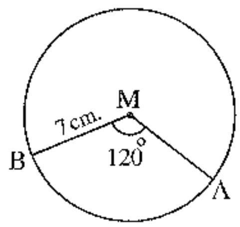
---

4 [a] **In the opposite figure :**

M is a circle with radius length 7 cm.

, m (∠ AMB) = 120°

Find : The length of $(\overset{\frown}{AB})$ $\left(\pi = \frac{22}{7}\right)$



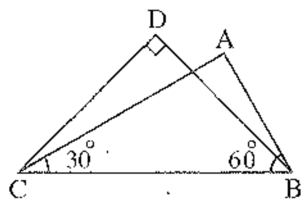
[b] **In the opposite figure :**

m (∠ BDC) = 90° , m (∠ ACB) = 30°

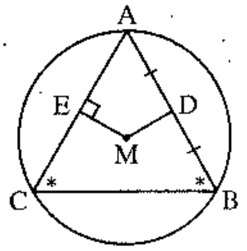
, m (∠ ABC) = 60°

Prove that :

The points A , B , C and D have one circle passing through them.



5 **[a] In the opposite figure :**



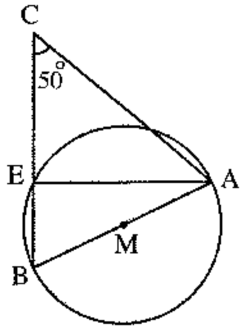
Triangle ABC is inscribed in the circle M , in which

m (∠ B) = m (∠ C) , D is the midpoint of $\overline{AB}$

, $\overline{ME} \perp \overline{AC}$

Prove that : MD = ME

[b] In the opposite figure :



$\overline{AB}$ is a diameter of the circle M

, m (∠ C) = 50°

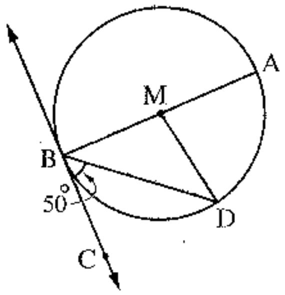
Find with proof : m (∠ CAE)

10 Port Said Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) **In the opposite figure :**



If m (∠ CBD) = 50°

, then m (∠ AMD) = ...

(a) 40° (b) 50°

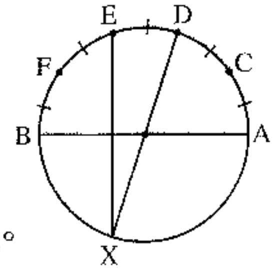
(c) 80° (d) 100°

(2) A circle with diameter length (2 X + 5) cm. , the straight line L is distant from its centre by (X + 2) cm. where $X > 0$, then the straight line is ....

(a) a secant to the circle at two points. (b) lying outside the circle.

(c) a tangent to the circle. (d) an axis of symmetry to the circle.

(3) **In the opposite figure :**



If $\overline{AB}$ is a diameter in circle

, m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{DE}$) = m ($\overset{\frown}{EF}$) = m ($\overset{\frown}{FB}$)

, then m (∠ DXE) = ...

(a) 72° (b) 54° (c) 36° (d) 18°

(4) M and N are two intersecting circles their radii lengths are 5 cm. , 2 cm. , then MN ∈ ....

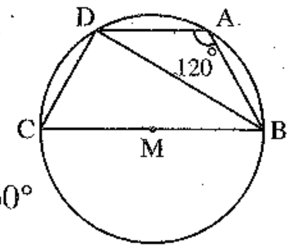
(a) [3 , 7[(b)]3 , 7[(c)]3 , 7] (d) [3 , 7]

(5) **In the opposite figure :**

If m (∠ BAD) = 120°

, then m (∠ CBD) = ...

(a) 15° (b) 30° (c) 45° (d) 60°



(6) The number of all common tangents drawn to two distant circles equals ....

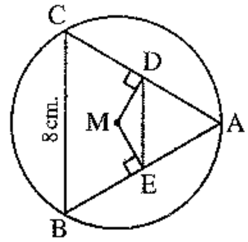
(a) 4 (b) 3 (c) 2 (d) 1

---

2 **[a] Using the given data in the opposite figure :**

(1) **Prove that :** $\overline{DE} \parallel \overline{CB}$

(2) **Find :** DE



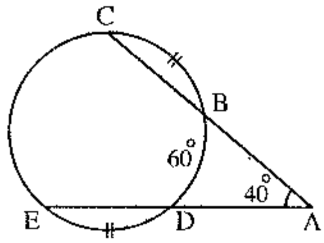
[b] In the opposite figure :

m (∠ A) = 40° , m ($\overset{\frown}{BD}$) = 60°

and m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{DE}$)

Find with proof :

m ($\overset{\frown}{EC}$) and m ($\overset{\frown}{BC}$)



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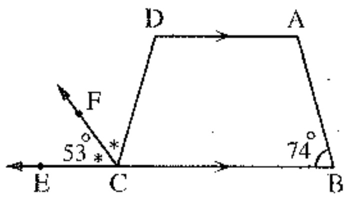
3 **[a] Using the given data in the opposite figure :**

Prove that :

ABCD is a cyclic quadrilateral.



[b] ABCD a parallelogram in which AC = BC

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circumcircle of the triangle ABC

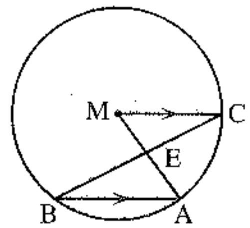
---

4 **[a] In the opposite figure :**

$\overline{AB}$ is a chord in the circle M

, $\overline{CM} \parallel \overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$

Prove that : BE > AE



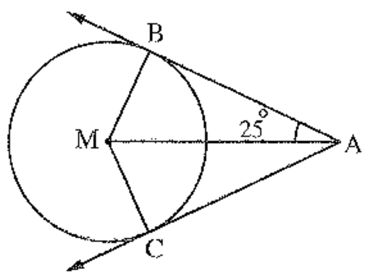
[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M

touch it at B and C respectively and m (∠ BAM) = 25°

(1) **Prove that :** $\overrightarrow{MA}$ bisects (∠ BMC)

(2) **Find :** m (∠ BMC) .



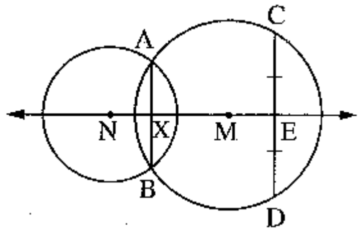
5 **[a] In the opposite figure :**

The two circles M and N intersect at A and B

, $\overline{CD}$ is a chord in the circle M cuts $\overleftrightarrow{MN}$ at E

, if E is the midpoint of $\overline{CD}$

Prove that : $\overline{AB} \parallel \overline{CD}$



[b] ABCD is a square , $\overrightarrow{AX}$ bisects ∠ BAC and intersects $\overline{BD}$ at X

and $\overrightarrow{DY}$ bisects ∠ CDB and intersects $\overline{AC}$ at Y

Prove that : AXYD is a cyclic quadrilateral.

11 Damietta Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from the given ones :**

(1) ABC is a triangle having one symmetric line and its side lengths are 10 , 5 and X cm. , then X = ... cm.

(a) 5 (b) 8 (c) 10 (d) 12

(2) If the two circles M , N are touching internally , the length of one radius of them is 3 cm. , MN = 8 cm. , then the length of the radius of the other circle is .... cm.

(a) 5 (b) 11 (c) 6 (d) 12

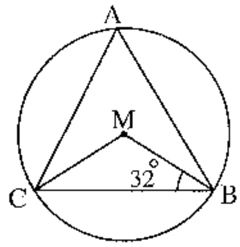
(3) If the ratio between the measures of the angles of a triangle is 2 : 3 : 4 , then the measure of the greatest angle is ....

(a) 40° (b) 90° (c) 45° (d) 80°

(4) **In the opposite figure :**

M is a circle , m (∠ MBC) = 32°

, then m ($\overset{\frown}{BC}$ the minor) = ...



(a) 116° (b) 23° (c) 58° (d) 64°

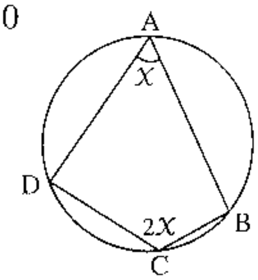
(5) A rectangular picture its length is 60 cm. and its width is 40 cm. We need to make a wooden frame its width is 5 cm. , then its total area is .... cm^2.

(a) 3050 (b) 3500 (c) 2925 (d) 3250

(6) **In the opposite figure :**

m (∠ A) = X° , m (∠ C) = 2 X°

, then X = ...



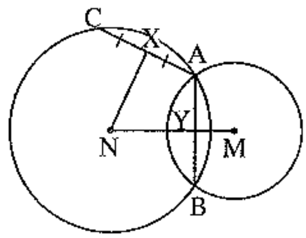
(a) 60° (b) 50° (c) 80° (d) 20°

2 [a] A , B are two points where AB = 6 cm. , draw a circle of radius length 5 cm. and passes through the two points A , B

Find : (1) The number of circles can be drawn.

(2) The distance from the centre to $\overline{AB}$ by proof.

[b] In the opposite figure :



M , N are two intersecting circles at A , B , $\overleftrightarrow{MN} \cap \overline{AB} = \{Y\}$

, AB = AC , if X is the midpoint of $\overline{AC}$

Prove that : NX = NY

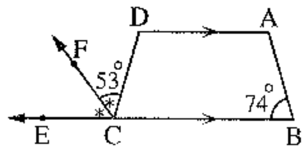
---

3 [a] $\overline{AB}$, $\overline{AC}$ are two chords in a circle

If X and Y are the two midpoints of $\widehat{AB}$, $\widehat{AC}$ respectively , $\overline{XY}$ cuts $\overline{AB}$ at D , $\overline{AC}$ at H

Prove that : AD = AH

[b] In the opposite figure :



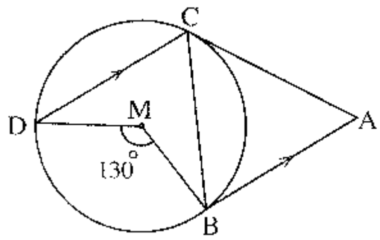
$\overline{AD}$ // $\overline{BC}$, m (∠ B) = 74° , m (∠ DCF) = 53°

, $\overrightarrow{CF}$ bisects ∠ DCE

Prove that : ABCD is a cyclic quadrilateral.

---

4 [a] **In the opposite figure :**

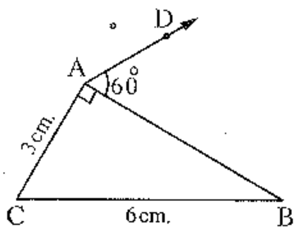


$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB}$ // $\overline{CD}$, m (∠ BMD) = 130°

Prove that : $\overrightarrow{CB}$ bisects ∠ ACD

[b] In the opposite figure :



m (∠ BAC) = 90° , m (∠ DAB) = 60°

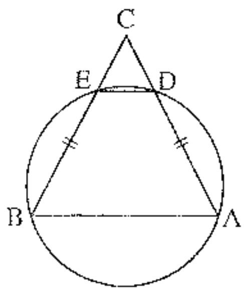
AC = 3 cm. , BC = 6 cm.

Prove that :

$\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC

---

5 [a] **In the opposite figure :**



$\overline{AD}$ and $\overline{BE}$ are two equal chords in length in the circle

, $\overrightarrow{AD} \cap \overrightarrow{BE} = \{C\}$

Prove that : CD = CE

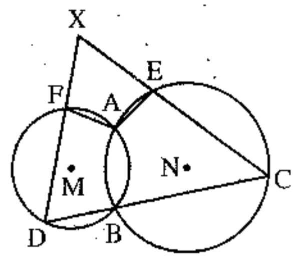
[b] In the opposite figure :

Two intersecting circles at A and B

, $\overline{CD}$ passes through the point B and intersects

the two circles at C and D

Prove that : AFXE is a cyclic quadrilateral.



12 Kafr El-Sheikh Governorate

Answer the following questions : ***(Calculator is allowed)***

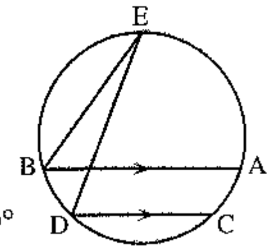
1 [a] Choose the correct answer from those given :

(1) **In the opposite figure :**

If m $(\widehat{AC}) = 30°$, $\overline{AB} \parallel \overline{CD}$

, then m (∠ BED) = ...

(a) 10° (b) 15° (c) 30° (d) 60°



(2) The two tangents drawn from the two ends of a diameter of a circle are ....

(a) parallel. (b) equal in length. (c) congruent. (d) intersecting.

(3) M and N are two intersecting circles their radii lengths are 5 cm. , 2 cm.

, then MN ∈ ....

(a)]3 , 7[(b) [3 , 7[(c)]3 , 7] (d) [3 , 7]

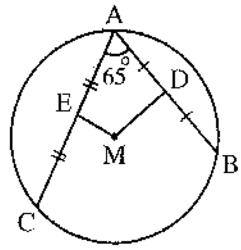
[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two chords in the circle M ,

D , E are the two midpoints of $\overline{AB}$, $\overline{AC}$ respectively

and m (∠ BAC) = 65°

Find : m (∠ DME)



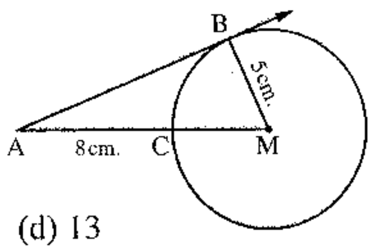
2 [a] Choose the correct answer from those given :

(1) **In the opposite figure :**

$\overrightarrow{AB}$ is a tangent to the circle M

, if MB = 5 cm. , AC = 8 cm. , then AB = ... cm.

(a) 5 (b) 10 (c) 12 (d) 13



(2) The centre of the circumcircle of any triangle is the point of intersection of ....

(a) the interior bisectors of its angles. (b) the exterior bisectors of its angles.

(c) its heights. (d) the symmetric axes of its sides.

(3) The measure of the arc which represents $\frac{1}{3}$ the measure of the circle equals ....

(a) 60° (b) 90° (c) 120° (d) 240°

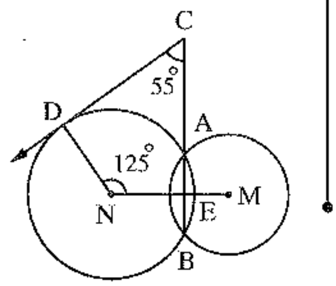
[b] In the opposite figure :

M and N are two intersecting circles at A and B

, C ∈ $\overrightarrow{BA}$, D ∈ the circle N

, m (∠ MND) = 125° and m (∠ BCD) = 55°

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle N at D



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3 **[a]** State three cases of the cyclic quadrilateral.

[b] ABCD is a quadrilateral in which AB = AD , m (∠ ABD) = 30° and m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.

---

4 **[a] Prove that :** The two tangent-segments drawn to a circle from a point outside it are equal in length.

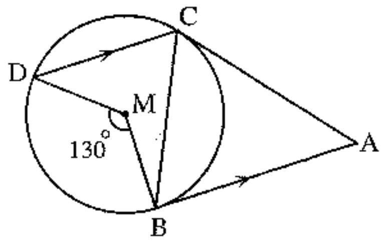
[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB}$ // $\overline{CD}$, m (∠ BMD) = 130°

(1) **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD

(2) **Find :** m (∠ A) with proof.



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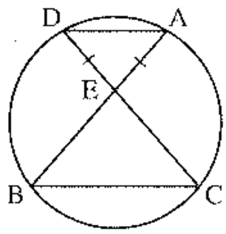
5 **[a] In the opposite figure :**

$\overline{AB}$ ∩ $\overline{CD}$ = {E} , EA = ED

Prove that : EB = EC



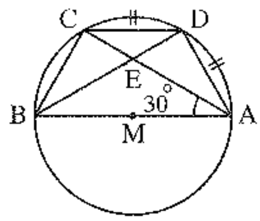
[b] In the opposite figure :

$\overline{AB}$ is a diameter of a circle M , C ∈ the circle

, m (∠ CAB) = 30° , D is the midpoint of $\overset{\frown}{AC}$, $\overline{DB}$ ∩ $\overline{AC}$ = {E}

(1) **Find :** m (∠ BDC) , m (∠ ABD) with proof.

(2) **Prove that :** Δ ABE is an isosceles triangle.



13 El-Beheira Governorate

Answer the following questions : (Calculators are permitted)

1 **Choose the correct answer from those given :**

(1) The distance between the two points (6 , 0) , (– 4 , 0) equals .... length units.

(a) – 10 (b) 10 (c) 2 (d) 24

(2) If the length of a diameter of a circle is 7 cm. , and the straight line L at a distance of 3.5 cm. from its centre , then L is ....

(a) a secant to the circle at two points. (b) lying outside the circle.
(c) a tangent to the circle. (d) an axis of symmetry to the circle.

(3) If $\overline{AB}$ is a diameter of a circle , where A (3 , – 5) , B (5 , 1) , then the centre of the circle is ....

(a) (4 , – 2) (b) (4 , 2) (c) (2 , 2) (d) (8 , – 2)

(4) The inscribed angle which is opposite to the minor arc in a circle is ....

(a) reflex. (b) right. (c) obtuse. (d) acute.

(5) It is possible to draw a circle passing through the vertices of a ....

(a) trapezium. (b) rhombus. (c) parallelogram. (d) rectangle.

(6) The number of tangents can be drawn from a point lies on a circle equals ....

(a) one. (b) two. (c) four. (d) infinite number.

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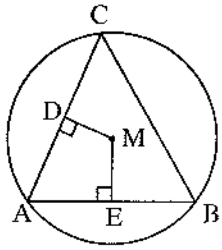
2 **[a] In the opposite figure :**



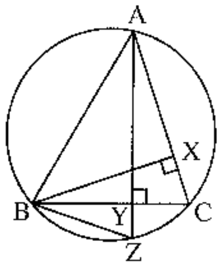
ABC is a triangle drawn inside a circle of centre M

, $\overline{MD} \perp \overline{AC}$, $\overline{ME} \perp \overline{AB}$

, BC = 8 cm.

(1) **Prove that :** $\overline{DE} \parallel \overline{CB}$ (2) **Find :** DE

[b] In the opposite figure :



ABC is a triangle drawn inside a circle , $\overline{BX} \perp \overline{AC}$

$\overrightarrow{AY} \perp \overline{BC}$ cuts it at Y and cuts the circle at Z

Prove that :

(1) ABYX is a cyclic quadrilateral.

(2) $\overline{BC}$ bisects ∠ XBZ

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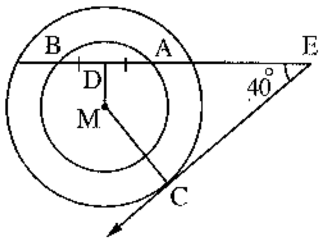
3 **[a] In the opposite figure :**



Two concentric circles of centre M

, $\overrightarrow{EC}$ is a tangent to the greater circle

, $\overline{EB}$ cuts the smaller circle at A , B

, D is the midpoint of $\overline{AB}$ and m (∠ CED) = 40°

Find with proof : m (∠ DMC)

[b] $\overline{AB}$, $\overline{CD}$ are two parallel chords in a circle M , E is the midpoint of $\overline{AB}$, $\overleftrightarrow{EM}$ is drawn to cut $\overline{CD}$ at F **Prove that :** FC = FD

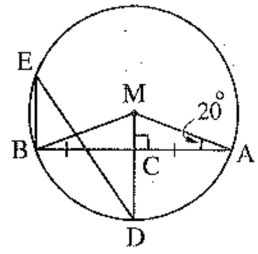
4 [a] **In the opposite figure :**

$\overrightarrow{MC} \cap \overline{AB} = \{C\}$, $\overrightarrow{MC} \perp \overline{AB}$

, $\overrightarrow{MC}$ intersects the circle at D

, m ($\angle$ MAB) = 20°

Find : (1) m $(\widehat{AD})$ (2) m ($\angle$ DEB)



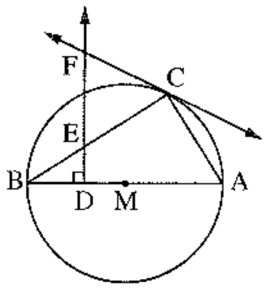
[b] **In the opposite figure :**

$\overline{AB}$ is a diameter of a circle M

, $\overleftrightarrow{CF}$ is a tangent of the circle at C and $\overrightarrow{DE} \perp \overline{AB}$

Prove that : (1) ADEC is a cyclic quadrilateral.

(2) FE = FC



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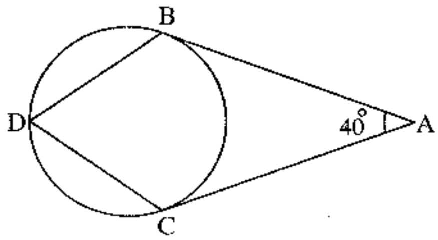
5 [a] Find the measure of the arc which represents $\frac{1}{3}$ its circle , then calculate the length of this arc if the length of the radius is 7 cm. $\left(\pi = \frac{22}{7}\right)$

[b] **In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two tangents to the circle at B , C

and m ($\angle$ A) = 40°

Find with proof : m ($\angle$ D)



14 El-Fayoum Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

(1) If the straight line L is a tangent to the circle of diameter 8 cm. , then the distance between L and the centre equals .... cm.

(a) 3 (b) 4 (c) 6 (d) 8

(2) The angle whose measure is 50° complements an angle of measure ....

(a) 90° (b) 130° (c) 50° (d) 40°

(3) The inscribed angle which is opposite to the minor arc in a circle is ....

(a) reflex. (b) obtuse. (c) right. (d) acute.

(4) ABC is a triangle in which AB = AC , m ($\angle$ C) = 40° , then m ($\angle$ A) = ...

(a) 40° (b) 80° (c) 100° (d) 120°

(5) The number of the symmetry axes of square is ....

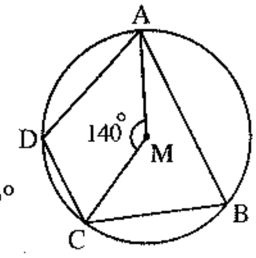
(a) 1 (b) 2 (c) 3 (d) 4

(6) **In the opposite figure :**

In the circle M , if m (∠ AMC) = 140°

, then m (∠ ADC) = ...

(a) 40° (b) 70° (c) 110° (d) 140°



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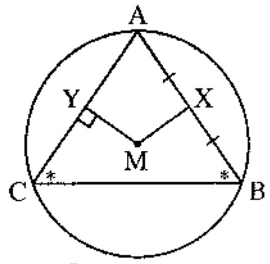
2 **[a] In the opposite figure :**

Triangle ABC is inscribed in circle M , in which :

m (∠ B) = m (∠ C) , X is the midpoint of $\overline{AB}$

, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

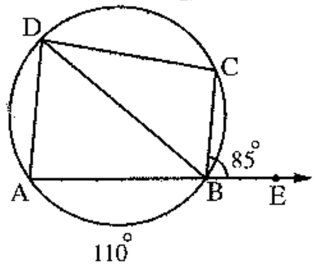


[b] In the opposite figure :

E ∈ $\overrightarrow{AB}$, E ∉ $\overline{AB}$, m ($\overset{\frown}{AB}$) = 110°

, m (∠ CBE) = 85°

Find : m (∠ BDC)



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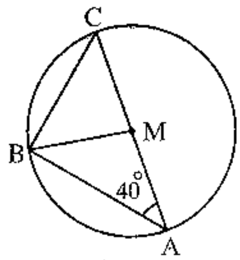
3 **[a] In the opposite figure :**

$\overline{AC}$ is a diameter in a circle M , B ∈ the circle M

, m (∠ BAC) = 40°

Find : m (∠ CBM)

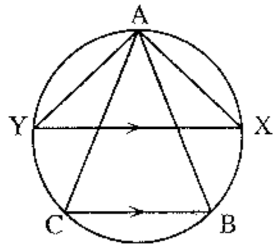


[b] In the opposite figure :

ABC is an inscribed triangle inside a circle

, $\overline{XY}$ // $\overline{BC}$

Prove that : m (∠ XAC) = m (∠ BAY) .



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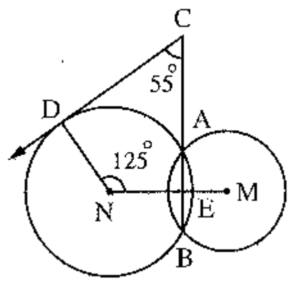
4 **[a] In the opposite figure :**

M and N are two intersecting circles at A and B , C ∈ $\overrightarrow{BA}$

, D ∈ the circle N and m (∠ MND) = 125°

, m (∠ BCD) = 55°

Prove that : $\overleftrightarrow{CD}$ is a tangent to circle N at D

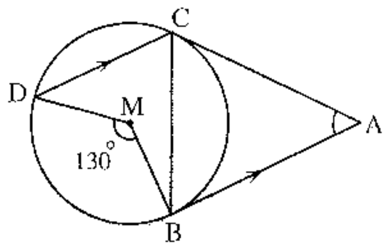


[b] In the opposite figure :

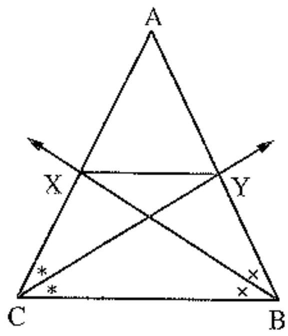
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB}$ // $\overline{CD}$, m (∠ BMD) = 130°

(1) **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD (2) **Find :** m (∠ A)



5 **[a] In the opposite figure :**



ABC is a triangle in which AB = AC

, $\overrightarrow{BX}$ bisects ∠ B and intersect $\overline{AC}$ at X

, $\overrightarrow{CY}$ bisects ∠ C and intersect $\overline{AB}$ at Y

Prove that : BCXY is a cyclic quadrilateral

and prove that : $\overline{XY}$ // $\overline{BC}$

[b] ABC is a triangle inscribed in a circle , $\overleftrightarrow{AD}$ is a tangent to the circle at A , X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$ **Prove that :** $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

15 Beni Suef Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

(1) It is impossible to draw a circle passing through the vertices of ....

(a) a triangle. (b) a square. (c) a rhombus. (d) a rectangle.

(2) If m_1 and m_2 are the slopes of two perpendicular straight lines , then ....

(a) $m_1 + m_2 = 0$ (b) $m_1 - m_2 = -1$ (c) $m_1 = m_2$ (d) $m_1 \times m_2 = -1$

(3) M and N are two circles touching internally , their radii lengths are 3 cm. , and 5 cm. , then MN = ... cm.

(a) 2 (b) 3 (c) 5 (d) 8

(4) The point of concurrence of the medians of the triangle divides each median in the ratio .... from its base.

(a) 2 : 1 (b) 1 : 2 (c) 2 : 3 (d) 1 : 3

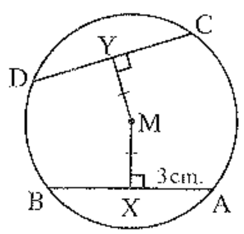
(5) The measure of the arc which represents $\frac{1}{3}$ the measure of the circle equals ....

(a) 60° (b) 90° (c) 120° (d) 240°

(6) The area of the rhombus whose diagonal lengths are 8 cm. and 10 cm. equals .... cm^2.

(a) 2 (b) 18 (c) 40 (d) 80

2 **[a] In the opposite figure :**



$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$, MX = MY

and AX = 3 cm.

Find : The length of $\overline{CD}$

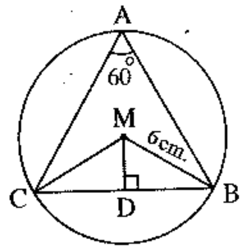
[b] Two concentric circles M , $\overline{AB}$ is a chord in the larger circle and intersects the smaller circle at C , D , draw $\overline{ME} \perp \overline{AB}$ **Prove that :** AC = BD

3 [a] **In the opposite figure :**

In the circle M , m (∠ A) = 60°

, $\overline{MD} \perp \overline{BC}$, MB = 6 cm.

Find with proof : The length of $\overline{MD}$

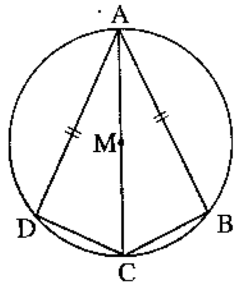


[b] In the opposite figure :

$\overline{AC}$ is a diameter in the circle M

, AB = AD

Prove that : m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{CD})$

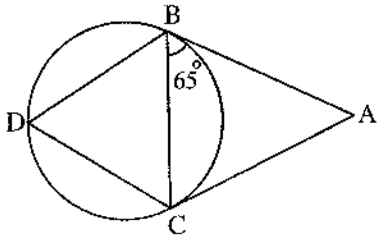


4 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C

, m (∠ ABC) = 65°

Find with proof : m (∠ A) and m (∠ D)

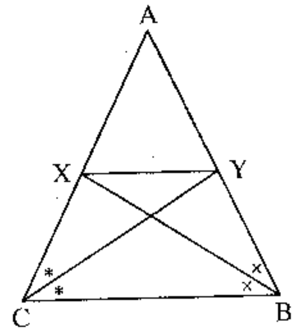


[b] In the opposite figure :

ABC is a triangle in which AB = AC , $\overrightarrow{BX}$ bisects ∠ B and intersects $\overline{AC}$ at X

, $\overrightarrow{CY}$ bisects ∠ C and intersects $\overline{AB}$ at Y

Prove that : The figure BCXY is a cyclic quadrilateral.

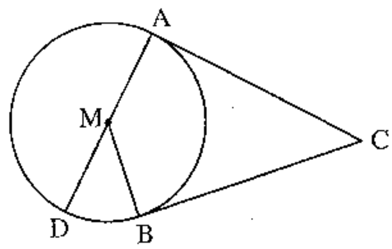


5 [a] **In the opposite figure :**

$\overline{AD}$ is a diameter in a circle of centre M

, $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are two tangents to the circle at A , B

Prove that : m (∠ DMB) = m (∠ ACB)



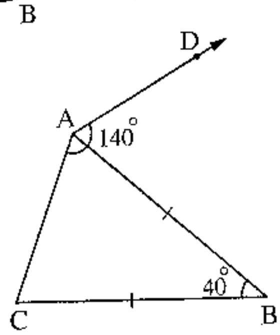
[b] In the opposite figure :

BA = BC , m (∠ DAC) = 140°

and m (∠ B) = 40°

Prove that :

$\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC



16 El-Menia Governorate

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

(1) The two angles A and C in the right-angled triangle at B are ....

(a) complementary. (b) supplementary.

(c) adjacent. (d) vertically opposite angles.

(2) The length of the opposite to the angle of measure 30° in the right-angled triangle is .... the length of the hypotenuse.

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

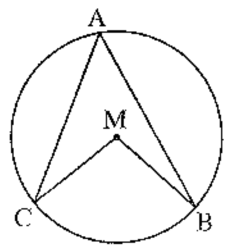
(3) The area of the rhombus whose diagonal lengths are 6 cm. , 8 cm. is .... cm^2

(a) 2 (b) 14 (c) 24 (d) 48

(4) The number of circles passing through three non-collinear points is ....

(a) 1 (b) zero (c) 2 (d) 3

(5) **In the opposite figure :**



In the circle M ,

if m (∠ M) − m (∠ A) = 50°

, then m (∠ A) = ...

(a) 40° (b) 50° (c) 100° (d) 130°

(6) Which of the following shapes is a cyclic quadrilateral ?

(a) rhombus (b) rectangle (c) parallelogram (d) trapezium

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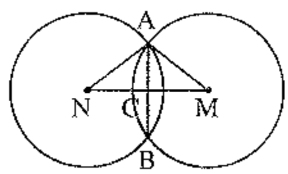
2 [a] **In the opposite figure :**

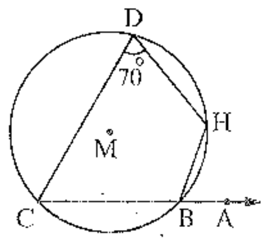


Two congruent circles M and N are intersecting at A and B

If MA = 10 cm. , AB = 12 cm.

Find by proof : The length of $\overline{MN}$

[b] **In the opposite figure :**



BCDH is a cyclic quadrilateral in the circle M

, m (∠ D) = 70° , A ∈ $\overrightarrow{CB}$, m (∠ C) = $\frac{1}{2}$ m (∠ H)

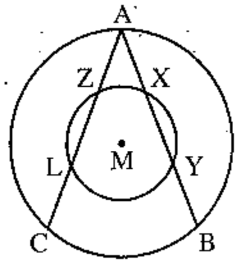
Find by proof : m (∠ ABH) , m (∠ H)

3 **[a] In the opposite figure :**

Two concentric circles at M

, AB = AC

Prove that : XY = ZL

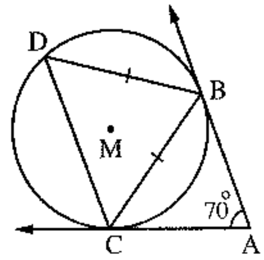


[b] In the opposite figure :

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle M

, m (∠ BAC) = 70° , BC = BD

Find : m (∠ ABD)



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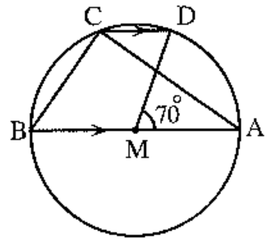
4 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overline{DC}$ // $\overline{AB}$, m (∠ AMD) = 70°

Find by proof : m (∠ ACD) , m (∠ ABC)



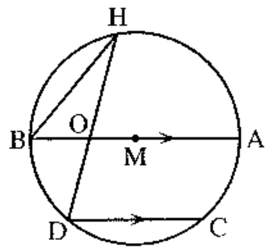
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overline{AB}$ // $\overline{DC}$, m ($\widehat{DC}$) = 80°

, m ($\widehat{AH}$) = 100°

Find by proof : m (∠ DHB) , m (∠ AOH)



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5 **In the opposite figure :**

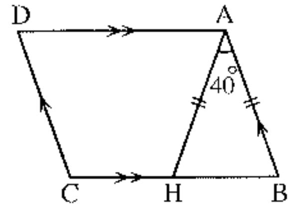
ABCD is a parallelogram

, H ∈ $\overline{BC}$ such that AB = AH , m (∠ BAH) = 40°

(1) **Find :** m (∠ AHB) , m (∠ D)

(2) **Prove that :** AHCD is a cyclic quadrilateral.

(3) **Prove that :** $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABH



17 Assiut Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer :**

(1) The chord which passes through the centre of the circle is called ....

(a) tangent. (b) diameter. (c) radius. (d) side.

(2) The number of symmetry axes of a square ....

(a) 2 (b) 3 (c) 4 (d) 5

(3) The inscribed angle which is opposite to the minor arc in a circle is ....

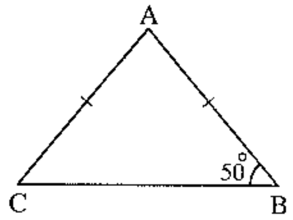
(a) reflex. (b) right. (c) obtuse. (d) acute.

(4) **In the opposite figure :**

ABC is a triangle , AB = AC

, m (∠ B) = 50°

, then m (∠ A) = ...



(a) 100° (b) 90° (c) 80° (d) 70°

(5) A tangent to a circle of diameter length 8 cm. is at a distance of .... cm. from its centre.

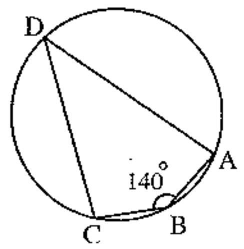
(a) 4 (b) 3 (c) 8 (d) 6

(6) **In the opposite figure :**

m (∠ B) = 140°

, then m (∠ D) = ...

(a) 40° (b) 60° (c) 30° (d) 50°



---

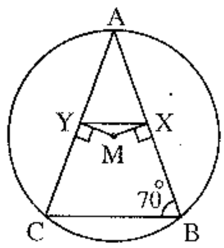
2 [a] **In the opposite figure :**

A circle M , $\overline{MX} \perp \overline{AB}$

, $\overline{MY} \perp \overline{AC}$, m (∠ B) = 70°

(1) **Prove that :** $\overline{XY}$ // $\overline{BC}$

(2) **Find with proof :** m (∠ YXM)



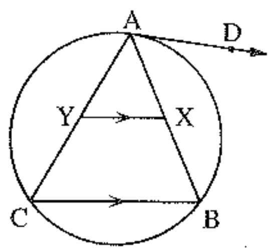
[b] **In the opposite figure :**

$\overline{XY}$ // $\overline{CB}$,

$\overrightarrow{AD}$ is a tangent to the circle at A

Prove that :

$\overrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y



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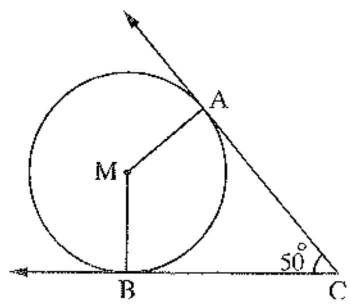
3 [a] **In the opposite figure :**

$\overrightarrow{CA}$, $\overrightarrow{CB}$ are two tangents to the circle M

, m (∠ C) = 50°

Find with proof : m (∠ AMB)



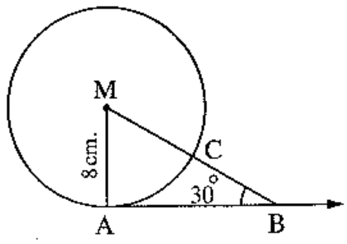
[b] In the opposite figure :

$\overrightarrow{AB}$ is a tangent to the circle M at A and MA = 8 cm.

, m (∠ ABM) = 30°

Find : (1) The length of $\overline{MB}$

(2) m ($\overset{\frown}{CA}$)



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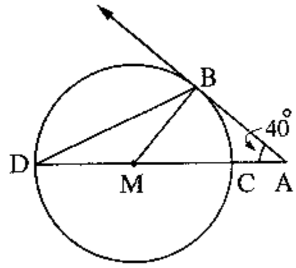
4 **[a] In the opposite figure :**

$\overrightarrow{AB}$ is a tangent to the circle at B , m (∠ A) = 40°

, $\overrightarrow{AM}$ intersects the circle M at C and D

Find with proof : m (∠ BDC)



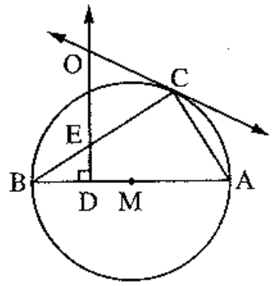
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overleftrightarrow{CO}$ is a tangent to the circle at C and $\overrightarrow{DO} \perp \overline{AB}$

Prove that : (1) ADEC is a cyclic quadrilateral.

(2) OE = OC



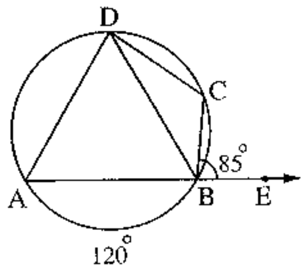
---

5 **[a] In the opposite figure :**

E ∈ $\overrightarrow{AB}$, E ∉ $\overline{AB}$

, m ($\overset{\frown}{AB}$) = 120° , m (∠ CBE) = 85°

Find : m (∠ BDC)

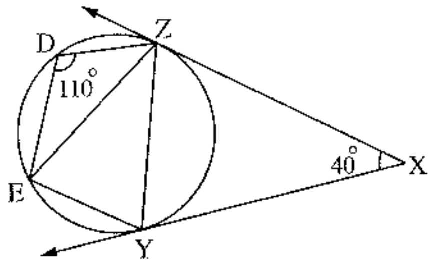


[b] In the opposite figure :

$\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle from the point X , m (∠ X) = 40°

, m (∠ D) = 110°

Prove that : m ($\overset{\frown}{ZE}$) = m ($\overset{\frown}{ZY}$)



18 Souhag Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer :**

(1) The two tangents which are drawn from the two endpoints of a diameter of a circle are ....

(a) parallel. (b) equal in length. (c) congruent. (d) intersecting.

(2) The number of the axes of symmetry in the equilateral triangle = ...

(a) 1 (b) 2 (c) 3 (d) an infinite number.

(3) M and N are two intersecting circles, their radii lengths are 5 cm. , 2 cm. , then MN ∈ ....

(a) [3 , 7] (b) [3 , 7[(c)]3 , 7] (d)]3 , 7[

(4) The number of common tangents of two distant circles is ....

(a) 1 (b) 2 (c) 3 (d) 4

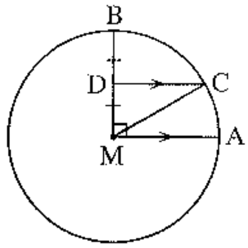
(5) The length of side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

(6) **In the opposite figure :**

$\overline{AM} // \overline{CD}$, MD = DB

, m (∠ AMB) = 90° , then m $(\overset{\frown}{AC})$ = ...



(a) 45° (b) 60°

(c) 30° (d) 90°

---

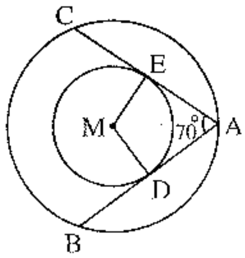
2 [a] Find the measure of the arc which represents $\frac{1}{2}$ its circle , then calculate the length of this arc if the length of the radius is 7 cm. $\left(\pi = \frac{22}{7}\right)$

[b] **In the opposite figure :**

Two concentric circle at M , $\overline{AB}$ and $\overline{AC}$ are two tangents to the smaller circle at D and E , m (∠ A) = 70°

(1) **Find :** m (∠ DME)

(2) **Prove that :** AB = AC



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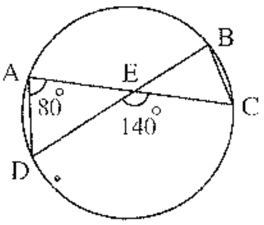
3 [a] **In the opposite figure :**

m (∠ CED) = 140°

, m (∠ A) = 80°

Find : m (∠ C)

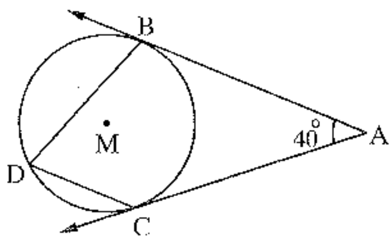


[b] **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 40°

Find with proof : m (∠ D)



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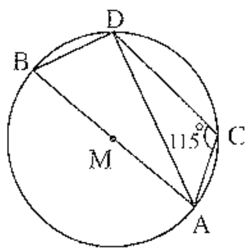
4 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M ,

m (∠ ACD) = 115°

Find with proof : m (∠ DAB)



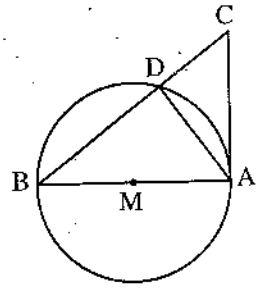
[b] In the opposite figure :

$\overline{AB}$ is a diameter of the circle M

, $\overline{AC}$ is a tangent touches it at A

, if AC = 9 cm. and BM = 6 cm.

Find : The lengths of $\overline{BC}$ and $\overline{AD}$



5 [a] State three cases of cyclic quadrilateral.

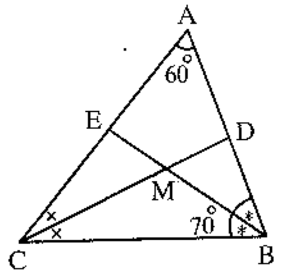
[b] In the opposite figure :

m (∠ A) = 60° , $\overrightarrow{BE}$ bisects ∠ ABC

, m (∠ B) = 70° , $\overrightarrow{CD}$ bisects ∠ ACB

(1) **Find :** m (∠ BMC)

(2) **Prove that :** ADME is a cyclic quadrilateral.



19 Qena Governorate

Answer the following questions : (Calculators are Permitted)

1 Choose the correct answer :

(1) If the area of the circle M = 16 π cm^2 , A is a point on its plane where MA = 8 cm. , then A is ....

(a) outside the circle. (b) inside the circle.

(c) on the circle. (d) on the centre of the circle.

(2) A tangent to a circle of diameter length 6 cm. is at distance of .... cm. from its centre.

(a) 6 (b) 12 (c) 3 (d) 2

(3) The centre of the circumcircle of the triangle is the intersection point of its ....

(a) altitudes of triangle. (b) medians of a triangle.

(c) perpendicular bisectors of the sides of a triangle. (d) bisectors of its angles.

(4) The inscribed angle drawn in a semicircle is .... angle.

(a) acute. (b) obtuse. (c) right. (d) straight.

(5) The two tangent-segments drawn from a point outside a circle are ....

(a) equal in length. (b) not equal in length.

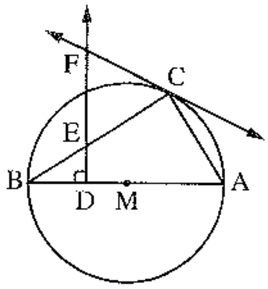
(c) perpendicular. (d) parallel.

(6) The figure is said to be cyclic quadrilateral if the measure of any exterior angle at any vertex equal to .... of the interior angle at the opposite vertex.

(a) the measure. (b) half the measure.

(c) twice the measure. (d) third the measure.

2 **[a] In the opposite figure :**



$\overline{AB}$ is a dimeter in the circle M

, $\overleftrightarrow{CF}$ is a tangent to the circle at C , $\overrightarrow{DE} \perp \overline{AB}$

Prove that :

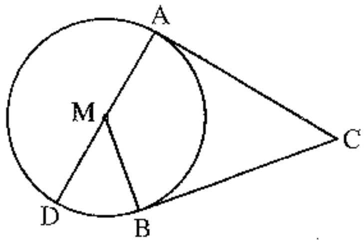
(1) ADEC is a cyclic quadrilateral.

(2) FE = FC

[b] The length of $\overline{AB}$ is 4 cm. , draw a circle of radius length 3 cm. and passes through the two points A , B how many circles can be drawn ? Find the radius length of the smallest circle that can be drawn to pass through the two points A , B

---

3 **[a] In the opposite figure :**

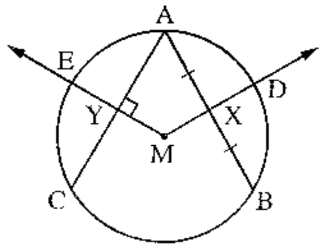


$\overline{AD}$ is a diameter in the circle M

, $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are two tangents to the circle M at A and B respectively

Prove that : m (∠ DMB) = m (∠ ACB)

[b] In the opposite figure :



$\overline{AB}$ and $\overline{AC}$ are two equal chords in length in circle M

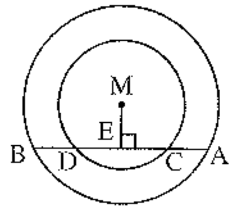
and X is the midpoint of $\overline{AB}$, $\overrightarrow{MX}$ intersects the circle at D

, $\overrightarrow{MY} \perp \overline{AC}$ intersects it at Y and intersects the circle at E

Prove that : XD = YE

---

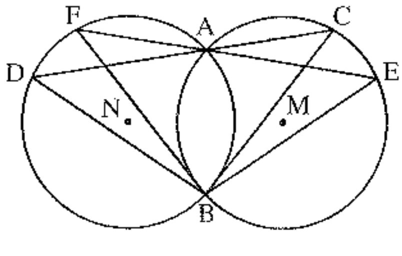
4 **[a] In the opposite figure :**



Two concentric circles M

, $\overline{AB}$ is a chord in the larger circle intersecting the smaller circle at C and D , $\overline{ME} \perp \overline{AB}$ **Prove that :** AC = BD

[b] In the opposite figure :



M and N are two intersecting circles at A and B

, $\overleftrightarrow{AC}$ intersects the circle M at C and intersects the circle N at D ,

$\overleftrightarrow{AE}$ intersects the circle M at E and intersects the circle N to F

Prove that : m (∠ EBC) = m (∠ FBD)

---

5 ABC is an acute-angled triangle drawn inside a circle , draw $\overrightarrow{AD} \perp \overline{BC}$ to cut $\overline{BC}$ at D and cuts the circle at E , then draw $\overrightarrow{CN} \perp \overline{AB}$ to cut $\overline{AB}$ at N

Porve that : (1) ANDC is a cyclic quadrilateral. (2) m (∠ BND) = m (∠ BED)

20 Luxor Governorate

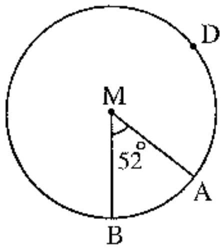
Answer the following questions :

1 Choose the correct answer :

(1) The sum of measures of the accumulative angles at a point = ...°

(a) 80 (b) 120 (c) 360 (d) 630

(2) **In the opposite figure :**



If m (∠ AMB) = 52°

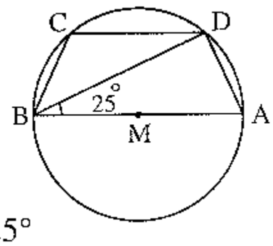
, then m $(\widehat{ADB})$ = ...°

(a) 52 (b) 104 (c) 128 (d) 308

(3) The length of side opposite to the angle of measure 30° in the right-angled triangle equals .... the hypotenuse length.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) 2

(4) **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

, m (∠ ABD) = 25°

, then m (∠ C) = ...

(a) 50° (b) 100° (c) 115° (d) 125°

(5) The sum of lengths of any two sides of a triangle .... the length of the third side.

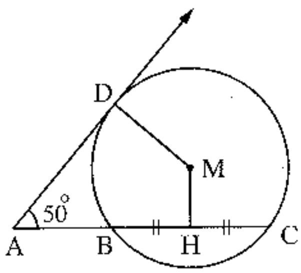
(a) < (b) > (c) = (d) ≤

(6) The number of circles pass by three non-collinear points = ...

(a) infinite number. (b) 3 (c) 1 (d) 0

---

2 [a] In the opposite figure :



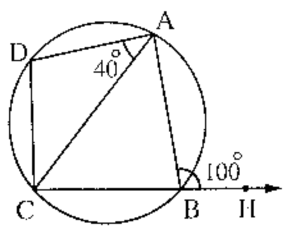
$\overrightarrow{AD}$ is a tangent to the circle at D ,

H is the midpoint of $\overline{BC}$

, m (∠ A) = 50°

Find with proof : m (∠ DMH)

[b] In the opposite figure :



m (∠ ABH) = 100°

, m (∠ DAC) = 40°

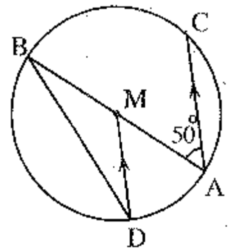
Prove that : m $(\widehat{CD})$ = m $(\widehat{AD})$

3 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overline{AC}$ // $\overline{MD}$, m (∠ CAB) = 50°

Find : m (∠ MDB)



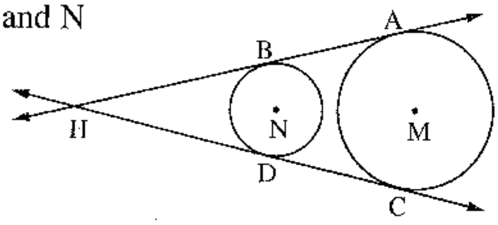
[b] In the opposite figure :

$\overleftrightarrow{AH}$ and $\overleftrightarrow{CH}$ are two tangents to the two circles M and N

touch the circle M at A and C

touch the circle N at B and D

Prove that : AB = CD



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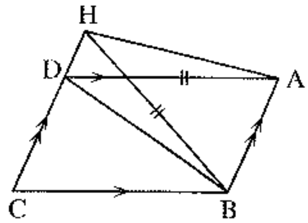
4 **[a] In the opposite figure :**

ABCD is a parallelogram H ∈ $\overrightarrow{CD}$

where BH = AD

prove that : ABDH is a cyclic quadrilateral.



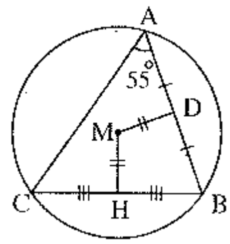
[b] In the opposite figure :

D is the midpoint of $\overline{AB}$

, H is the midpoint of $\overline{BC}$,

m (∠ A) = 55° , MD = MH

Find : m (∠ B)



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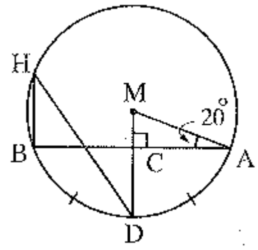
5 **[a] In the opposite figure :**

$\overrightarrow{MC}$ ⊥ $\overline{AB}$ and intersects the circle M at D

which is the midpoint of $\overset{\frown}{AB}$

, m (∠ MAB) = 20°

Find : (1) m ($\overset{\frown}{AD}$)   (2) m (∠ DHB)

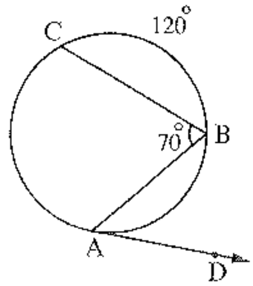


[b] In the opposite figure :

$\overrightarrow{AD}$ is a tangent to the circle at A

, m (∠ B) = 70° , m ($\overset{\frown}{BC}$) = 120°

Find : m (∠ BAD)

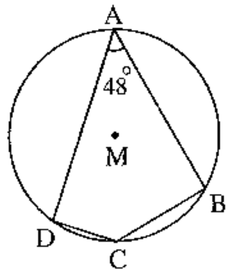


21 Aswan Governorate

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from the given ones :

(1) **In the opposite figure :**

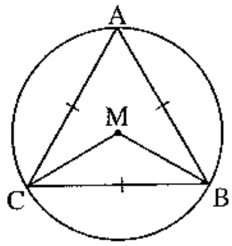


m (∠ A) = 48° , then

the measure of major arc $\widehat{BD}$ = ...

(a) 260° (b) 265° (c) 264° (d) 262°

(2) **In the opposite figure :**

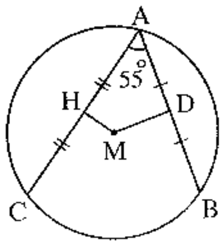


ABC is an equilateral triangle inscribed in circle M

, then m (∠ BMC) = ...

(a) 50° (b) 120° (c) 60° (d) 100°

(3) **In the opposite figure :**



D is the midpoint of $\overline{AB}$, H is the midpoint of $\overline{AC}$

, m (∠ A) = 55°

, then m (∠ DMH) = ...

(a) 120° (b) 130° (c) 135° (d) 125°

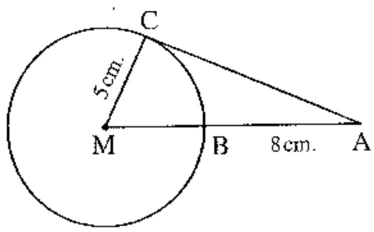
(4) Number of axes of symmetry of the circle = ...

(a) zero (b) one (c) infinite number. (d) 4

(5) The length of side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) 2

(6) **In the opposite figure :**



$\overline{AC}$ is a tangent to circle M at C

if MC = 5 cm. , AB = 8 cm.

, then AC = ... cm.

(a) 5 (b) 10 (c) 13 (d) 12

---

2 [a] M and N are two circles of radii length 9 cm. and 4 cm. respectively.

Show the position of each of them with respect to the other if :

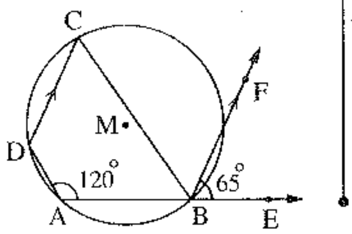
(1) MN = 5 cm. (2) MN = 10 cm.

[b] In the opposite figure :

ABCD is a quadrilateral inscribed in circle M

, $\overrightarrow{BF} \parallel \overline{DC}$, m ($\angle$ EBF) = 65° , m ($\angle$ BAD) = 120°

Find : m ($\angle$ ADC)



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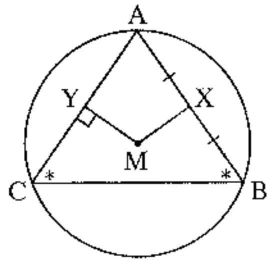
3 **[a] In the opposite figure :**

ABC is a triangle inscribed in circle M ,

m ($\angle$ B) = m ($\angle$ C) , X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

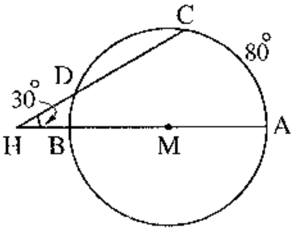


[b] In the opposite figure :

$\overline{AB}$ is a diameter in circle M , $\overrightarrow{AB} \cap \overrightarrow{CD} = \{H\}$,

m ($\angle$ AHC) = 30°, m ($\overset{\frown}{AC}$) = 80°

Find : m ($\overset{\frown}{CD}$)



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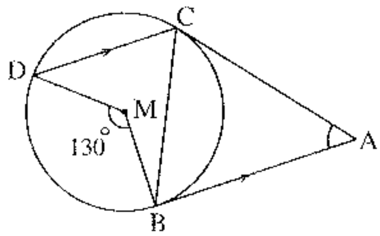
4 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

at B and C , $\overline{AB} \parallel \overline{CD}$, m ($\angle$ BMD) = 130°

(1) **Find :** m ($\angle$ ABC)

(2) **Prove that :** $\overrightarrow{CB}$ bisects $\angle$ ACD

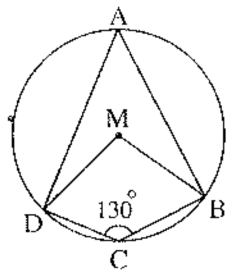


[b] In the opposite figure :

In the circle M ,

if m ($\angle$ BCD) = 130°

Find : m ($\angle$ BMD)



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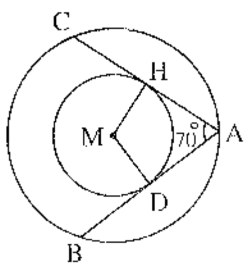
5 **[a] In the opposite figure :**

Two concentric circles at M

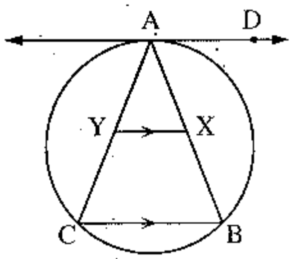
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to smaller circle at D and H

, m ($\angle$ BAC) = 70°

Prove that : (1) AB = AC (2) **Find :** m ($\angle$ DMH)



[b] In the opposite figure :



ABC is a triangle inscribed in a circle ,

$\overleftrightarrow{AD}$ is a tangent to a circle at A

, X $\in \overline{AB}$, Y $\in \overline{AC}$, $\overline{XY} \parallel \overline{BC}$

Prove that :

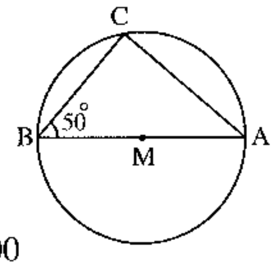
$\overleftrightarrow{AD}$ is a tangent to the circle which passes through the points A , X , Y

22 South Sinai Governorate

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

m (∠ ABC) = 50° , then m $(\overset{\frown}{BC})$ = ... °

(a) 40 (b) 50 (c) 80 (d) 100

(2) The rhombus in which the lengths of diagonals are 6 cm. and 8 cm. its area = ... cm^2

(a) 12 (b) 14 (c) 24 (d) 48

(3) If M is a circle of radius length r cm. , then the length of the simicircle = ... cm.

(a) $2\pi r$ (b) $\frac{1}{4}\pi r$ (c) $\frac{1}{2}\pi r$ (d) πr

(4) The longest chord in the circle is called ....

(a) diameter. (b) tangent. (c) secant. (d) radius.

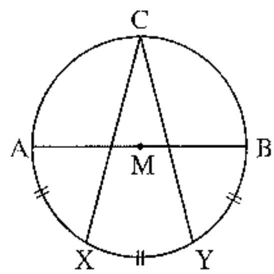
(5) The image of the point (2 , 3) by rotation R (O , 180°) is the point ....

(a) (2 , 3) (b) (– 2 , 3) (c) (2 , – 3) (d) (– 2 , – 3)

(6) The sum of measures of the two opposite angles in the cyclic quadrilateral equal .... °

(a) 180 (b) 120 (c) 100 (d) 30

2 [a] In the opposite figure :



$\overline{AB}$ is a diameter in the circle M

, the length of $(\overset{\frown}{AX})$ = the length of $(\overset{\frown}{XY})$ = the length of $(\overset{\frown}{BY})$

find with proof : m (∠ C)

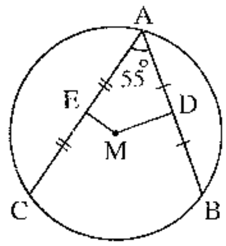
[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two chords in the circle M

, D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$,

m (∠ BAC) = 55°

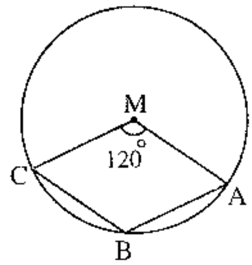
Find with proof : m (∠ DME)



3 **[a] In the opposite figure :**

M is a circle and m (∠ AMC) = 120°

Find with proof : m (∠ ABC)



[b] Two circles M and N with radii lengths of 7 cm. and 4 cm. respectively

Show the position of each of them respect to the other in the following cases :

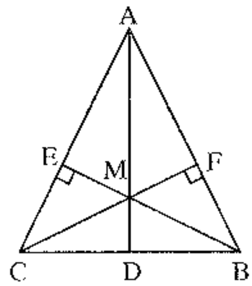
(1) MN = 8 cm. (2) MN = 3 cm. (3) MN = 12 cm.

4 **[a] In the opposite figure :**

Δ ABC , $\overline{BE} \perp \overline{AC}$, $\overline{CF} \perp \overline{AB}$

$\overrightarrow{AM} \cap \overleftrightarrow{BC} = \{D\}$

Prove that : MDCE is a cyclic quadrilateral.

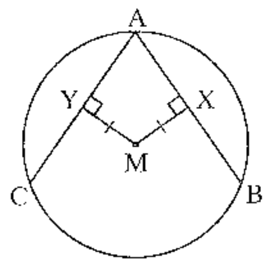


[b] In the opposite figure :

M is a circle , $\overline{AB}$ and $\overline{AC}$ are two chords ,

$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$, AB = 6 cm. , MX = MY

Find with proof : The length of $\overline{AY}$



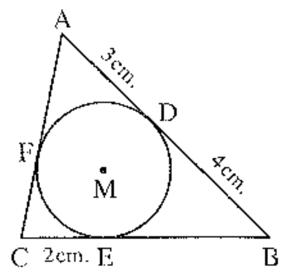
5 **[a] In the opposite figure :**

M is an inscribed circle in the triangle ABC

and touches its sides at D , E and F

, AD = 3 cm. , CE = 2 cm. , BD = 4 cm.

Find with proof : The perimeter of Δ ABC

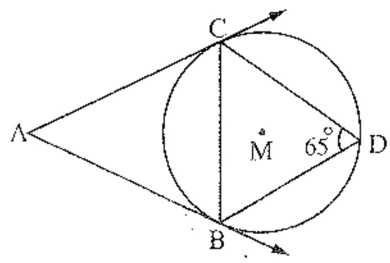


[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents of the circle M

, m (∠ D) = 65°

Find with proof : m (∠ A)



23 Red Sea Governorate

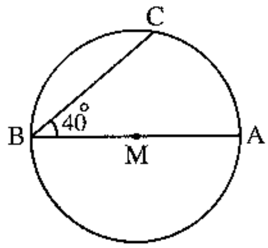
Answer the following questions :

1 Choose the correct answer from the given ones :

(1) Number of the circles that pass through three non-collinear points equals ....

(a) zero (b) one (c) three (d) an infinite number

(2) **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

, m (∠ ABC) = 40° , then m $(\overset{\frown}{BC})$ = ...

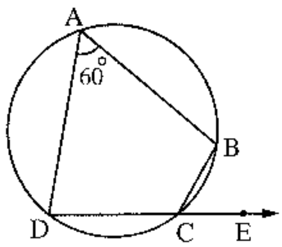
(a) 40° (b) 50°

(c) 90° (d) 100°

(3) If the two circles M and N are touching externally , their radii lengths are 9 cm. , r cm. , and MN = 14 cm. , then r = ... cm.

(a) 5 (b) 7 (c) 10 (d) 23

(4) **In the opposite figure :**

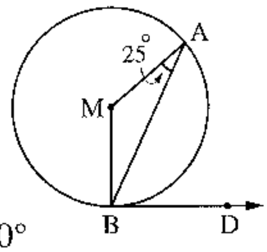


If m (∠ BAD) = 60° , then m (∠ BCE) = ...

(a) 30° (b) 60°

(c) 80° (d) 120°

(5) **In the opposite figure :**



If $\overrightarrow{BD}$ is a tangent to the circle M

, m (∠ BAM) = 25°

, then m (∠ ABD) = ...

(a) 25° (b) 50° (c) 65° (d) 120°

(6) Circumference of a circle is 6 π cm. , L is a straight line at a distance of 3 cm. from its centre , then L is ....

(a) a tangent to the circle. (b) a secant to the circle.

(c) outside the circle. (d) the diameter to the circle.

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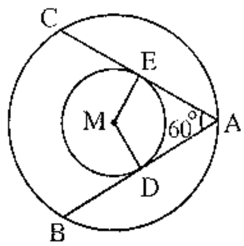
2 [a] In the opposite figure :



Two concentric circles M ,

$\overline{AB}$, $\overline{AC}$ are two tangents to the smaller circle , m (∠ A) = 60°

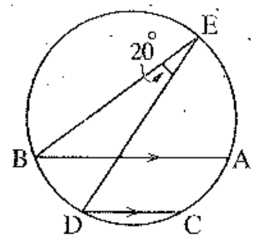
(1) **Find :** m (∠ DME) (2) **Prove that :** AB = AC

[b] In the opposite figure :

$\overline{AB}$, $\overline{CD}$ are two parallel chords

, m (∠ BED) = 20°

Find : m ($\overset{\frown}{AC}$)



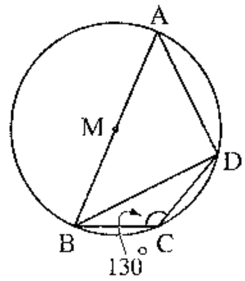
3 **[a] In the opposite figure :**

ABCD is a quadriteral inscribed in a circle M

where M ∈ $\overline{AB}$

, m (∠ BCD) = 130°

Find : m (∠ A) , m (∠ ABD)



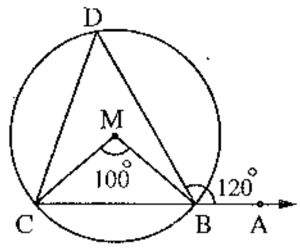
[b] In the opposite figure :

In the circle M :

m (∠ BMC) = 100°

, m (∠ ABD) = 120°

Find with proof : m (∠ DCB)



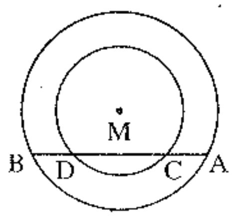
4 **[a] In the opposite figure :**

Two concentric circle M

, $\overline{AB}$ is a chord in the large circle intersecting

the small circle at C and D

Prove that : AC = BD

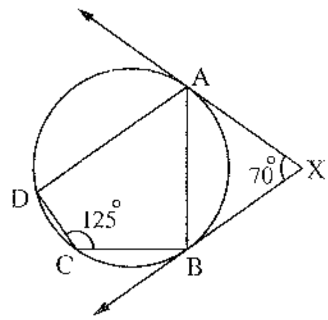


[b] In the opposite figure :

$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to a circle at A and B

, m (∠ AXB) = 70° , m (∠ DCB) = 125°

Prove that : $\overrightarrow{AB}$ bisects ∠ DAX

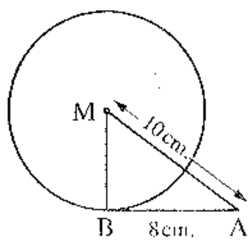


5 **[a] In the opposite figure :**

$\overline{AB}$ is a tangent to a circle M at B

, AB = 8 cm. , AM = 10 cm.

Find : The area of Δ ABM



[b] ABC is a triangle inscribed in a circle , $\overleftrightarrow{BD}$ is a tangent to the circle at B

, X ∈ $\overline{AB}$, Y ∈ $\overline{BC}$ where $\overline{XY}$ // $\overline{BD}$

Prove that : AXYC is a cyclic quadrilateral.

24 Matrouh Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer :

(1) The perimeter of the square whose area is 81 cm^2 is ....

(a) 24 cm. (b) 8 cm. (c) 9 cm. (d) 36 cm.

(2) The two opposite angles in the cyclic quadrilateral are ....

(a) equal. (b) complementary. (c) supplementary. (d) alternate.

(3) ABC is a triangle where $(AB)^2 = (AC)^2 + (BC)^2$, m ($\angle$ B) = 40° , then m ($\angle$ A) = ...

(a) 40° (b) 50° (c) 90° (d) 130°

(4) The measure of the arc which represents $\frac{1}{3}$ the measure of the circle equals ....

(a) 60° (b) 90° (c) 120° (d) 240°

(5) The area of the triangle whose base length is 10 cm. and its height is 6 cm. equals .... cm^2

(a) 6 (b) 10 (c) 30 (d) 60

(6) If the two circles M , N are touching internally , the radius length of one of them is 3 cm. , and MN = 8 cm. , then the radius length of the other circle equals ....

(a) 5 cm. (b) 6 cm. (c) 11 cm. (d) 12 cm.

---

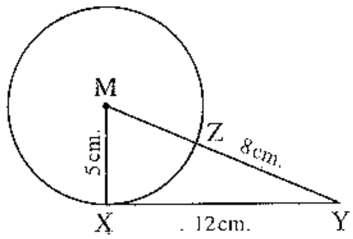
2 [a] In the opposite figure :

M is a circle whose radius length is 5 cm.

, XY = 12 cm , $\overline{MY} \cap$ the circle M = {Z}

and ZY = 8 cm.

Prove that : $\overleftrightarrow{XY}$ is a tangent to the circle M at X



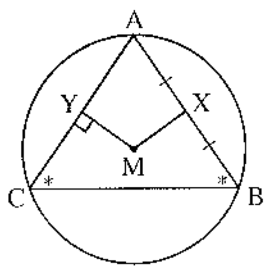
[b] In the opposite figure :

Δ ABC is inscribed in the circle M

, in which m ($\angle$ B) = m ($\angle$ C)

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY



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3 [a] Prove that : The measure of the angle of tangency is equal to the measure of the inscribed angle subtended by the same arc.

[b] ABCD is a quadrilateral drawn in a circle , $F \in \overline{AB}$

, draw $\overrightarrow{FE} \parallel \overline{CB}$ to cut $\overline{CD}$ at E , $\overrightarrow{DF} \cap \overrightarrow{CB} = \{X\}$

Prove that : (1) AFED is a cyclic quadrilateral. (2) m (∠ BXF) = m (∠ EAD)

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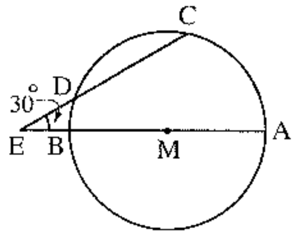
4 **[a] In the opposite figure :**



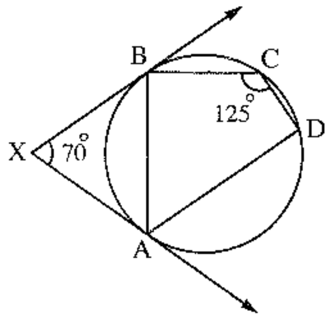
$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$

, m (∠ AEC) = 30° , m $(\overset{\frown}{AC})$ = 80°

Find : m $(\overset{\frown}{CD})$

[b] In the opposite figure :



$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

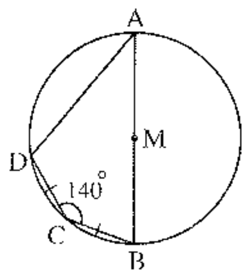
, m (∠ AXB) = 70° , m (∠ DCB) = 125°

Prove that : $\overrightarrow{AB}$ bisects ∠ DAX

---

5 **[a]** Mention three cases of the cyclic quadrilateral.

[b] In the opposite figure :



ABCD is a quadrilateral inscribed in the circle M

where $M \in \overline{AB}$, CB = CD

, m (∠ BCD) = 140°

Find : (1) m (∠ A) (2) m (∠ D)